Dr. Nestler - Math 2 - 3.3 - Complex Zeros of Polynomials

Examples: $x^2 + 16$ is irreducible in the real number system, since

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

Fundamental Theorem of Algebra. Every polynomial function $f(x)$ of degree $n \geq 1$ has at least one complex zero.

Complete Factorization Theorem. Every polynomial function $f(x)$ of degree $n \geq 1$ can be factored completely into linear factors:

$$f(x) = a(x - c_1)\cdots(x - c_n)$$

where $a, c_1, \ldots, c_n$ are complex (possibly real) numbers. So $f$ has exactly $n$ complex zeros, counted with multiplicity, and $f$ can be factored into a product of $n$ linear polynomials.

Examples: $x^2 + 16 = (x - 4i)(x + 4i)$

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

Conjugate Pairs Theorem. Let $f$ be a polynomial function with real coefficients. If $z = a + bi$ is a complex zero of $f$, then $\bar{z} = a - bi$ is also a zero of $f$.

Proof: Suppose $0 = f(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$. Take conjugates:

$$0 = f(\bar{z}) = a_n \bar{z}^n + a_{n-1} \bar{z}^{n-1} + \cdots + a_1 \bar{z} + a_0$$

$$= a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = f(z).$$