1. Three curves are shown below. One curve represents the position function of a particle moving in one dimension. Another curve represents its velocity, and the remaining curve matches its acceleration. Determine which curve represents its acceleration.

(a) I  (b) II  (c) III.

Show work on remaining problems.

2. Use definition of \( \cosh x \) to find the limit: \( \lim_{x \to -\infty} e^{x \cosh x} \).

3. Find the derivative of \( y \) with respect to \( x \). \( y = \cos \sqrt{\ln(1-x)} \).

4. Use the following equation: \( y^3 - xy^2 = y + 1 \).
   (a) Find \( \frac{dy}{dx} \).
   (b) Find an equation of the tangent line at the point \((-1, -1)\).

5. Find \( y' \) when \( y = x \tan^{-1}(5x) \).
6. Let \( h(x) = \sqrt{2x-1} \). Find \( h''(5) \).

7. Use logarithmic differentiation to find \( \frac{dy}{dx} \). \( y = x^{x^2} \)

8. A yacht travels due east from point A to point C as shown in the diagram below. A speedboat remains due south of the yacht the entire time as the speedboat travels a straight diagonal from point B to point C. The distance from A to B is 3 miles, and from A to C it is 4 miles. The yacht travels at 40 mph. Find the speed of the speedboat so that the speedboat meets the yacht at point C.

9. Let \( f(x) = \frac{1}{x+2} \)

\( a \) Find the linearization \( L(x) \) at \( x = 0 \).

\( b \) State the linear approximation.

\( c \) Use linear approximation to estimate \( \frac{1}{2.1} \).
1. Ans (c)

2. \[
\lim_{x \to -\infty} e^x \cosh x = \lim_{x \to -\infty} \frac{e^x(e^x + e^{-x})}{2} = \frac{1}{2} (e^{2x} + 1) = \frac{1}{2} (e^{x+1})^2
\]

3. \[
y = \cos \left[ \ln(1-x) \right]^{-\frac{1}{2}}
\]
   \[
d'y = -\sin \left[ \ln(1-x) \right] \cdot \frac{1}{2} \left[ \ln(1-x) \right]^{-\frac{1}{2}} \cdot \frac{1}{1-x}
\]

4. \[
y^3 - xy^2 = y + 1
\]
   \[
3y^2 \frac{dy}{dx} - x \cdot 2y \cdot \frac{dy}{dx} = \frac{dy}{dx} + 0
\]
   \[
3y^2 \frac{dy}{dx} - 2xy \frac{dy}{dx} = y^2
\]
   \[
\frac{dy}{dx} = \frac{y^2}{3y^2 - 2xy - 1}
\]
   \[
\text{Slope at } (1,1) \text{ is undefined.}
\]
   \[
\text{Tangent line at } x = 1
\]

5. \[
h(x) = (2x-1)^{\frac{1}{2}}
\]
   \[
h'(x) = \frac{1}{2} (2x-1)^{-\frac{3}{2}} \cdot 2 = (2x-1)^{-\frac{3}{2}}
\]
   \[
h''(x) = -\frac{1}{2} (2x-1)^{-\frac{3}{2}} \cdot 2 = -(2x-1)^{-\frac{3}{2}}
\]
   \[
h'''(x) = -\frac{3}{2} (2x-1)^{-\frac{3}{2}} \cdot 2 = -\frac{3}{2} (2x-1)^{-\frac{3}{2}}
\]
   \[
h(1) = \left(2 \cdot 1 - 1 \right)^{-\frac{3}{2}} = \frac{1}{27}
\]

6. \[
y = x^x
\]
   \[
h' = x^x \ln x + x^x \cdot \frac{1}{x} = x^x (\ln x + 1)
\]
   \[
y' = y \left[ 2x \ln x + x \right]
\]
   \[
y' = x^x \left[ 2x \ln x + x \right]
\]

7. \[
\int f(x) = \frac{1}{x+2}
\]
   \[
f'(x) = -(x+2)^{-\frac{3}{2}}
\]
(b) \[ \frac{f(a + \Delta x) - f(a)}{\Delta x} \approx \frac{dy}{dx} \]

\[ f(a + \Delta x) \approx f(a) + \frac{dy}{dx} \cdot \Delta x \]

\[ f(0 + \Delta x) \approx f(0) + f'(0) \cdot \Delta x = L(0) \]

\[ f(0 + \Delta x) \approx \frac{1}{2} - \frac{1}{4} \cdot \Delta x \]

(c) \[ \frac{1}{2.1} = \frac{1}{0.1 + 2} = f(0 + 0.1) \]

\[ \frac{1}{2.1} \approx \frac{1}{2} - \frac{1}{4} \cdot 0.1 \approx \frac{1}{40} \]

\[ \frac{1}{2.1} \approx \frac{19}{40} \]

Extra: Error \[ \frac{1}{2.1} - \frac{19}{40} \approx 0.0025 \]

0.25% error