Chapter 4 Review

4.1 Mental Math

Compatible numbers are numbers whose sums, differences, products or quotients are easy to calculate mentally. Associate and commutative properties for addition and multiplication and the distributive property enable us to rearrange problems to take advantage of compatible numbers. (see ex. 4.1 on p. 135 and ex. 4.2 on p. 136)

Additive Compensation enables us to restate an addition problem using compatible numbers. This involves increasing one of the addends by a given number and decreasing the other added by the same number.

\[78 + 56 = \overline{78+2} + \overline{56-2}\] creates a revised problem

Example:

\[\begin{align*}
80 & + 54 \\
134 & \\
\end{align*}\]

Equals Addition Method (Compensation for Subtraction) enables us to restate a subtraction problem using compatible numbers. This involves adding the same number to both the subtrahend and the minuend to produce a simpler problem. Note that adding the same number to both the subtrahend and the minuend maintains the difference between them. Usually we would like the subtrahend to end in 0 to simply the problem.

\[92 - 37 = \left(\overline{92+3}\right) - \left(\overline{37+3}\right)\]

Example:

\[\begin{align*}
95 & - 40 \\
55 & \\
\end{align*}\]

Multiplicative Compensation enables us to restate a multiplication problem using compatible numbers. This involves rewriting one or more factors so that a pair of compatible numbers becomes part of the multiplication exercise.

\[24 \times 75 = \left(6 \times 4\right) \times \left(25 \times 3\right)\] factor 24 and 75

\[= 6 \times \left(4 \times 25\right) \times 3\] regroup - mult associativity

Example:

\[= \left(4 \times 25\right) \times 6 \times 3\] rearrange - mult commutativity

\[= 100 \times 18\] simplify

\[= 1800\]

Left-to-Right Methods

Addition: To add 247 + 312 from left to right, add the hundreds (200 + 300), then the tens (40 + 10) and finally the ones (7 + 2). This gives 500 + 50 + 9 = 559.

Addition involving “carrying”: To add 576 + 385, begin by adding the hundreds 500 + 300 = 800. Then add 800 + 70 + 80 = 800 + 150 = 950 and finish by adding 950 + 6 + 5 = 950 + 11 = 961.

Multiplication: To multiply 6 x 492, think of the problem as 6 x (400 + 90 + 2) = 6x400 + 6x90 + 6x2 and calculate from left to right to produce 2400 + 540 which gives 2940; add 12 more to get 2952.

NOTE: You should be able to explain the processes you use when doing “Mental Math”. 
4.1 Estimation
Be able to demonstrate three types of estimation: Range, One-Column/Two Column Front-End and Front-End with Adjustment. (see pp.137 & 138).

4.2 Written Algorithms for Whole-Number Operations
You should be able to demonstrate addition and subtraction using base 10 pieces, a chip abacus, place value representations, intermediate algorithms and the standard addition algorithm. Be sure you can explain how the standard algorithm for addition develops from the intermediate algorithms. Be sure you can explain how carrying (for addition) and borrowing (for subtraction) work. (see pages 149 – 152)

You should be able to demonstrate multiplication using a chip abacus, place value representations, a pictorial representation on a grid, intermediate algorithms and the standard multiplication algorithm. Be sure you can explain how the standard algorithm develops from the intermediate algorithms. Be able to demonstrate how to use the lattice method for multiplication. (see pages 154-155)

You should be able to demonstrate division using base 10 pieces, the scaffold method, intermediate algorithms and the standard division algorithm. Be sure you can explain how the standard algorithm for division develops from the intermediate algorithms. (see pp155-158)

4.3 Perform operations of addition, subtraction, multiplication and division in other number bases.

5.1 a) You should be able to determine if a given number is prime or composite.  
b) If a number is composite, find its prime factorization.  
c) Remember that the following statements all have the same meaning: “a divides b”, “a is a factor of b”, “a is a divisor of b”, “b is a multiple of a”, and “b is divisible by a”. Notation: \( a \mid b \) means “a divides b”.  
d) You should be able to apply tests for divisibility by 2, 5, 10, 4, 8, 3, 9, 11, and 6.

5.2 Counting Factors: To determine the number of factors of any composite number, first write the composite number in prime factored form. The number of factors of the composite number depends on the exponents (powers) of the prime factors. Add 1 to each of the exponents involved and then calculate the product of the “increased” exponents.

Ex. \( 1350 = 2^1 x 3^3 x 5^2 \) so the powers involved are 1, 3, and 2.  
The number of factors of 1350 = \((1 +1)(3 + 1)(2 +1)\) = 2 x 4 x 3 = 24 factors.

You should be able to find the Greatest Common Factor (GCF) and Least Common Multiple (LCM) of two or more numbers using both the set intersection method and the prime factorization method. Be sure you are clear on the difference between the GCF and the LCD of two numbers (see pp. 192 – 198).

6.1 This section defines the concept “fraction” and provides several models for representing fractions. You should understand the terms “numerator”, “denominator”, and “simplified fraction” . You should be able to test whether fractions are equal/unequal using the cross multiplication property (p. 212 and p. 215-216)  
You should be able to find a fraction between any two given fractions using the theorem given on p. 217.

6.2 This section provides models for the addition and subtraction of fractions as well as algorithms for these operations.
Be sure you can illustrate addition and subtraction of fractions with like and unlike denominators using fraction bars or fraction strips (see p. 222 – 23 and p. 226).

The definitions for Addition and Subtraction of Fractions with Common Denominators are given on p. 223 and p.226, respectively. Be sure you explain how to derive the theorems (algorithms) for Addition and Subtraction of Fractions with Unlike Denominators (see pp. 223 and 227).

Be sure to review properties for addition of fractions (see p.224 – 25). These are useful in doing mental math.

6.3 This section provides models for the multiplication and division of fractions as well as algorithms for these operations.

Be sure you can illustrate multiplication of fractions using a rectangular model (see p. 233).

Be sure to review the definition of fraction multiplication (see p. 233) and the properties for multiplication of fractions (see p. 236).

Note the definition for Division of Fractions with Common Denominators (p. 237). It says that

\[ \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} \].

This says that when two fractions with like denominators are divides we can find the answer by simply dividing the numerators!

Be sure to review the theorem for Division of Fractions with Unlike Denominators – Invert the Divisor and Multiply. Note that we can prove this works by using the following logic:

\[ \frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c} \] rewrite as a product of fractions
\[ = \frac{a \cdot d}{b \cdot c} \] divide the numerators (def of Division)
\[ = \frac{a}{b} \times \frac{d}{c} \].

*Work on problems 1 – 6 from Chapter review 6.1 on pp. 249.
*Work on problems 1 – 5 from Chapter review 6.2 on pp. 249.
*Work on problems 1 – 7 from Chapter review 6.3 on pp. 249 – 250.
*Work on problems 1 – 22 from the Chapter Test on pp. 251.