Graphing Polynomial and Rational Functions

Steps:
1. Make the side with x's into a single fraction.
2. Now do steps 3 through 9 from the handout "Quadratic and Rational Inequalities.
3. Plot a zero as points on the x-axis. (These points are also called x-intercepts.)
4. Draw vertical asymptotes through bottom zeros (undefined points).
5. Draw horizontal asymptote.
6. Plot the y-intercept.
7. Draw smooth curve where + means draw above the x-axis and - means draw below x-axis.

Example: \( y = f(x) = \frac{2x^2 - 10x}{x^2 - 9} \)

\[ y = \frac{2x(x - 5)}{(x - 3)(x + 3)} \] step 3

top

\[ \begin{align*}
2x &= 0 & x - 5 &= 0 & x - 3 &= 0 & x + 3 &= 0 \\
\quad x &= 0 & \quad x &= 5 & \quad x &= 3 & \quad x &= -3 \\
\infty & & -3 & & 0 & & 3 & & 5 & & \infty
\end{align*} \] step 4

bottom

Intervals
\((-\infty, -3)\) \((-3, 0)\) \((0, 3)\) \((3, 5)\) \((5, \infty)\) step 5

Test pts
\(-4\) \(-1\) \(1\) \(4\) \(6\) step 6

Plug in
\[ \begin{align*}
\frac{2(-4)(-4-5)}{(-4-3)(-4+3)} & & \frac{2(0)(-1-5)}{(-1-3)(0+3)} & & \frac{2(1)(-1-5)}{(1-3)(1+3)} & & \frac{2(4)(4-5)}{(4-3)(4+3)} & & \frac{2(6)(6-5)}{(6-3)(6+3)}
\end{align*} \] step 7

Sign
+ - + - + step 8

See Fig. Page 184/185 (Graph).
Horizontal asymptote: \( y = \frac{2x^2}{x^2} \)

\( y = 2 \)

y-intercept: \((0, 0)\)

Steps 10 & 11

Steps 12 & 13

Step 14
Example: \( f(x) = -2x^4 - 8x^3 + 10x^2 \)

Step 1: not apply because already have a single fraction 
\[ \frac{-2x^4 - 8x^3 + 10x^2}{1} \]

\[ y = \frac{-2x^2(x^2 + 4x - 5)}{1} \]

\[ y = \frac{-2x^2(x + 5)(x - 1)}{1} \]

1. **Top**  
   - \(-2x^2 = 0\)  
   - \(x + 5 = 0\)  
   - \(x - 1 = 0\)  

2. **Bottom**  
   - \(x = 0\)  
   - \(x = -5\)  
   - \(x = 1\)  
   - \(1 \neq 0\)

**Step 5**

\[ -\infty \), -5, 0, 1, \), \infty \]

<table>
<thead>
<tr>
<th>Intervals</th>
<th>((-\infty, -5))</th>
<th>((-5, 0))</th>
<th>((0, 1))</th>
<th>((1, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test pts</td>
<td>-6</td>
<td>-1</td>
<td>(\frac{1}{2})</td>
<td>2</td>
</tr>
<tr>
<td>Sub in</td>
<td>(-) (-) (-)</td>
<td>(-)(+)(-)</td>
<td>(+)(-)(-)</td>
<td>(-)(+)(+)</td>
</tr>
<tr>
<td>Sign</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

\[ y \]

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