Review for EXAM 5

Practice Problems

1.) Find the length of the arc subtended by a central angle of 30° on a circle of radius 2 feet. What is the area of the sector?

2.) The minute hand of a clock is 6 inches long. How far does the tip of the minute hand move in 15 minutes?

3.) Find the remaining angle(s) and side(s) of each triangle, if they exist. If no triangle exists, write "No such triangle." Also, if the triangle exists, find its area.
   a.) \( \angle A = 50^\circ, \angle B = 30^\circ, a = 1 \)
   b.) \( a = 2, c = 5, \angle A = 60^\circ \)
   c.) \( a = 3, \angle A = 10^\circ, b = 4 \)

4.) A ship is just offshore of New York City. A sighting is taken of the Statue of Liberty, which is about 305 feet tall. If the angle of elevation to the top of the statue is 20°, how far is the ship from the base of the statue?

5.) A student stands a point \( A \) and measures the angle of elevation to the top of a building as 30°. She then walks straight toward the building to a point \( B \) where the angle to the top of the building is 45°. If the distance she walks is 100 feet, what is the height of the building?

6.) Obtain an equation of the conic described. Graph the equation.
   a.) Parabola; focus at \((-2, 0)\); directrix is the line \( x = 2 \)
   b.) Ellipse; center at \((-1, 2)\); focus at \((0, 2)\); vertex at \((2, 2)\).
   c.) Hyperbola; vertices at \((4, 0)\) and \((4, 4)\); an asymptote is the line \( y + 2x = 10 \).

7.) Identify each conic section and graph it.
   a.) \( y^2 + 4x + 3y - 8 = 0 \)
   b.) \( x^2 + 2y^2 + 4x - 8y + 2 = 0 \)
   c.) \( x^2 - y^2 - 2x - 2y - 1 = 0 \)

8.) Write down the first five terms of each sequence.
   a.) \( \left\{ ( -1)^n \left( \frac{n + 3}{n + 2} \right) \right\} \)
   b.) \( \left\{ (2n + 3) \right\} \)
   c.) \( \left\{ \frac{2^n}{n^2} \right\} \)
   d.) \( a_1 = 4; a_n = - \frac{1}{2} a_{n-1} \)

9.) Determine if the sequence is arithmetic, geometric or neither. If it is arithmetic, find the common difference and the sum of the first \( n \) terms. If it is geometric, find the common ratio and the sum of the first \( n \) terms.
   a.) \( 0, 4, 8, 12, ... \)
   b.) \( 1, -3, -7, -11, ... \)
   c.) \( \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, ... \)
10.) Evaluate each sum.
   a.) \( \sum_{k=1}^{5} (k^2 + 12) \)  b.) \( \sum_{k=1}^{10} (3k - 9) \)  c.) \( \sum_{k=1}^{7} \left( \frac{1}{3} \right)^{k-1} \)  d.) \( \sum_{k=1}^{\infty} 4 \left( \frac{1}{2} \right)^{k-1} \)

11.) Find the 40th term of the sequence 1, -1, -3, -5, ...

12.) Use the Principle of Mathematical Induction to show the given statement is true for all natural numbers.
   a.) \( 3 + 6 + 9 + \cdots + 3n = \frac{3n}{2} (n + 1) \)  b.) \( 2 + 6 + 10 + \cdots + (4n - 2) = 2n^2 \)

13.) Expand each using the binomial theorem.
   a.) \( (x + 2)^5 \)  b.) \( (x - 3)^4 \)  c.) \( (3x - 4)^4 \)

14.) Find the coefficient of \( x^6 \) in the expansion of \( (2x + 1)^8 \).

15.) A child is spinning a rock at the end of a 2-foot rope at the rate of 180 revolutions per minute. Find the linear speed of the rock when it is released.

**SOLUTIONS TO REVIEW FOR EXAM 5**

1.) \( A = \frac{1}{2} r^2 \theta = \frac{1}{2} (2)^2 (\frac{\pi}{5}) = \frac{2\pi}{5} \text{ ft}^2 \)  \( S = r \theta = 2 \left( \frac{\pi}{5} \right) = \frac{2\pi}{5} \text{ ft} \)

2.) \( s = r \theta = (6) \left( \frac{\pi}{5} \right) = 3\pi \text{ inches} \)

3.) a.) \( \angle C = 100^\circ, b \approx .65, c \approx 1.29 \)
   b.) No such triangle
   c.) Two triangles possible: \( \angle B = 13.4^\circ, \angle C = 156.6^\circ, c \approx 6.86 \)
   or \( \angle B = 166.6^\circ, \angle C = 3.4^\circ, c \approx 1.02 \)

4.) 837.98 feet

5.) \( \frac{100}{\sqrt{3}-1} \approx 136.6 \text{ feet} \)

6.) a.) Equation for the parabola is \( y^2 = -8x \)
   b.) Equation for the ellipse is \( \frac{(x + 1)^2}{9} + \frac{(y - 2)^2}{8} = 1 \)
   c.) Equation for the hyperbola is \( \frac{(y - 2)^2}{4} - \frac{(x - 4)^2}{1} = 1 \)

7.) a.) Equation is a parabola: \( (y + \frac{3}{2})^2 = -4(x - \frac{41}{16}) \)
   b.) Equation is an ellipse: \( \frac{(x + 2)^2}{10} + \frac{(y - 2)^2}{5} = 1 \)
   c.) Equation is a hyperbola: \( (x - 1)^2 - (y + 1)^2 = 1 \)

8.) a.) \( -\frac{4}{3}, -\frac{5}{4}, -\frac{6}{7}, -\frac{7}{8} \)
   b.)5, 7, 9, 11, 13
c.) 2, 1, \( \frac{8}{9}, \frac{16}{25} \)  
d.) -2, 1, -\( \frac{1}{2} \), \( \frac{1}{4} \)

9.) a.) Arithmetic sequence. The common difference \( d = 4 \), \( a = 0 \). The \( nth \) term is \( a_n = 0 + (n - 1)(4) = 4n - 4 \). So the sum is \( S_n = \frac{n}{2} (a + a_n) = 2n^2 - 2n \).

b.) Arithmetic sequence. The common difference is \( d = -4 \), \( a = 1 \). The \( nth \) term is \( a_n = 1 + (n - 1)(-4) = -4n + 5 \). So the sum is \( S_n = -2n^2 + 3n \).

c.) Geometric sequence. The common ratio is \( r = \frac{1}{2} \), \( a = 3 \). The sum is \( S_n = \frac{a(1 - r^n)}{1 - r} = \frac{3(1 - \left(\frac{1}{2}\right)^n)}{1 - \frac{1}{2}} = 6 \left(1 - \left(\frac{1}{2}\right)^n\right) \)

10.) a.) 115  b.) 75  c.) \( \frac{1093}{729} \)  d.) 8

11.) \( d = -2 \), \( a_n = 1 + (n - 1)(-2) = -2n + 3 \). So \( a_{40} = -77 \)

12.) a.) Condition I: When \( n = 1 \), \( 3(1) = 3 \) and \( \frac{3}{2}(1 + 1) = 3 \). Thus it is satisfied.
Condition II: Assume it is true for \( n = k \), i.e. \( 3 + 6 + \cdots + 3k = \frac{3}{2}k(k + 1) \).
We will show that it is true for \( n = k + 1 \), i.e. \( 3 + 6 + \cdots + 3k + 3(k + 1) = \frac{3}{2}(k + 1)(k + 2) \).

\[
3 + 6 + \cdots + 3k + 3(k + 1) \\
= \frac{3}{2}k(k + 1) + 3(k + 1) \\
= \frac{3}{2}(k^2 + 3k + 2) \\
= \frac{3}{2}(k + 1)(k + 2)
\]

Thus the statement is true since both conditions are satisfied.

b.) Condition I: When \( n = 1 \), \( 4(1) - 2 = 2 \) and \( 2(1)^2 = 2 \). Thus it is satisfied.
Condition II: Assume it is true for \( n = k \), i.e. \( 2 + 6 + \cdots + (4k - 2) = 2k^2 \).
We will show that it is true for \( n = k + 1 \), i.e. \( 2 + 6 + \cdots + (4k - 2) + (4(k + 1) - 2) = 2(k + 1)^2 \).

\[
2 + 6 + \cdots + (4k - 2) + (4k + 1 - 2) \\
= 2k^2 + (4k + 1 - 2) \\
= 2k^2 + 4k + 2 \\
= 2(k^2 + 2k + 1) = 2(k + 1)^2
\]

Thus the statement is true since both conditions are satisfied.

13.) a.) \( x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32 \)  
b.) \( x^4 - 12x^3 + 54x^2 - 108x + 81 \)  
c.) \( 81x^4 - 432x^3 + 864x^2 - 768x + 256 \)

14.) 1792  15.) \( 720\pi \) \( \text{ft/min} \approx 2262 \text{ft/min} \)