Review for EXAM 1

1. Solve the following in equalities. Write your solution in interval notation.

(a) \( \frac{2}{3}(x - 6) \geq x - 1 \)  
(b) \( 2(x + 3) > 2x + 1 \)  
(c) \( \frac{x + 3}{12} + \frac{x - 5}{15} < \frac{2}{3} \)

(d) \( -1 \leq \frac{2x}{3} + 5 \leq 2 \)  
(e) \( -2x - 5 < -3 \) or \( 6x < 0 \)

(f) \( 3x + 2 \leq 5 \) or \( 7x > 84 \)

2. Find all real solutions of the equation.

(a) \( \frac{x - 5}{2} - \frac{2x + 5}{3} = \frac{5}{6} \)  
(b) \( \frac{x + 1}{x - 1} = \frac{2x - 1}{2x + 1} \)  
(c) \( 3x^2 + 5x - 2 = 0 \)

(d) \( (x + 5)^2 - 3(x + 5) - 10 = 0 \)

3. Phyllis invested $12,000, a portion earning a simple interest rate of \( 4\frac{1}{2}\% \) per year and the rest earning a rate of 4% per year. After one year the total interest earned on these investments was $525. How much money did she invest at each rate?

4. Suppose that \( w \) varies directly as the square of \( z \) and inversely as \( x \) and \( y \). Suppose we know that when \( w = 10 \), \( x = 15 \), \( y = 2 \) and \( z = 5 \). Find the constant of variation \( k \). Also find \( z \) when \( w = 2 \), \( x = 8 \) and \( y = 27 \). Assume that \( z > 0 \).

5. Test the equation for symmetry, find its intercepts(if any) and sketch its graph.

(a) \( 2x - y + 1 = 0 \)  
(b) \( y = 16 - x^2 \)  
(c) \( \frac{x}{2} - \frac{y}{3} = 1 \)  
(d) \( y^2 = 4 - x^2 \)

6. Find \( f(0) \), \( f(2) \), \( f(-2) \), \( f(a) \), \( f(x + h) \) and evaluate \( \frac{f(x + h) - f(x)}{h} \) for each function \( f \):

a.) \( f(x) = 1 - 3x \)  
b.) \( f(x) = x^2 - x + 1 \)

c.) \( f(x) = \frac{1}{x + 3} \)

7. Find the average rate of change of \( f(x) = 8x^2 - x \) from 1 to 2.
8. The graph of $f$ is given below.

![Graph of $f$]

a.) If the domain of $f$ is $(-\infty, \infty)$, find the range.
b.) Find the coordinates of the intercepts.
c.) Find all local maximum values, if any.
d.) Find all local minimum values, if any.
e.) On what intervals is the function increasing? On what intervals is the function decreasing?
f.) Find $f(1) - f(0) + f(-1)$.
g.) For what values of $x$ is $f(x) = 3$.
h.) Is $f$ even, odd or neither? Explain.

9. Find the domain of the following functions:

(a) $f(x) = \sqrt{x} + 3$
(b) $g(x) = \frac{2x}{2x^2 - 5x - 3}$
(c) $h(t) = \frac{1}{\sqrt{t}}$

(d) $k(x) = \sqrt{2x + 4}$
(e) $p(x) = \frac{5\sqrt{x}}{x^2 + 1}$

10. Sketch the graph of the functions below:

(a) $f(x) = \sqrt{x} + 3$    (b) $g(x) = 1 - 2x$    (c) $h(x) = 1 - \frac{1}{2}x^2$    (d) $y = 2(x + 1)^3$

(e) $f(x) = \frac{1}{x} + 1$    (f) $g(x) = -\sqrt{x} + 5$    (g) $y = -2|x + 3| - 1$

(h) $f(x) = \begin{cases} 1 - 2x & \text{if } x \leq 0 \\ 2x - 1 & \text{if } x > 0 \end{cases}$
(i) $f(x) = \begin{cases} x + 6 & \text{if } x < -2 \\ x^2 & \text{if } x \geq -2 \end{cases}$

(j) $r(x) = \sqrt{-x} + 2$
11. Determine whether the functions below are even, odd, or neither.

\[ (a) \ f(x) = 2x^5 - 3x^2 + 2 \quad (b) \ f(x) = x^3 - x^7 \quad (c) \ f(x) = \frac{1-x^2}{1+x^2} \]

12. Find the slope of the line containing \((7, 4)\) and \((-3, 2)\). Write the equation of the line through these two points in slope-intercept form. Is the line \(-5x + 25y = 1\) parallel or perpendicular to this line?

13. Let \(f(x) = \frac{5}{x}\). Find and simplify \(\frac{f(x) - f(1)}{x - 1}\).

14. Suppose that you are trying to decide between two long distance services. Company A will allow you to call your mother for $0.15 per minute with a $1 connection charge. Company B will charge you $0.25 per minute, but there is no connection charge.

(a) Write the linear function \(A(x)\) that represents the cost of an \(x\) minute phone call to your mother using company A.

(b) Write the linear function \(B(x)\) that represents the cost of an \(x\) minute phone call to your mother using company B.

(c) If you talk to your mother for 45 minutes, how much will company A charge you? How about company B?

(d) If you have $5 to spend, how long will company A allow you to talk? How about company B?

(e) Graph \(A(x)\) and \(B(x)\) on the same \(x-y\) plane.

15. Let \(f(x) = \frac{x-10}{x+1}\) and \(g(x) = \frac{x+1}{x-2}\). Find \(f + g, f - g, f \cdot g, f/g\) and their domains.

16. A manufacturer sells a product for $10 per unit. The manufacturer's fixed costs are $1200 per month, and the variable costs are $2.50 per unit. How many units must be produced each month to break even?

17. The revenue a company earns is directly proportional to the number of articles sold. If the company has a revenue of $50,000 on the sale of 2500 cell phone headsets, what revenue can the company expect on the sale of 3000 headsets?

18. The revenue (in dollars) from the sale of \(x\) car seats for infants is given by \(R(x) = 60x - 0.0025x^2\). Find the average rate of change of revenue if production is changed from 1000 to 1050 car seats.

19. A plant can manufacture 80 golf clubs per day for a total daily cost of $7,647 and 100 golf clubs per day for a total daily cost of $9,147. Assuming the daily cost and production are linearly related, find the total daily cost of producing \(x\) golf clubs. Interpret the slope and \(y\)-intercept of this cost equation.

20. Find the \(x\)-intercept(s) of each function:

(a) \(f(x) = |2x - 5| - 21\) \quad (b) \(g(x) = \sqrt{x + 4} - 3\) \quad (c) \(h(x) = (x + 4)^{3/2} - 27\)
21. Sketch the graph of \( f(x) = 2 \text{int}(x + 1) \).
Answers to the Review for Exam 1

1.) (a) \(-\infty, -\frac{7}{3}\)  (b) \(-\infty, \infty\)  (c) \(-\infty, 5\)  
(d) \([-9, -\frac{9}{2}\])  (e) \(-\infty, \infty\)  (f) \(-\infty, 1\) \cup (12, \infty)  

2.) (a) \(x = -30\)  (b) \(x = 0\)  (c) \(x = \{-2, \frac{1}{3}\}\)  
(d) \(x = \{-7, 0\}\)  

3.) She should invest $3000 at the 4% rate and $9000 at the 4.5% rate.  

4.) \(w = k \cdot \frac{x^2}{xy}\). When \(w = 10\), \(x = 15\), \(y = 2\) and \(z = 5\). So  
\[10 = k \cdot \frac{25}{30} \Rightarrow k = 12\]  
Thus \(w = \frac{12x^2}{xy}\). To find \(z\), we solve using the fact that \(w = 2\), \(x = 8\) and \(y = 27\):  
\[2 = \frac{12x^2}{216} \Rightarrow z^2 = 36 \Rightarrow z = 6\]  

5.) (a) no symmetry, \(-\)-intercept \((-\frac{1}{2}, 0\), \(-\)-intercept \((0, 1)\). The graph is a line with slope \(2\).  
(b) symmetric with respect to the \(-\)-axis, \(-\)-intercepts \((4, 0)\) and \((-4, 0)\), \(-\)-intercept \((0, 16)\). The graph is a parabola reflected about the \(-\)-axis and shifted up 16 units.  
(c) no symmetry, \(-\)-intercept \((2, 0)\), \(-\)-intercept \((0, -7)\). The graph is a line with slope \(\frac{7}{2}\).  
(d) symmetric with respect to both axes and the origin, \(-\)-intercepts \((-2, 0), (2, 0)\) and the \(-\)-intercepts \((0, -2), (0, 2)\). The graph is a circle centered at the origin with radius 2.  

6.) (a) \(f(0) = 1, f(2) = -5, f(a) = 1 - 3a, f(x + h) = 1 - 3(x + h),\)  
\[\frac{f(x + h) - f(x)}{h} = \frac{1 - 3(x + h) - (1 - 3x)}{h} = \frac{-3h}{h} = -3\]  
(b) \(f(0) = 1, f(2) = 3, f(-2) = 7, f(a) = a^2 - a + 1,\)  
\(f(x + h) = (x + h)^2 - (x + h) + 1,\)  
\[\frac{f(x + h) - f(x)}{h} = \frac{(x + h)^2 - (x + h) + 1 - (x^2 - x + 1)}{h} = 2x + h - 1\]  
(c) \(f(0) = \frac{1}{3}, f(2) = \frac{1}{5}, f(-2) = 1, f(a) = \frac{1}{a+3}, f(x + h) = \frac{1}{(x+h)+3}\)
\[
\frac{f(x + h) - f(x)}{h} = \frac{1}{(x + h) + 3} - \frac{1}{x + 3} = \frac{x + 3 - (x + h + 3)}{h(x + 3)(x + h + 3)} = \frac{-h}{h(x + 3)(x + h + 3)}
\]

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8.) (a) \([0, \infty)\) (b) \((-1, 0), (1, 0), (0, 2)\) (c) 2 is the local max (d) 0 is the local min

(e) Increasing: \((-1, 0) and (1, \infty)\)

Decreasing: \((-\infty, -1) and (0, 1)\)

(f) \(-2\) (g) \(x = \frac{3}{2}, x = -\frac{3}{2}\)

(h) even, symmetric with the y-axis.

9.) (a) \([-3, \infty)\) (b) \((-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 3) \cup (3, \infty)\)

(c) \((0, \infty)\) (d) \((-\infty, \infty)\) (e) \([0, \infty)\)

10.)
11.) (a) neither  (b) odd  (c) even

12.) slope is $\frac{1}{5}$. Equation of the line is $y = \frac{1}{5}x + \frac{13}{5}$. Parallel.

13.) \[
\frac{f(x) - f(1)}{x - 1} = \frac{\frac{5}{x} - \frac{5}{1}}{x - 1} = \frac{5 - 5x}{x(x - 1)} = \frac{5(1 - x)}{x(x - 1)} = -\frac{5}{x}
\]

14.) (a) $A(x) = .15x + 1$       (b) $B(x) = .25x$       (c) $A(45) = 7.75, B(45) = 11.25$

(d) Company $A$ will allow you approximately 26.67 minutes and Company $B$ will allow you 20 minutes.
15. 
\( (f + g)(x) = f(x) + g(x) = \frac{x - 10}{x + 1} + \frac{x + 1}{x - 2} = \frac{(x - 10)(x - 2) + (x + 1)^2}{(x + 1)(x - 2)} = \frac{2x^2 - 10x + 21}{(x + 1)(x - 2)} \)

\( (f - g)(x) = f(x) - g(x) = \frac{x - 10}{x + 1} - \frac{x + 1}{x - 2} = \frac{(x - 10)(x - 2) - (x + 1)^2}{(x + 1)(x - 2)} = \frac{-14x + 19}{(x + 1)(x - 2)} \)

\( (f \cdot g)(x) = f(x) \cdot g(x) = \frac{x - 10}{x + 1} \cdot \frac{x + 1}{x - 2} = \frac{x - 10}{x - 2} \)

The domain for these three new functions is whatever is in the domain of \( f \) and the domain of \( g \). Domain of \( f \) is \((-\infty, -1) \cup (-1, \infty)\). Domain of \( g \) is \((-\infty, 2) \cup (2, \infty)\). Therefore the domain of \( f + g, f - g, \) and \( f \cdot g \) is \((-\infty, -1) \cup (-1, 2) \cup (2, \infty)\).

\( \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{(x - 10)(x - 2)}{(x + 1)^2} \). Domain is \(( -\infty, -1) \cup (-1, 2) \cup (2, \infty)\).

16. The total revenue for \( x \) units of the product is \( 10x \), so the equation for total revenue is \( R = 10x \). The fixed costs are $1200, so the total cost for \( x \) units is \( 2.50x + 1200 \). Thus the equation for total cost is \( C = 2.50x + 1200 \). The company will break even when \( R = C \). So we have

\[
10x = 2.50x + 1200 \\
7.5x = 1200 \\
x = 160
\]

Thus the manufacturer will break even if 160 units are produced per month.

17. $60,000  
18. $8.75  
19. \( C = 75x + 1647 \), The \( y \)-intercept, $1647, is the fixed cost and the slope, $75, is the cost per club.

20. (a) \((13, 0)\)  
(b) \((5, 0)\)  
(c) \((5, 0)\)

21. To be given in class.