If we take \( dx = \Delta x = x - a \) and \( dy = \Delta y = y - b \) in Equation 10, then the differential of \( z \) is
\[
dz = f_x(a, b)(x - a) + f_y(a, b)(y - b)
\]
So, in the notation of differentials, the linear approximation (4) can be written as
\[
f(x, y) \approx f(a, b) + dz
\]
Figure 7 is the three-dimensional counterpart of Figure 6 and shows the geometric interpretation of the differential \( dz \) and the increment \( \Delta z \): \( dz \) represents the change in height of the tangent plane, whereas \( \Delta z \) represents the change in height of the surface \( z = f(x, y) \) when \( (x, y) \) changes from \( (a, b) \) to \( (a + \Delta x, b + \Delta y) \).

**EXAMPLE 4**

(a) If \( z = f(x, y) = x^2 + 3xy - y^2 \), find the differential \( dz \).

(b) If \( x \) changes from 2 to 2.05 and \( y \) changes from 3 to 2.96, compare the values of \( \Delta z \) and \( dz \).

**SOLUTION**

(a) Definition 10 gives
\[
dz = \frac{\partial z}{\partial x} \, dx + \frac{\partial z}{\partial y} \, dy = (2x + 3y) \, dx + (3x - 2y) \, dy
\]

(b) Putting \( x = 2, \, dx = \Delta x = 0.05, \, y = 3, \, \text{and} \, dy = \Delta y = -0.04 \), we get
\[
dz = [2(2) + 3(3)]0.05 + [3(2) - 2(3)](-0.04)
\]
\[
= 0.65
\]
The increment of \( z \) is
\[
\Delta z = f(2.05, 2.96) - f(2, 3)
\]
\[
= [(2.05)^2 + 3(2.05)(2.96) - (2.96)^2] - [2^2 + 3(2)(3) - 3^2]
\]
\[
= 0.6449
\]
Notice that \( \Delta z \approx dz \) but \( dz \) is easier to compute.

**EXAMPLE 5** The base radius and height of a right circular cone are measured as 10 cm and 25 cm, respectively, with a possible error in measurement of as much as 0.1 cm.