1.) Find the following limits:

(a) \( \lim_{x \to \infty} \frac{\sqrt{x^2 - 9}}{x - 3} \)  
(b) \( \lim_{x \to 0} \frac{2\sin x - x}{x} \)  
(c) \( \lim_{x \to \infty} \frac{x}{x - \frac{y}{x}} \)  
(d) \( \lim_{y \to 0} \frac{\sin^6 y}{6y^6} \)  

(d) \( \lim_{w \to 0} \frac{\sqrt{2 + 3w} - \sqrt{2 - 3w}}{w} \)  
(e) \( \lim_{x \to 1} \frac{2|x - 1|}{x - 1} \)  
(f) \( \lim_{x \to \infty} \frac{1 - x}{x - 2} \)

2.) Let

\[ f(x) = \frac{x^2 + 6x - 16}{x - 2} \]

Find the value \( f(2) \) that would make \( f \) continuous at 2.

3.) Let \( f(x) = \sqrt{x} \). Show that the line \( 4y = x + 4 \) is tangent to the graph of \( f \). What is the point of tangency?

4.) Compute the derivative of the following:

(a) \( y = x^2 \sin \frac{1}{x} \)  
(b) \( y = (3x - 5)^{5/9} \)  
(c) \( \int_{2}^{5x^2} t \cos(3t) \, dt \)

5.) Find the equation of the line tangent to the graph of \( f(x) = x\sqrt{x - 1} \) at the point (5, 10).

6.) Find an equation of the line tangent to the graph of the equation \( xy^3 - x^2 = y^2 + xy - 5 \) at the point (2, 1).

7.) A shadow in the shape of an equilateral triangle is growing 9 square inches per minute. At what rate does the height of the triangle grow when the area is \( \sqrt{3} \) square inches.

8.) Suppose the velocity of an object is given by \( v(t) = \frac{1}{4} t^2 + \sin t \) for \( 0 \leq t \leq \frac{\pi}{2} \). For what value of \( t \) in \([0, \frac{\pi}{2}]\) is the acceleration maximal?

9.) Use a linear approximation to approximate \( \sqrt{31} \).

10.) Use Newton's Method to approximate a \( \sqrt{31} \).

11.) Find the domain, intercepts, local extrema, inflection points and asymptotes of the following functions. Then sketch their graphs.
12.) Find the maximum value and the minimum value of \( f(x) = -(x^2 - 12x) \) on the interval \([1, 3]\).

13.) Find the area of the largest rectangle that can be drawn with its base on the \( x \) - axis and with two vertices on the graph \( y = \frac{8}{x^2 + 4} \).

14.) Let \( f(x) = x^3 + 8 \) and let \( P = \{-2, -1, 0, 1, 2\} \) be a partition of the interval \([-2, 2]\). Find the upper sum \( A_{CP} \) and the lower sum \( A_{IP} \).

15.) Evaluate the following integrals:
   (a) \( \int_{1}^{32} \frac{4}{\sqrt{x^3}} dx \)
   (b) \( \int_{0}^{\pi/4} \frac{\sec x \tan x}{(4 + \sec x)^2} dx \)
   (c) \( \int_{1}^{4} |x - 2| dx \)

16.) Find the area of the region bounded by the graphs \( f(x) = x^3 - 3x + 2 \) and \( g(x) = x + 2 \).

17.) Find the volume of the solid obtained by revolving the region between \( f(x) = \cos x \) and \( g(x) = \sec x \) on the interval \([0, \frac{\pi}{4}]\) around the \( x \) - axis. Hint : \( \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x \)

18.) Find the length of the curve \( f(x) = \frac{2}{3}x^{3/2} \) on the interval \([0, 3]\).

19.) A cylindrical well 20 feet deep and 3 feet in radius is dug. Assuming that the soil weighs 150 pounds per cubic foot, calculate the work \( W \) required to raise the soil to ground level.

20.) Use Trapezoidal Rule and Simpson's Rule to approximate \( \int_{1}^{2} \frac{1}{x} dx \) using 10 subintervals.

NOTE THAT PROBLEMS ON THE EXAM ARE NOT LIMITED TO THESE.
Solutions

1.) Find the following limits:

(a) \( \lim_{x \to \infty} \frac{\sqrt{x^2 - 9}}{x - 3} = 1 \)
(b) \( \lim_{x \to 0} \frac{2\sin x - x}{x} = 1 \)
(c) \( \lim_{x \to \infty} \frac{x}{x^2} = 1 \)
(d) \( \lim_{y \to 0} \frac{\sin^6 y}{6y^6} = \frac{1}{6} \)

(d) \( \lim_{w \to 0} \frac{\sqrt{2 + 3w} - \sqrt{2 - 3w}}{w} = \frac{3\sqrt{2}}{2} \)
(e) \( \lim_{x \to 1} \frac{2|x - 1|}{x - 1} = -2 \)
(f) \( \lim_{x \to 2^+} \frac{1 - x}{x - 2} = -\infty \)

2.) \( f(x) = \frac{x^2 + 6x - 16}{x - 2} = \frac{(x - 2)(x + 8)}{x - 2} = x + 8 \), for \( x \neq 2 \)

\( \lim_{x \to 2} f(x) = 10 \). Thus \( f(2) \) must equal 10 for \( f \) to be continuous at \( x = 2 \).

3.) The slope of the line is \( \frac{1}{4} \). Therefore \( f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{4} \), when \( x = 4 \). So the point of tangency is \( (4, 2) \).

4.) Compute the derivative of the following:

(a) \( \frac{dy}{dx} = 2x \sin \frac{1}{x} - \cos \frac{1}{x} \)
(b) \( \frac{dy}{dx} = \frac{15}{9} (3x - 5)^{-4/9} \)
(c) \( \frac{d}{dx} \int_2^{5x^2} t \cos(3t) \, dt = 50x^3 \cos(15x^2) \)

5.) \( f(x) = x \sqrt{x - 1} \) So \( f'(x) = \frac{x}{2\sqrt{x - 1}} + \sqrt{x - 1} \).
\( f'(5) = \frac{13}{4} \). Thus the equation of the tangent line is \( y - 10 = \frac{13}{4} (x - 5) \).

6.) \( y - 1 = 2(x - 2) \).

7.) In an equilateral triangle with base \( b \), the height \( h = \frac{\sqrt{3}}{2} b \). Thus \( b = \frac{2}{\sqrt{3}} h \). So
\( A = \frac{1}{2} bh = \frac{b^2}{\sqrt{3}} \). So \( \frac{dA}{dt} = \frac{2}{\sqrt{3}} h \frac{dh}{dt} \). So we want \( \frac{dh}{dt} \) when \( A = \sqrt{3} \). Note that the height \( h = \sqrt{3} \) at that area. Thus \( \frac{dh}{dt} = \frac{9}{2} \).

8.) \( t = \frac{\pi}{6} \)
9.) \( \sqrt{31} \approx \sqrt{32} + \frac{1}{5(32)}(31 - 32) = \frac{159}{80} \)

10.) \( \frac{159}{80} \)

11.) Find the domain, intercepts, local extrema, inflection points and asymptotes of the following functions. Then sketch their graphs.

(a) \( f(x) = \frac{5x}{x^2 - 1} \)

(b) \( f(x) = \frac{8}{x^2 + 4} \)
12.) Maximum is $f(3) = 27$ and the minimum is $f(1) = 11$.

13.) $x = 2$ will yield the maximum area.

14.) The upper sum is $A_{CP} = f(-1) + f(0) + f(1) + f(2)$ and the lower sum is $A_{IP} = f(-2) + f(-1) + f(0) + f(1)$.

15.) Evaluate the following integrals:
(a) $\int_{1}^{32} \frac{4}{\sqrt[3]{x}} \, dx = 30$
(b) $\int_{0}^{\pi/4} \frac{\sec x \tan x}{(4 + \sec x)^2} \, dx = \frac{1}{5} - \frac{1}{4 + \sqrt{2}}$
(c) $\int_{1}^{4} |x - 2| \, dx = \frac{5}{2}$

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17.) $\frac{3}{4} - \frac{\pi}{8}$

18.) $\frac{14}{3}$

19.) $W = \int_{-20}^{0} 150(0 - x) \cdot 9 \pi \, dx$

20.) $T_{10} \approx .69377 \quad S_{10} \approx 0.69315$