Choose the best answer.

1. Find the derivative of \( f(x) = (x-1)(3-5x) \).
   (a) \( 8-10x \)  
   (b) \( -2-10x \)  
   (c) \( \frac{(x-1)(3-5x)}{5} \)  
   (d) \( \frac{(x^2-x)(3x-5x^2)}{5} \)  
   (e) \( -2 \)

2. Let \( f(x) = \sqrt{x} \), \( g(x) = \sqrt{x} \), \( k(x) = x^2 \), \( l(x) = x^3 \). Choose the function having the largest average rate of change as \( x \) goes from 1 to 2.
   (a) \( f \)  
   (b) \( g \)  
   (c) \( h \)  
   (d) \( k \)  
   (e) cannot be determined.

3. Given the same functions of problem 2, which function gives the largest instantaneous rate of change at \( x = 1 \).
   (a) \( f \)  
   (b) \( g \)  
   (c) \( h \)  
   (d) \( k \)  
   (e) cannot be determined.

4. Select all statements below that are (always) true for any function \( f(x) \).
   I. If \( \lim_{x \to a} f(x) = f(a) \), then \( f \) is differentiable at \( x = a \).
   II. If \( f \) is differentiable at \( x = a \), then \( \lim_{x \to a} f(x) = f(a) \).
   III. If \( \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \), then \( \lim_{x \to a} f(x) = f(a) \).
   (a) I  
   (b) II  
   (c) III  
   (d) I, III  
   (e) II, III

5. Let \( f(x) = \frac{(x-1)(x-2)}{x+5} \). Select the answer below that identifies all true statements in the list:
   I. The maximum domain of \( f \) is \( \mathbb{R} \), \( x \neq -5, x \neq 1, x \neq 2 \).
   II. The \( x \)-intercepts are \((1,0)\) and \((2,0)\).
   III. The vertical asymptote is \( x = -5 \).
   IV. The horizontal asymptote is \( y = 1 \).
   (a) I, II, IV  
   (b) I, III, IV  
   (c) II, III, IV  
   (d) I, II, III  
   (e) II, III
6. Let \( f(x) = \begin{cases} -x, & x \leq 0 \\ 1-x, & x > 0 \end{cases} \).

At \( x=0 \), what type of discontinuity does \( f \) have?

(a) removable  (b) jump  (c) infinite  (d) oscillatory  (e) cannot be determined

7. Let \( f(x) = ax + b \), where \( a \) and \( b \) are constants, and where \( a > 0 \).

Select all statements from the list that are true for \( f \).

I. \( f'(x) \leq 0 \) for all real numbers \( x \).
II. \( f \) increases for all real numbers \( x \).
III. \( x = -a \) is a critical number.

(a) I    (b) II    (c) III    (d) I and II    (e) I and III

8. Let \( f(x) = ax^2 + bx + c \), where \( a, b, \) and \( c \) are constants, and where \( a > 0 \) and \( b > 0 \).

Select all statements from the list that are true about \( f \).

I. \( f'(x) \leq 0 \) on \( (-\infty, \infty) \).
II. \( f \) increases on \( (-\infty, \infty) \).
III. \( f''(x) > 0 \) on \( (-\infty, \infty) \).

(a) I    (b) II    (c) III    (d) I and II    (e) I and III

9. Consider the following statement: if \( a^3 > g(a) > 0 \), then \( \frac{1}{a^3} < \frac{1}{g(a)} \).

Assume \( g(x) \) is a function, and \( a \) is a constant.

Choose the reason that would justify the statement.

(a) \( f(x) = \frac{1}{x} \) increases on \( (0, \infty) \)  (b) \( f(x) = \frac{1}{x} \) decreases on \( (0, \infty) \).
(c) \( g(x) \) increases on \( (-\infty, \infty) \)  (d) \( g(x) \) decreases on \( (-\infty, \infty) \)

10. Which condition below ensures that the discontinuity of \( h \) at \( x=1 \) is removable? \( h(x) = \frac{(x-1)^n (x+2)}{(x-1)^m} \).

(a) \( \lim_{x \to 1} h(x) = \infty \)  (b) \( m > n \)  (c) \( m \leq n \)  (d) \( \lim_{x \to -2} h(x) = 0 \).
11. Find the limit. \( \lim_{n \to \infty} \frac{1}{n} \left[ \frac{n(n+1)(2n+1)}{6n^2} - \frac{n(n+1)+4n}{2n} \right] \).
(a) 0    (b) 4    (c) 11/3    (d) 23/6    (e) \( \infty \)

Show all work on remaining problems.

12. Find the derivative. \( f(x) = 3x + \sin(\cos(x^2 - 1)) \).

13. Integrate. \( \int \frac{5}{x^2} \sqrt{\frac{2-x}{x}} \, dx \).

14. (a) Use the fact that \( \lim_{x \to \infty} \frac{1}{x^p} = 0 \) \((p > 0)\) to find the limit.
\( \lim_{x \to \infty} \frac{1 - x}{\sqrt[4]{x^2 + x + 1}} \)

(b) Use \( x = -1.25 \), \(-1000\), and \(-8000\) in a table to guess the limit.
\( \lim_{x \to -\infty} (1-x)x^{1/3} \)

15. Consider the equation \( y^3 - xy^2 + x^2 = -1 \).
(a) Find \( \frac{dy}{dx} \) in terms of \( x \) and \( y \), assuming the equation determines a differentiable function of \( x \).
(b) Find the equation of the line tangent to the above equation at \((1, -1)\).
(c) Use differentials to approximate the change in \( y \) \((dy)\) when \( x \) changes from 1 to 1.1 in the equation of part (a).

16. Let \( f(x) = 3 + x \sqrt{x + 1} \).
(a) Show \( f \) does not satisfy the hypothesis of the Mean Value Theorem on \([-2, 0]\).
(b) Show \( f \) does satisfy the hypothesis of MVT on \([-1, 0]\).
(c) Find all numbers \( c \) in \((-1, 0)\) satisfying the conclusion of MVT.
17. Consider the statement: \[ \int_0^\pi \left[ 1 + 2x - \cos(2x) \right] dx > 0. \]

Name \( f(x) = 1 + 2x - \cos(2x) \).

(a) Prove the statement using the Closed Interval Method.

(b) Prove the statement another way. Use inequalities to establish \( f(x) > 0 \) on \([0, \pi]\), starting with finding numbers \( c \) and \( d \) such that \( c \leq \cos(2x) \leq d \).

(Do not use derivatives in part (b).)

(c) Make a sign chart for \( f' \) and a sign chart for \( f'' \) on \([0, \pi]\) to find where \( f \) increases/decreases and its concavity.

(d) Graph \( f \) on \([0, \pi]\).

18. Let \( f(x) = 4x^{3/2} - 1 \).

(a) Find \( S(x) = \int_0^x \sqrt{1 + [f'(t)]^2} \, dt \). (Also evaluate integral.)

(b) Use part (a) to find the arc length of the graph from A \((0, -1)\) to B \((4, 3)\).

(c) Find \( ds \) and \( ds \) when \( x \) changes from \( x \) to \( x + \Delta x \).

19. Let \( f(x) = \begin{cases} -|x| + 2, & x \neq 0 \\ 1, & x = 0 \end{cases} \).

(a) Find the average rate of change of \( f \) as \( x \) changes from 0 to 6.

(b) Use the answer to part (a) to find \( f'(0) \).

(c) Use "short-cut" formulas for derivatives to find \( \lim_{x \to 0} f'(x) \).

(d) On a graph of \( f \), sketch a limit of secants to confirm the answer to (b).
1. \[ f(x) = (x-1)(3-5x) \]
   \[ f'(x) = 1 \cdot (3-5x) + (x-1) \cdot (-5) \]
   \[ = 3 - 5x - 5x + 5 \]
   \[ = 8 - 10x \]
   Answer: (a) 5

4. Answer: (b) 5

5. Answer: (c) 5

6. \[ f(x) = ax + b \]
   \[ f'(x) = a > 0 \]
   Answer: (b) 5

7. \[ f(x) = ax^2 + bx + c \]
   \[ f'(x) = 2ax + b \]
   \[ f''(x) = 2a > 0 \]
   Answer: (c) 5

8. Graph shows answer in (d). 5
   Also get answer by differentiating.
   \[ f'(x) = \frac{1}{x}, \quad g'(x) = \frac{1}{2}, \quad h'(x) = 2 \]
   \[ k'(x) = 3. \]

9. Answer: (b)

10. b

11. d
8. (a) \[ \lim_{x \to \infty} \frac{1-x}{\sqrt{x^2 + x + 1}} \]
\[ = \lim_{x \to \infty} \frac{(1-x) \cdot \sqrt{x^2}}{\sqrt{x^2 + x + 1} \cdot \frac{1}{\sqrt{x^2}}} \]
\[ = \lim_{x \to \infty} \frac{\sqrt{1-x^2}}{1 + \frac{1}{x} + \frac{1}{x^2}} \]
\[ = \frac{0}{1+0+0} \]
\[ = 0 \]

(b) \[ \frac{\sqrt{1-x^2}}{x} \]
\[ = \frac{1}{\sqrt{3}} \]

(c) \[ \lim_{x \to \infty} \frac{1-x}{\sqrt{x^2 + x + 1}} \]
\[ = \lim_{x \to \infty} \frac{(1-x) \cdot \sqrt{x^2}}{\sqrt{x^2 + x + 1} \cdot \frac{1}{\sqrt{x^2}}} \]
\[ = \lim_{x \to \infty} \frac{\sqrt{1-x^2}}{1 + \frac{1}{x} + \frac{1}{x^2}} \]
\[ = \frac{0}{1+0+0} \]
\[ = 0 \]

9. (a) \[ f(x) = 3 + x \sqrt{x+1} \quad \text{dom} f = [-1, \infty) \]
\[ f'(x) = 0 + 1 \cdot \frac{1}{2 \sqrt{x+1}} + \frac{x}{2 \sqrt{x+1}} \]
\[ = \frac{2(x+1) + x}{2 \sqrt{x+1}} \]
\[ = \frac{3x+2}{2 \sqrt{x+1}} \]

(b) \[ f(x) \text{ is not defined in all of } [-2,0). \]

(c) \[ f'(x) = \frac{f(x)-f(a)}{x-a} \]
\[ \frac{3c+2}{2c+1} = 0 \]
\[ c = -\frac{2}{3} \]

10. (a) \[ f(x) = 1 + 2x - \cos(2x) \]
\[ f'(x) = 2 + 2 \sin(2x) \]
\[ f(c) = 0 \quad \text{min} \]

Set \( f'(c) = 0 \) on \([0, \pi]\). \[ f''(\frac{\pi}{2}) = 1 + \frac{3\pi^2}{2} \]
\[ \sin(2x) = -1 \]
\[ 2x = \frac{3\pi}{2}, \frac{7\pi}{2} \ldots \]
\[ x = \frac{3\pi}{4}, \frac{7\pi}{4} \]

(b) \[ -1 \leq \cos(2x) \leq 1 \]
\[ -1 \leq -\cos(2x) \leq 1 \]

(c) \[ f''(x) = 4 \cos(2x) \]
\[ f''(x) > 0 \text{ on } [0, \pi] \]
\[ x = \frac{\pi}{2}, \frac{3\pi}{2} \]

11. (a) \[ f(x) = 1 - 2x - \cos(2x) \]
\[ f'(x) = 2 + 2 \sin(2x) \]
\[ f(c) = 0 \quad \text{min} \]

Set \( f'(c) = 0 \) on \([0, \pi]\). \[ f''(\frac{\pi}{2}) = 1 + \frac{3\pi^2}{2} \]
\[ \sin(2x) = -1 \]
\[ 2x = \frac{3\pi}{2}, \frac{7\pi}{2} \ldots \]
\[ x = \frac{3\pi}{4}, \frac{7\pi}{4} \]
(a) Average rate of change

\[ f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \]

(b) \[ f(x) = \sqrt{x} \]

(c) \[ f'(x) = \frac{1}{2} x^{-1/2} \]

(d) \[ f^{-1}(x) = x^2 \]

(e) \[ \int f(x) \, dx = \frac{2}{3} x^{3/2} + C \]

(f) \[ \int x e^x \, dx = \left[ x^2 e^x \right] - 2 e^x + C \]

(g) \[ \int \frac{1}{x^2 + 1} \, dx = \arctan(x) + C \]

(h) \[ \int \frac{1}{x} \, dx = \ln|x| + C \]

(i) \[ \int \frac{1}{x^2} \, dx = \frac{-1}{x} + C \]

(j) \[ \int \frac{1}{x^3} \, dx = \frac{-1}{2x^2} + C \]

(k) \[ \int \frac{1}{x^4} \, dx = \frac{1}{3x^3} + C \]

\[ \text{Area: } \int_{0}^{1} x^2 \, dx = \frac{1}{3} \]

\[ \text{Volume: } \int_{0}^{1} \pi x^2 \, dx = \frac{\pi}{3} \]