Circle the best answer.

1. Find the slope of the tangent line to the graph of \( g(x) \) at \( x = 1 \).
   \[ g(x) = \frac{x^3 - 3x^2 + 4x - 1}{x + 1} \]
   (a) \( \frac{1}{4} \)  (b) \(-\frac{1}{4}\)  (c) \( \frac{3}{4}\)  (d) \(-\frac{3}{4}\)  (e) \(-\frac{5}{4}\)

2. Consider the graph of \( f \). Determine the relationship of \( \Delta y \) and \( dy \) given a change in the domain from \( x \) to \( x + \Delta x \).

3. Given \( h \) at right, find \( h'(c) \).
   \[ h(x) = \sin^2 \sqrt{1 - 3x} \]
   (a) \(-6 \sin 1 \cos 1\)  (b) \(-6 \sin 1\)  (c) \(-3 \sin 1 \cos 1\)  (d) \(-3 \sin 1\)

4. Explain why \( f \) is not differentiable at \( x = 0 \).
   \[ f(x) = \begin{cases} \sqrt[3]{x}, & 0 \leq x \\ x^2, & x < 0 \end{cases} \]
   (a) \( f'_-(0) = -\infty \)  (b) \( f'_+(0) = 0 \neq f'_-(0) \)  (c) \( f'_+(0) = -\infty \)
   (d) \( \lim_{x \to 0^-} f'(x) = 0 \neq f'_-(0) \)  (e) \( \lim_{x \to 0^+} f'(x) = \infty \)

Show all work on remaining problems.

5. (a) Use the limit definition of the derivative to find \( f'(x) \).
   \[ f(x) = \frac{1}{x+2} \]
   (b) Find the equation of the tangent line to \( f \) at \( x = 0 \).
   (c) Graph \( f \) and the tangent line.
6. Use the Intermediate Value Theorem to show the equation has a solution on the interval \((0, 1)\). Hint: define a function.

\[ x^5 - x^2 + 3x - 1 = 0 \]

7. Find the linear approximation for \( f(b) \) if the independent variable changes from \( a \) to \( b \).

\[ f(x) = (x + 1) \cos x \]
\[ a = 0^\circ, \quad b = 2^\circ \]

8. Consider the equation.

Find \( \frac{dy}{dx} \).

\[ x^3 - \sqrt{xy} + y^2 = 1 \]

9. Consider the function \( f \) given:

\[ f(x) = \begin{cases} 
1, & x \leq 0 \\
\sqrt{x}, & 0 < x 
\end{cases} \]

(a) Use the limit definition of one-sided derivatives to find \( f'_+(0) \) and \( f'_-(0) \).

(b) Does \( f'_x(0) \) exist? Explain.

(c) Evaluate \( \lim_{x \to 0^+} f'_x(x) \).

(d) **Extra Credit**: graphically explain the answers to parts (a) and (c).
\[ g(x) = \frac{x^3 - 3x^2 + 4x - 1}{x+1} \]
\[ g'(x) = \frac{(x+1)(3x^2 - 6x + 4) - (x^2-3x^2+4x-1)x}{(x+1)^2} \]
\[ g'(x) = \frac{(2x+1) - x(2x+1)}{(x+1)^2} = \frac{x}{4}, \text{ Ans (a)} \]

\[ h(x) = \sin^2(1-3x)^{1/2} \]
\[ h'(x) = 2 \sin(y-3x) \cdot \cos(y-3x) \cdot \frac{-3}{2x-3x} \]
\[ h'(0) = 2 \sin 1 \cdot \cos 1 \cdot \frac{-3}{2} = -3 \sin 1 \cdot \cos 1, \text{ Ans (c)} \]

\[ f(x) = \begin{cases} x^{1/3}, & 0 \leq x \\ x^2, & x < 0 \end{cases} \]
\[ f'(x) = \begin{cases} \frac{1}{3x^{2/3}}, & 0 < x \\ 2x, & x < 0 \end{cases} \]
\[ \lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} \frac{1}{3x^{2/3}} = +\infty, \text{ Ans (e)} \]

\[ f(x) = x^5 - x^2 + 3x - 1 \]
To show \( f(c) = 0 \) for some \( c \) in (0,1).

\[ f(0) = -1, \quad f(1) = 2 \Rightarrow f(0) < f(1). \]
\[ f \text{ being a poly is cont on } (\infty, \infty) \]
and so certainly is cont on the subset [0,1]. We have established

\[ \text{the hypothesis of IVT. By the conclusion of IVT, there exists } c \text{ in (0,1) where } f(c) = 0 = 0. \]

(Note that \( f(0) \neq 0 \Rightarrow c \neq 0 \)
and \( f(1) \neq 0 \Rightarrow c \neq 1. \)
\[ \Rightarrow c \text{ is not an endpoint.} \)
\[ f'(0) = \frac{f(h) - f(0)}{h} \quad \text{as } h \to 0 \]

\[ f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{f(0) + 0.5h^2 - f(0)}{h} = \lim_{h \to 0} \frac{0.5h^2}{h} = \lim_{h \to 0} 0.5h = 0 \]

\[ f(x) = \begin{cases} 1, & x = 0 \\ x^2, & x \neq 0 \end{cases} \]

\[ f'(x) = \begin{cases} 2x, & x \neq 0 \\ \text{undefined}, & x = 0 \end{cases} \]

\[ f(x) = 1 + \tan(x) \]

\[ f(x) = \frac{1}{x} \cos x \]

\[ f(x) = \frac{1}{x} \sin x \]

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