Math 2
Review for Chapter 2.6 and 3

1. Given that \( f(x) = -2x^2 + 16x - 26 \)
   a) Rewrite the above function in \( f(x) = a(x - h)^2 + k \) form.
   b) Using a series of transformations (shifting, compressing, stretching, and/or reflection), sketch its graph.

2. Among all rectangles that have a perimeter of 60 feet, determine the dimensions of the one with the largest area.

3. You want to put up a small fence around a rectangular part of your yard and then divide it into three rectangular plots by placing two fences parallel to one of the sides. If you can only afford 100 yards of fencing, determine the dimensions that will give maximum area.

4. Solve: \( \frac{(x-1)^2}{(x+1)(x+2)} \leq 0 \)

5. Given a polynomial function defined by \( f(x) = x^3(x+2)(x+5)^2(x-3) \),
   a) Determine its \( x \) - and \( y \) -intercepts.
   b) Determine whether its graph crosses or is tangent to the \( x \) -axis at each of the \( x \) -intercepts.
   c) Discuss left and right end behavior.
   d) Give the maximum number of turning points of the graph.
   e) Using the \( x \) -intercepts as “critical values,” determine the intervals on which the graph is above the \( x \) -axis and the intervals on which the graph is below the \( x \) -axis.
   f) Using the above information, sketch a possible graph.

For the following functions, determine, if any, the vertical, horizontal, and oblique asymptotes.

6. \( f(x) = \frac{3x^2 + 2x + 4}{2x^2 - 3x + 1} \)
7. \( g(x) = \frac{x-4}{x^2+3} \)
8. \( h(x) = \frac{x^3+8}{x^2-4x-21} \)

Using the guidelines discussed in class, graph the following functions. Include domain, intercepts, any holes, asymptotes (and any crossovers) and a sign analysis.

9. \( g(x) = \frac{x^2}{x-2} \)
10. \( h(x) = \frac{(3x^2+12x)(x-6)}{(x^2-x-6)(x-6)} \)
11. \( f(x) = \frac{x-1}{x^2+3x-4} \)

12. Given that \( f(x) = 4x^6 - 9x^3 + 5x^2 - 7x - 1 \).
   a) Determine, without doing the actual division, the remainder when \( f(x) \) is divided by \( x + 3 \).
   b) Determine whether or not \( x - 2 \) is a factor of \( f(x) \).

13. Given that \( f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4 \).
   a) List all the possible rational zeros of \( f(x) \).
   b) Use Descartes’ Rules of Signs to determine the possibilities for the number of positive, negative, and nonreal (complex) zeros.
   c) Determine least integral upper bound and greatest integral lower bound of real solutions.
   d) Using the above information and with the help of synthetic division, give all zeros of \( f(x) \).
   e) Give the function in simplest factored form.
   f) Using the above information as well as knowledge derived from the end behavior of \( f(x) \), Intermediate Value Theorem, and possibly other crucial points, sketch the graph of \( y = f(x) \).

14. Evaluate and write in \( a + bi \) form.
   a) \( i^{27} \)
   b) \( (5 + 3i)(-1 + i) \)
   c) \( \frac{3 + 2i}{4 - i} \)
15. Find a polynomial function $f(x)$ of degree 4 whose coefficients are real numbers, two of its zeros are $3 - i$ and $4i$, and $f(0) = 80$.

(Answers)

1. a) $f(x) = -2(x - 4)^2 + 6$

2. 15 feet x 15 feet (a square)

3. $12 \frac{1}{2}$ yards x 25 yards

4. $(-2, -1) \cup \{1\}$

5. a) $x$-intercepts at $-5, -2, 0, 3$; $y$-intercept at 0
   b) crosses at $x = -2, x = 3$; tangent at $x = 0, x = -5$
   c) as $x \to -\infty$, $f(x) \to +\infty$;
      as $x \to +\infty$, $f(x) \to +\infty$
   d) 5
   e) above $x$-axis on $(-\infty, -5) \cup (-5, -2) \cup (3, +\infty)$;
      below $x$-axis on $(-2, 0) \cup (0, 3)$

6. VA at $x = \frac{1}{2}, x = 1$; HA at $y = \frac{1}{2}$

7. No VA; HA at $y = 0$

8. VA at $x = -3, x = 7$; oblique of $y = x + 4$

9. $g(x) = \frac{x^2}{x - 2}$
   
   domain: $(-\infty, 2) \cup (2, +\infty)$
   
   intercept at $(0,0)$
   
   symmetry: none
   
   asymptotes: VA at $x = 2$
   
   oblique at $y = x + 2$

10. $h(x) = \frac{3x^2 + 12x}{x^2 - x - 6} = \frac{3(x+4)(x-6)}{(x-3)(x+2)(x-6)}$
    
    domain: $(-\infty, -2) \cup (-2, 3) \cup (3, 6) \cup (6, +\infty)$
    
    hole at $(6, 7 \frac{1}{2})$
    
    intercepts: $(0,0), (-4,0)$
    
    symmetry: none
    
    asymptotes: VA at $x = -2, x = 3$
    
    HA at $y = 3$
    
    crossover at $x = -\frac{1}{3}$
11. \( f(x) = \frac{x - 1}{x^2 + 3x - 4} \)

domain: \((-\infty, -4) \cup (-4, 1) \cup (1, +\infty)\)

hole at \((1, \frac{1}{2})\)

symmetry: none

asymptote: VA at \(x = -4\)

HA at \(y = 0\)

12. a) 3224, b) not a factor, since \(f(2) = 189 \neq 0\)

13. a) -4, -2, -1, -\(\frac{1}{2}\), 1, 2, 4

b) 3 positive, 1 negative, and no complex roots

OR 1 positive, 1 negative, and 2 complex roots

c) least integral upper bound is 1;

greatest integral lower bound is -1

d) -\(\frac{1}{2}\), 1, \(\pm 2i\)

e) \(f(x) = (2x + 1)(x - 1)(x^2 + 4)\)

14. a) 0 - \(i\) b) -8 + 2\(i\) c) \(\frac{10}{17} + \frac{11}{17}i\)

15. \(f(x) = \frac{1}{2} (x^2 + 16) \left[ (x - 3)^2 + 1 \right] = \frac{1}{2} x^4 - 3x^3 + 13x^2 - 48x + 80\)