MATH 26 Review Problems for Final Exam

**EQUATIONS:** Solve the following. If you omit a solution, explain why.

1.) \( \frac{x - 1}{x + 1} = \frac{2x + 2}{x - 2} \)  
2.) \( \sqrt{2x - 1} - x = 2 \)  
3.) \( 2\sqrt{x^2} - \sqrt{x} = 1 \)  
4.) \( |2x + 3| = 7 \)  
5.) \( e^{1-2x} = 4^x \)  
6.) \( \log_4 x + \log_4(x - 3) = 1 \)  
7.) \( e^{2x} - e^x - 2 = 0 \)  
8.) \( \log_3 x - \log_3(x - 1) = 1 \)  
9.) \( 2^{x+2} = 4^{x-1} \)

**FUNCTIONS/GRAPHS:** Identify each and graph showing all intercepts, asymptotes, vertices and/or starting points.

1.) \( f(x) = x^2 + x - 2 \)  
2.) \( f(x) = -3\sqrt{x - 2} + 6 \)  
3.) \( f(x) = 2|x + 1| - 7 \)  
4.) \( f(x) = 1 - \frac{1}{x} \)  
5.) \( f(x) = x^2(x - 1)(x + 2)^3 \)  
6.) \( f(x) = -e^x + 2 \)  
7.) \( f(x) = x^6 - x^5 - 2x^4 \)  
8.) \( f(x) = \frac{2x^2 - 4x}{x^2 - x - 12} \)  
9.) \( f(x) = 6^x - 3 \)  
10.) \( f(x) = -\ln(x - 6) \)  
11.) \( f(x) = \log_3(x+3)+1 \)  
12.) \( f(x) = \begin{cases} 3 & \text{if } x < -2 \\ 2 - \frac{1}{2}x & \text{if } x \geq -2 \end{cases} \)

For problems 1-10, find each of the following:

1.) \( (f \circ g)(x) \) where \( f(x) = \frac{8}{x - 1} \) and \( g(x) = \frac{1}{x} \). Also find the domain of \( f \circ g \).

2.) Inverse of \( f(x) = \frac{x - 1}{x + 3} \). What is the range of \( f^{-1} \)?

3.) Domain of \( fg \) where \( f(x) = \frac{3}{5x - 7} \) and \( g(x) = \sqrt{3x + 1} \)

4.) Range of \( f(x) = -5e^x + 25 \)

5.) Intercepts of \( f(x) = -\log_2(x + 4) + 1 \)

6.) Asymptotes and holes of \( f(x) = \frac{x^2 - 9}{(x - 3)(x + 1)(x - 1)} \)

7.) The graph obtained by reflecting \( y = x^2 \) about the \( x \)-axis, stretching it by 2, horizontally shifting it to the right by 3 and vertically shifting it down by 12

8.) Polynomial \( p(x) \) of degree 4 with \( x \)-intercepts \((-1, 0), (0, 0) \) and \((3, 0)\) with the property that \( p(x) \geq 0 \) only for \( x \) in the interval \([-1, 3]\)

9.) \( \frac{f(x + h) - f(x)}{h} \) where \( f(x) = x^2, f(x) = \frac{1}{x - 1} \)

10.) Write the following expressions as a single quotient in which only positive exponents and/or radicals appear:

i. \( \frac{(9 - x^2)^{1/2} + x^2(9 - x^2)^{-1/2}}{9 - x^2} \)

ii. \( -x^{1/3} + \frac{1}{3}x^{-2/3}(8 - x) \)
APPLICATIONS:

1.) The manager of a weekend flea market knows from past experience that if he charges $x$ dollars for a rental space at the flea market, then the number $y$ of spaces he can rent is given by the equation $y = 200 - 4x$.

(a) Sketch a graph of this linear equation.
(b) What do the slope, the $y$-intercept, and the $x$-intercept represent?

2.) Suppose $p = -2x^2 + 80$ is a price supply equation and $p = 15x + 30$ is a price demand equation. Find the equilibrium quantity and price if $x$ represents quantity demanded in units of a thousand and $p$ is the price in dollars.

3.) Patricia wishes to have a rectangular-shaped garden in her backyard. She has 80 feet of fencing material with which to enclose her garden. Letting $x$ denote the width of the garden, find a function $A$ in variable $x$ giving the area of the garden. What is its domain?

4.) Phillip, the proprietor of a vineyard, estimates that if 10,000 bottles of wine were produced this season, then the profit would $5 per bottle. But if more than 10,000 bottles were produced, then the profit per bottle would drop by $0.0002 for each additional bottle sold. Assume that at least 10,000 bottles of wine are produced and sold and let $x$ denote the number of bottles produced and sold above 10,000. Find a function $P$ giving the profit in terms of $x$.

5.) The revenue of a charter bus company depends on the number of unsold seats. If the revenue $R$ is given by $R = 5000 + 50x - x^2$, where $x$ is the number of unsold seats, find the maximum revenue and the number of unsold seats which produce maximum revenue.

6.) Suppose $10,000 is deposited at 12% interest per annum. Find the amount in the account after 3 years if the interest is compounded (a) annually, (b) quarterly, (c) daily, (d) continuously.

7.) Find the present value of $10,000 due in 5 years at an annual interest rate of 10% compounded continuously.

8.) The height(in feet) of a certain kind of tree is approximated by

$$ h(t) = \frac{160}{1 + 240e^{-0.2t}} $$

where $t$ is the age of the tree in years. Estimate the age of an 80 foot tree.

9.) Phosphorus 32 has a half-life of 14.2 days. If 100 g of this substance are present initially, find the amount present after $t$ days. What amount will be left after 7.1 days? How long will it take 66% of the initial sample to decompose?

10.) Find the value of the annuity with annual payments of $1000 for 9 years at 8% interest compounded annually.

INEQUALITIES: Solve the following.

1.) $2(7x - 3) \leq 12x + 16$  
2.) $x^2 - 3x - 18 \leq 0$

3.) $\frac{3 + x}{3 - x} \geq 1$  
4.) $\frac{-5(x - 1)^2}{x^2 + 3x + 2} > 0$

5.) $|5x + 3| < 3$  
6.) $1 < \frac{4m - 5}{2} < 9$
**IDENTITIES/LAWS:** Do the following:

1.) Express \( \ln \left( \frac{x\sqrt{x^2 + 1}}{x - 3} \right) \) as the sum and/or difference of logarithms.

2.) Express the following as a single logarithm: \( 8 \log_2 \sqrt{3x - 2} - \log_2 \left( \frac{4}{x} \right) + \log_2 4 \)

3.) Find the exact value of the following:
   (a) \( \log_2 \frac{1}{32} \)  
   (b) \( \ln e^{\sqrt{3}} \)  
   (c) \( \log 4 + \log 5 - \log 2 \)  
   (d) \( 125^{\log_5 10} \)

**SEQUENCES/SUMMATION NOTATION:** Do the following:

1.) Write out the first six terms of the sequence: \( \left\{ \frac{(-1)^{n-1}(n + 1)}{n + 2} \right\} \)

2.) Find the formula for the \( n \)th term of a geometric sequence whose first term is 3 and whose common ratio is 3. Also find the sum of first 7 terms, i.e. \( S_7 \).

3.) Use properties of summations to evaluate: \( \sum_{k=1}^{50} (2k^3 + 6) \)