INVERSE FUNCTIONS

1.) Prove algebraically that \( f(x) = \frac{3x - 4}{5 - 6x} \) is one to one.

2.) Let \( g(x) = x^2 - 3 \).
   
   (a) Prove \( g \) is not one to one.

   (b) What is the smallest value for \( a \) such that \( g \) has an inverse on \([a, \infty)\)?

   (c) Now let \( g \) be defined on the interval obtained in part (b). Find the inverse of \( g \) and verify it using composition.
3.) Show that the graph of $f^{-1}$ is the reflection of the graph of $f$ through the line $y = x$ by verifying the following conditions:

i. If $P(a, b)$ is on the graph of $f$, then $Q(b, a)$ is on the graph of $f^{-1}$.

ii. The midpoint of line segment $PQ$ is on the line $y = x$.

iii. The line $PQ$ is perpendicular to the line $y = x$.

4.) What condition(s) need to be placed on constants $a$, $b$ or $c$ so that $f(x) = \frac{ax + b}{cx - a}$ is its own inverse? In other words, $(f \circ f)(x) = x$. 