Math 22

Review for Chapters 1; 2; 3.1-3.5

Solve, using the method indicated or the most appropriate method. Check when necessary.

1. \((3x+1)(x-5) = x(1+3x)\)

2. \(5x^2 - 4x - 3 = 0\) (using quadratic formula)

3. \(2(3x+1)^2 = 12\) (using Square Root Theorem)

4. \(\frac{2x}{x-3} + \frac{4}{x+3} = \frac{-24}{x^2 - 9}\)

5. \(\sqrt{5x+6} + \sqrt{3x+4} = 2\)

Solve, expressing answer on a number line, in interval notation, and as an inequality.

7. \(x \geq -4\) or \(x \geq 2\)

8. \(x \geq -4\) and \(x \geq 2\)

9. \(-4 \leq 10 - 2x < 7\)

10. The monthly payment \(R\) on a loan varies directly with the amount borrowed \(P\). The monthly payment on a 5-year car loan is $20 for every $1000 borrowed.
   a) Find a linear equation that relates the monthly payment \(R\) to the amount borrowed \(P\) for a loan with similar terms.
   b) Find the monthly payment when the amount borrowed is $20,000.

11. Use the adjacent graph of a function to answer the following:
    a) What is the domain of \(g\)?
    b) What is the range of \(g\)?
    c) Determine \(g(-2)\) and \(g(0)\).
    d) Give the location of any \(x\)-intercept and \(y\)-intercepts.
    e) On what interval (if any) is \(g\) increasing?
    f) On what interval (if any) is \(g\) decreasing?
    g) On what interval (if any) is \(g\) constant?
    h) For what values of \(x\) is \(g(x) \leq 0\)?
    i) For what values of \(x\) is \(g(x) = -1\)?

12. \(x^2 - y = 25\)

13. \(2x^2 + y^4 + 4x = 16\)

14. \(7x^2 + 4y^2 = 28\)

15. \(y = \frac{1-x^2}{x}\)

16. \(y = \frac{4}{x+3}\)

17. A manufacturer of computer components has fixed costs of $12,000 and unit costs of $15 per unit.
   a) Assuming a linear relation, give the total cost function \(C\) that relates the cost to the number of computer components \(x\).
   b) Find the cost of manufacturing 10,000 computer components.
   c) If the manufacturer has a budget of $132,000, how many computer components can he provide?

18. Find the domain of the following functions.
   a) \(f(x) = x^2 - 4x - 21\)
   b) \(f(x) = \frac{x}{x^2 - 4x - 21}\)
   c) \(f(x) = \frac{\sqrt{x+4}}{x}\)
19. If \( f(x) = x^2 + 5x \), find: a) \( f(-2) \) b) \( f(3x) \) c) \( \frac{f(x+h) - f(x)}{h} \).

20. For the function \( f(x) = x^2 - 1 \), compute the average rate of change from
   a) 2 to 2.1 b) 2 to 2.01 c) 2 to 2.001 d) 2 to \( x \)

21. The graph of \( y = f(x) \) is shown. Sketch (on separate axes) and give the transformation (shift, reflection, stretch, compression, and to what degree) involved for each of the following. Give at least two coordinates for each graph.
   a) \( y = f(x+3) \) e) \( y = f(\frac{1}{2}x) \)
   b) \( y = f(x)+3 \) f) \( y = f(-x) \)
   c) \( y = f(x-3) \) g) \( y = |f(x)| \)
   d) \( y = 2f(x) \) h) \( y = -f(x+2) - 1 \)

22. Given that \( f(x) = \lfloor x \rfloor \) Give the function that shows
   a) \( f \) is reflected about the \( x \)-axis. d) \( f \) is vertically compressed by a factor of 3.
   b) \( f \) is shifted down 4 units. e) \( f \) is horizontally shifted 4 units to the right.
   c) \( f \) is horizontally compressed by a factor of 3. f) \( f \) is horizontally shifted 1 unit to the left and up 5

23. Graph: \( g(x) = \begin{cases} -|x|, & \text{if } x \leq 1 \\ x^2 + 2, & \text{if } x > 1 \end{cases} \) [Indicate at least two points on each “piece”]

24. Given that \( h(x) = \frac{x^4 - 13x^2 + 36}{x^2 - 1} \)
   a) Give its domain (in interval notation).
   b) Determine whether \( h \) is even, odd, or neither.
   c) Give symmetry, if any exist.
   d) Give the intercepts of \( h \).

### (Answers)

1. \( x = -\frac{1}{3} \) 2. \( x = \frac{2}{5} \pm \frac{\sqrt{69}}{5} \) 3. \( x = -\frac{1}{3} \pm \frac{\sqrt{6}}{3} \) 4. \( x = -2 \) 5. \( x = -1 \) 6. \( R = \frac{4-P}{Pt} \)

<table>
<thead>
<tr>
<th>number line</th>
<th>interval notation</th>
<th>Inequality</th>
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<tbody>
<tr>
<td>( \bullet ) ( \longrightarrow ) 4 2</td>
<td>([-4, +\infty))</td>
<td>({x : x \geq -4})</td>
</tr>
<tr>
<td>( \bullet ) ( \longrightarrow ) 4 2</td>
<td>([2, +\infty))</td>
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</tr>
<tr>
<td>( \bullet ) ( \longrightarrow ) (-\frac{1}{2} ) 7</td>
<td>((\frac{1}{2}, 7])</td>
<td>({x : \frac{1}{2} &lt; x \leq 7})</td>
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10. a) \( R = 0.02P \)  
   b) $400

11. a) \([-4, +\infty)\]  
   b) \((-\infty, 3]\)  
   c) \(g(-2) = 1; \ g(0) = -2\)  
   d) x-intercepts at \((-\frac{1}{2}, 0), (\frac{3}{2}, 0), \) and \((4, 0)\); y-intercept at \((0, -2)\)  
   e) increasing on \((0, 3)\)  
   f) decreasing on \([-4, 0) \cup (2, +\infty)\)  
   g) no interval \(h) \ (\frac{-\sqrt{3}}{2}, \frac{\sqrt{3}}{2}) \cup (4, +\infty)\)  
   i) \(x = -1; \ x = 1\)

12. x-intercepts at \((\pm 5, 0)\); y-intercept at \((0, -25)\); symmetry with respect to y-axis

13. x-intercepts at \((-4, 0)\) and \((2, 0)\); y-intercepts at \((0, \pm 2)\); symmetry with respect to x-axis

14. x-intercepts at \((\pm 2, 0)\); y-intercepts at \((0, \pm \sqrt{7})\); symmetry with respect to x-axis, y-axis, and origin

15. x-intercepts at \((\pm 1, 0)\); no y-intercepts; symmetry with respect to origin

16. no x-intercepts; y-intercept at \((0, \frac{3}{2})\); no symmetry

17. a) \(C(x) = 15x + 12,000\)  
   b) $162,000  
   c) 8,000

18. a) \((-\infty, +\infty)\)  
   b) \((-\infty, -3) \cup (-3, 7) \cup (7, +\infty)\)  
   c) \([-4, 0) \cup (0, +\infty)\)

19. a) -6  
   b) \(9x^2 + 15x\)  
   c) \(2x + h + 5\)

20. a) 4.1  
   b) 4.01  
   c) 4.001  
   d) \(x + 2\)

21. d) \((-2, -2)\) \((2, 2)\) \((2, 1)\) \((-2, 1)\) \((-4, -1)\)

22. a) \(y = -[x]\)  
   b) \(y = [x] - 4\)  
   c) \(y = 3x\)  
   d) \(y = \frac{1}{3}[x]\)  
   e) \(y = [x - 4]\)  
   f) \(y = [x + 1] + 5\)

23.

24. a) \((-\infty, -1) \cup (-1, 1) \cup (1, +\infty)\)  
   b) even  
   c) y-axis  
   d) \((0, -36), (-2, 0), (2, 0), (-3, 0), (3)\)