1. Given \( \triangle ABC \) with \( A = 67^\circ, a = 100, \) and \( c = 125. \)
   a) Determine the number of solutions.
   b) Find all of the remaining parts, if possible.
2. Given \( \triangle ABC \) with \( a = 12.4, b = 8.7, \) and \( B = 36.7^\circ. \)
   a) Determine the number of solutions.
   b) Find all of the remaining parts, if possible.
3. From a point \( P \) on level ground, the angle of elevation of the top of a tower is \( 26^\circ 50' \). From a point 25.0 meters closer to the tower and on the same line with \( P \) and the base of the tower, the angle of elevation of the top is \( 53^\circ 30' \). Approximate the height of the tower.
4. A ship leaves port at 12:00 p.m. and travels in the direction of \( S 35^\circ E \) at the rate of 24 miles per hour. A second ship leaves the same port at 12:30 p.m. and travels \( S 20^\circ W \) at 18 miles per hour. Approximately how far apart are the two ships at 2:00 p.m.?
5. Given \( \triangle ABC \) with \( XY = 180, AX = 100, XC = 220, AY = 240, \) & \( CY = 80. \)
   a) Find \( BX \) and \( BY. \)
   b) Find the height of \( \triangle ABC. \)
6. A pilot on a training flight is to leave an airport and fly for 100 miles in the direction of \( N 40^\circ E \) and then turn around and fly directly back to the airport. On the return flight, after traveling 60 miles, the pilot realizes that he had been traveling in the direction of \( S 25^\circ W. \)
   a) How far is he from the airport?
   b) In what direction must he now fly to return to the airport?

For the following conic sections
7. \( 7x^2 + 7y^2 - 14x + 42y = 42 \)
   a) Name the conic section
   b) Put in standard form
   c) Give the pertinent information for that graph
   d) Sketch the graph
8. \( 2y^2 - 16y - x + 35 = 0 \)
   a) Name the conic section
   b) Put in standard form
   c) Give the pertinent information for that graph
   d) Sketch the graph
9. \( 25y^2 - 4x^2 - 250y + 24x + 489 = 0 \)
   a) Name the conic section
   b) Put in standard form
   c) Give the pertinent information for that graph
   d) Sketch the graph
10. \( 16x^2 + 25y^2 - 64x + 100y - 236 = 0 \)

11. Determine whether the following sequences are arithmetic, geometric, or neither. Explain why.
   a) \( 27, -9, 3, -1, ..., \)
   b) \( \frac{18}{4}, 4, \frac{7}{2}, 3, ..., \)
   c) \( \frac{1}{3}, \frac{1}{6}, \frac{1}{12}, ..., \)
   d) \( \ln 2, \ln 4, \ln 16, \ln 256, ..., \)
12. Give the first five terms of the following recursively defined sequences.
   a) \( a_1 = 3; a_{n+1} = 2a_n + 1 \)
   b) \( a_1 = 1; a_2 = 2; a_{n+2} = -2a_{n+1} + 3a_n \)
   c) \( a_1 = 16; a_{n+1} = \sqrt{a_n} \)
13. Evaluate.
   a) \( \sum_{i=0}^{4} 5i^2 \)
   b) \( \sum_{n=1}^{4} \frac{1}{n(n+1)} \)
   c) \( \sum_{n=1}^{4} (-1)^{n+1} \frac{n}{n+1} \)
   d) \( \sum_{n=0}^{\infty} \left( \frac{1}{7} \right)^n \)
14. Rewrite the following sums using sigma notation (\( \sum \)).
   a) \( 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + ... \)
   b) \( 3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \frac{3}{256} \)
15. An elastic object is dropped from a height of 21 centimeters. It rebounds \( \frac{2}{3} \) of the distance from which it fell previously and continues to rebound in this manner.

a) Find the height of the object after the 3\(^{rd} \) bounce?

b) If this process continues until it comes to rest, determine the total distance this object will have traveled.

16. If the membership of an organization that now has 320 members increases by 50\% each year (and assuming no one drops out), determine its membership five years from now.

17. Prove by math induction: \( (1 \cdot 2) + (3 \cdot 4) + (5 \cdot 6) + \ldots + (2n-1)(2n) = \frac{n(n+1)(4n-1)}{3} \) for all natural numbers \( n \).

18. Consider the expansion of \( (2x-3y)^{12} \).

a) Give the number of terms in its expansion.

b) Determine the coefficient of the 10\(^{th} \) term.

c) Using the Binomial Theorem, expand and simplify the first four terms.

19. If \( f(x) = x^3 - 3x \), use the Binomial Theorem to find \( \frac{f(x+h)-f(x)}{h} \).

20. Using sum formulas and properties of sequences, evaluate \( \sum_{k=1}^{28} (4k^2 + 5k - 8) \).

(Answers)

1. no triangle possible
2. a) 2 sets of triangles  b) \( A_1 = 58.4^\circ, C_1 = 84.9^\circ, c_1 = 14.5; \ A_2 = 121.6^\circ, C_2 = 21.7^\circ, c_2 = 5.4 \)
3. 20.2 meters
4. 39 miles

5. a) \( BX = 242; \ BY = 233 \)  b) 304
6. a) 45 miles  b) S 60\(^{\circ} \) W

7. circle; center (1,-3), radius: 4
8. parabola; vertex (3,4); focus \( \left( 3\frac{1}{2}, 4 \right) \); eq of directrix: \( x = \frac{23}{8} \)

9. hyperbola; center of box (3,5); vertices (3,3) & (3,7); foci \( \left( 3, 5 \pm \sqrt{29} \right) \); eq of asymptotes: \( y - 5 = \pm \frac{3}{2} (x - 3) \)
10. ellipse; center (2,-2); vertices (-3,-2) & (7,-2); foci (-1,-2) & (5,-2)

11. a) geometric; ratio is \( -\frac{1}{2} \)  b) arithmetic; difference is \( \frac{1}{2} \)  c) neither  d) geometric; ratio is 2

12. a) 3,7,15,31,63  b) 1,2,-1,5,-13  c) 16,4,2,\( \sqrt{2}, \sqrt{4/2} \)

13. a) 150  b) \( \frac{4}{9} \)  c) \( -\frac{37}{60} \)  d) \( 4\frac{1}{6} \)

14. a) \( \sum_{n=1}^{\infty} \frac{1}{n^5} \)  b) \( \sum_{n=0}^{4} \left(-1\right)^n 3 \left(\frac{1}{2}\right)^n \)

15. a) \( \frac{56}{9} \) cm  b) 105 cm

16. 2430

18. a) 13 terms  b) \( \left(\frac{12}{9}\right) (2x)^3 (-3y)^9 = -34,642,080 x^3 y^9 \)

c) \( (2x)^{12} + \left(\frac{12}{1}\right) (2x)^{11} (-3y)^1 + \left(\frac{12}{2}\right) (2x)^{10} (-3y)^2 + \left(\frac{12}{3}\right) (2x)^9 (-3y)^3 + \ldots \)
\[= 4096x^{12} - 73,728x^{11} y + 608,256x^{10} y^2 - 304,128x^9 y^3 + \ldots \]

19. \( 5x^4 + 10x^3 h + 10x^2 h^2 + 5xh^3 + h^4 - 3 \)

20. 32,662