Center of Mass

One dimension

\[ \text{Torque}_1 = \text{Torque}_2 \]

\[ \text{Force}_1 \cdot \text{distance}_1 = \text{Force}_2 \cdot \text{distance}_2 \]

\[ m_1g \cdot (x - x_1) = m_2g \cdot (x_2 - x) \]

\[ m_1x + m_2x = m_1x_1 + m_2x_2 \]

\[ x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{M_0}{m} \]

Two dimensions

\[ x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \]

\[ y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} \]

Principle: Experiment shows that motions (velocities) which are at right angles are independent and so may be solved in separate equations. Also, forces, and therefore torques, which are at right angles are independent and so may be solved in separate equations.
In one-dimension, \( \mathbf{M}_0 = \text{moment of the center of mass } \overline{x} \text{ about origin} \)

\[
= m_1 x_1 + m_2 x_2
\]

= sum of torques relative to the origin as if the origin were the fulcrum.

In two-dimensions, \( \mathbf{M}_y = \text{moment of the center of mass } (\overline{x}, \overline{y}) \text{ about } y\text{-axis} \)

\[
= m_1 x_1 + m_2 x_2
\]

= sum of torques relative to the \( y\)-axis as if the \( y\)-axis were the fulcrum.

\[\text{Diagram showing forces } T_1 \text{ and } T_2 \text{ acting on } m_1 \text{ and } m_2.\]