Choose the best answer.

1. Which curves in the list have horizontal asymptotes?

   I. \( y = \cos x \)
   II. \( y = \ln x \)
   III. \( y = \tan^{-1} x \)
   IV. \( y = \frac{1}{x^2} \)

(a) I, II  (b) I, III  (c) II, III  (d) II, IV  (e) III, IV

2. Find the limit as \( x \to -\infty \) of \( y = 4x^2(x-1)(2-x)^3 \).

(a) \( \infty \)  (b) \( -\infty \)  (c) 0  (d) 4  (e) -4

3. Choose the reason why \( g \) is discontinuous at \( x = 0 \).

\[ g(x) = \begin{cases} 
   x^3 - 1, & x > 0 \\
   -2x + 1, & x < 0
\end{cases} \]

(a) \( \lim_{x \to 0^+} g(x) = 0 \)  (b) \( \lim_{x \to 0^+} g(x) \neq g(0) \)  (c) \( g(0) \) does not exist
(d) \( \lim_{x \to 0} g(x) \) exists but \( \lim_{x \to 0^-} g(x) \) does not exist
(e) \( \lim_{x \to 0^+} g(x) \) and \( \lim_{x \to 0^-} g(x) \) exist but are \( \neq \).

4. \( f(x) = \sqrt[3]{x} \) is not differentiable at \( x = 0 \) because

(a) \( f(x) \) is not continuous at \( x = 0 \)
(b) \( f'_+(0) \neq f'_-(0) \)
(c) \( \lim_{x \to 0} |f'(x)| = \infty \)
(d) \( \lim_{x \to 0} |f(x)| = \infty \)
5. Let \( f(x) \) be a function defined on \([-2,2]\). Assume \( f(x) \) may or may not be continuous on the interval. If \( f(-2) < 0 \) and \( f(2) > 0 \) then there exists \( c \) in \((-2,2)\) such that \( f'(c) = 0 \).
(a) True  (b) False

6. Choose the true statement about \( h'(x) \) when \( h(x) = \begin{cases} -x, & x \leq 0 \\ 1-x, & x > 0 \end{cases} \)
(a) \( \lim_{x \to 0^-} h'(x) = -1 \) and \( \lim_{x \to 0^+} h'(x) = \infty \)  
(b) \( \lim_{x \to 0^-} h'(x) = -1 \) and \( \lim_{x \to 0^+} h'(x) = -\infty \)  
(c) \( \lim_{x \to 0^-} h'(x) = \infty \) and \( \lim_{x \to 0^+} h'(x) = -1 \)  
(d) \( \lim_{x \to 0^-} h'(x) = -\infty \) and \( \lim_{x \to 0^+} h'(x) = -1 \)  
(e) \( \lim_{x \to 0} h'(x) = -1 \).

Show all work on remaining problems.

7. Use the definition of derivative to find \( f'(a) \) when \( f(x) = \sqrt{1-x} \).

8. Let \( C(x) = \frac{\cos x}{x} \) be the unit cost function.
(a) Find the marginal unit cost function.
(b) Find \( C'(5\pi) \) and explain its meaning. What does it predict?
(c) Compare \( C'(5\pi) \) with \( C\left(\frac{11\pi}{2}\right) - C(5\pi) \).
(d) Use the calculator to sketch the graph of \( C(x) \) on the interval \([0,20]\).

9. Find the graph of \( f'(x) \) from the graph of \( f(x) \).

10. Use algebra to find the limit. \( \lim_{x \to \infty} \frac{3-x^2}{\sqrt{1-5x} + 16x^4} \).

11. Suppose \( f(1) = -1 \), \( f'(1) = 3 \), \( g(1) = 4 \), and \( g'(1) = -5 \). Find \( (fg)'(1) \).

Extra Credit: Find \( \lim_{\theta \to 0} \frac{1 - \cos(5\theta)}{5\sin\theta} \).
1. \[ \frac{4x^2(x-1)(x-3)}{x-0} \]

\[ \lim_{x \to 0^+} g(x) = 1 \]

\[ \lim_{x \to 0^-} g(x) = -1 \]

Answer: \(-\infty\)

2. (a) Not correct: \(\lim_{x \to 0} f(x) = 0 = f(0) \Rightarrow f \text{ is cont. at } x = 0\).

(b) OK answer: \(f'(0) = \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} \frac{3}{h} = +\infty\)

And \(f'(0) = +\infty\) also.

3. There is no \(c\) in \((-2, 2)\) where \(f(c) = 1\).

4. (a) \(\lim_{x \to 0^+} \sqrt[3]{x} = +\infty\) in the sense that \(+\infty = +\infty\).

But they are \(\neq\) in the sense that \(+\infty\) cannot be equated because \(+\infty\) is not a real number.

(c) \(\lim_{x \to 0} \sqrt[3]{x} = \lim_{x \to 0} \left(\frac{1}{x} \right) = \lim_{x \to 0} \frac{1}{x^2} = \infty\). True.

5. (a) \(C(x) = \frac{d}{dx} \cos x = \frac{x \cos x - (\cos x)(x)}{x^2} = -\frac{x \sin x - \cos x}{x^2}

(b) \(C'(5\pi) = -\frac{5\pi \cdot 0 - (-1)}{(5\pi)^2} = \frac{1}{25\pi^2} \approx 0.0001\)

It predicts that the increase in the unit cost is \(0.0001\) when \(x\) changes from \(5\pi\) to \(5\pi + 1\).
5. \( C\left(\frac{\pi}{2}\right) - C(5\pi) = 0 - \frac{-1}{5\pi} = 1 \approx 0.06366 \)

Should be to compare \( C'(5\pi) \) with slope of secant

\[
\frac{C\left(\frac{11\pi}{2}\right) - C(5\pi)}{\frac{11\pi}{2} - 5\pi} = \frac{\frac{1}{2\pi}}{\frac{3\pi}{2}} = \frac{2}{5\pi^2}
\]

\[
\text{Slope of secant} = \frac{\frac{1}{2\pi^2}}{\frac{3\pi}{2}} = 10.
\]

Slope of secant is 10 times steeper.

6. \( \lim_{x \to \infty} \frac{3 - x^2}{\sqrt{1 - 5x + 16x^4}} \)

\[
= \lim_{x \to \infty} \frac{(3 - x^2) \frac{1}{x^2}}{\sqrt{1 - 5x + 16x^4} \cdot \frac{1}{x^2}}
\]

\[
= \lim_{x \to \infty} \frac{\frac{3}{x^2} - 1}{\sqrt{\frac{1}{x^4} - \frac{5}{x^3} + 16}}
\]

\[
= \frac{0 - 1}{\sqrt{0 - 0 + 16}} = 2
\]

\[
= \frac{-1}{4}
\]

\[\frac{17}{17} = 1 \]

\[\text{Extra Credit} \]

\[
\lim_{\theta \to \infty} \frac{1 - \cos(5\theta)}{\sin\theta}
\]

\[
= \lim_{\theta \to \infty} \frac{1 - \cos(5\theta)}{\sin\theta} \cdot \frac{1 + \cos(5\theta)}{1 + \cos(5\theta)}
\]

\[
= \lim_{\theta \to 0} \frac{\sin^2(5\theta)}{\sin\theta} \cdot \frac{1 + \cos(5\theta)}{1 + \cos(5\theta)}
\]

\[
= \lim_{\theta \to 0} \frac{5 \sin(5\theta)}{\sin(5\theta)} \cdot \frac{1}{1 + \cos(5\theta)}
\]

\[
= \frac{5 \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{5}{1} = 5
\]

Another way

\[
\lim_{t \to 0} \frac{\frac{1 - \cos(t)}{\sin(t)}}{5}
\]

\[
= \lim_{t \to 0} \frac{\frac{1 - \cos(t)}{\sin(t)}}{5}
\]

\[
= \frac{1}{5} \cdot 1 = \frac{1}{5} = \frac{1}{5}
\]