Lab 5 Homework-Shane Smith

1. Pick a polynomial function $f$ of degree $\geq 3$ with at least 3 terms.

$$\text{In}[7]:= \text{Clear}[f]$$

$$f[x_] := x^5 - 10x^4 + x + 10$$

2. Graph $f$.

$$\text{In}[9]:= \text{Plot}[f[x], \{x, -50, 50\}, \text{PlotRange} \to \{-10000, 10000\}]$$

$$\text{In}[10]:= \text{Plot}[f[x], \{x, -10, 15\}, \text{PlotRange} \to \{-100, 100\}]$$
In[11] := Plot[f[x], {x, 9, 11}]

In[12] := Plot[f[x], {x, -2, 2}, PlotRange -> {-10, 12}]

Out[12] = *Graphics*

Use Newton's method to find all real roots of \( f \).

In[13] := g[x_] = x - \frac{f[x]}{f'[x]} // Simplify

\[ N[NestList[g, 9.8, 10], 6] \]


Out[14] = \( n_1 \approx 9.998 \)

In[15] := g[x_] = x - \frac{f[x]}{f'[x]} // Simplify

\[ N[NestList[g, 1.1, 10], 8] \]

\[ \{1.1, 1.05702, 1.05435, 1.05434, 1.05434, 1.05434, 1.05434, 1.05434, 1.05434, 1.05434\} \]

Out[16] = \( n_2 \approx 1.05434 \)
In[18]:= g[x_] = x - \frac{f[x]}{f'[x]} // Simplify

N[NestList[g, -0.9, 10], 8]

\[x_0 \approx -0.9\]
\[n=10\]
8 decimal places

Out[18]= \frac{10 + 30 x^4 - 4 x^5}{-1 + 40 x^3 - 5 x^4}

Out[19]= \{-0.9, -0.958268, -0.953353, -0.953314, -0.953314,
-0.953314, -0.953314, -0.953314, -0.953314, -0.953314\}

In[20]:= Plot[f[x], {x, -4, 11}, Axes -> True, PlotRange -> {-9000, 1000}]

\[\approx -0.953314\]


EXTRA: some choices for \(x_0\) will not work. For example, when calculating \(r_2\), the choice \(x_0 = 1.0\) will not allow the Newton's Method formula to close in on the nearby root \(r_2\).

Out[23]= \frac{10 + 30 x^4 - 4 x^5}{-1 + 40 x^3 - 5 x^4}

Out[30]= \{0.1, -10.4143, -8.10526, -5.28298, -4.85072, -3.73045, -2.8595,
-2.16842, -1.6804, -1.31271, -1.07849, -0.973788, -0.953958, -0.953315,
-0.953314, -0.953314, -0.953314, -0.953314, -0.953314, -0.953314\}

In the above choice for \(x_0\), that is, \(x_0 = 0.1\), a different root than the intended root is found. We found \(r_3\) instead of \(r_2\). We can look at a series of tangents to why this happened.
\text{In}[56] := \text{Plot}[f[x], \{x, -12, 3\}, \text{PlotRange} \to \{-10, 12\}]

\text{Out}[56] = \text{Graphics}

\text{In}[55] := \text{Plot}[f[x], \{x, -12, 3\}, \text{PlotRange} \to \{-1000000, 1000000\}]

\text{Out}[55] = \text{Graphics}
In[63]:= \text{Plot}\{f[x], \{x, -7, 3\}, \text{PlotRange} \rightarrow \{-100000, 100000\}\}

Out[63]= -\text{Graphics}-

A choice for $x_0$ may result in no roots being found. For example,

In[23]:= \text{Clear}[h]
\text{h}[x_] := 2 x^3 + 3 x^2 - 12 x + 6

In[25]:= \text{g}[x_] = x - \frac{\text{h}[x]}{\text{h}'[x]} \quad \text{// Simplify}
\text{N[\text{NestList}[\text{g}, 1.0, 10], 8]}\]

Out[25]= \frac{-6 + 3 x^2 + 4 x^3}{6 (-2 + x + x^2)}

\text{Power::infy}: \text{Infinite expression \frac{1}{0} encountered. More...}

\text{::indet}: \text{Indeterminate expression -2 + ComplexInfinity + ComplexInfinity encountered. More...}

\text{::indet}: \text{Indeterminate expression -6 + ComplexInfinity + ComplexInfinity encountered. More...}

Out[26]= \{1., \text{ComplexInfinity}, \text{Indeterminate}, \text{Indeterminate}, \text{Indeterminate}, \text{Indeterminate}, \text{Indeterminate}, \text{Indeterminate}, \text{Indeterminate}, \text{Indeterminate}\}
In[34]:= Plot[h[x], {x, -7, 3}, PlotRange -> {-5, 5}]

Out[34]= - Graphics -

In[35]:= Date[]
Out[35]= {2003, 10, 24, 12, 21, 50.2400000}

tangent line never
cross x-axis.

\[
x - \frac{h(x)}{h'(x)}\]

becomes

\[
1 - \frac{h'(1)}{h'(x)} = 1 - \frac{-1}{0} = \text{indeterminate}
\]