The following is an example of how you should present a proof of a limit using the limit definition.

Prove \( \lim_{{x \to 5}} (4x - 1) = 19 : \)

For every \( \varepsilon > 0 \) there is a \( \delta > 0 \) such that if \( 0 < |x - 5| < \delta \) then \( |4x - 1 - 19| < \varepsilon \).

Find \( \delta \), If \( |4x - 1 - 19| < \varepsilon \) then

\[ |4x - 20| < \varepsilon \] then

\[ |4(x - 5)| < \varepsilon \] then

\[ |4||x - 5| < \varepsilon \text{ or } 4|x - 5| < \varepsilon \text{ * you may drop the absolute value around the 4 as we know the value is positive.} \]

then \( |x - 5| < \frac{\varepsilon}{4} \), so if we let \( \delta \leq \frac{\varepsilon}{4} \) then

If \( 0 < |x - 5| < \frac{\varepsilon}{4} \) then

\[ 0 < 4|x - 5| < \varepsilon \] then

\[ 0 < |4(x - 5)| < \varepsilon \] then

\[ 0 < |4x - 1 - 19| < \varepsilon \text{ and the proof is complete.} \]