Part I: Problems

Simplify (without the use of calculators)

1. \( \log \sqrt[3]{1000} \)  
2. \( e^{\ln 3} \)  
3. \( \log_4 \left( \frac{8\sqrt{32}}{2} \right) \)  
4. \( \log_4(\log_3 25) \)

5. \( \sum_{i=1}^{20} 4 \)  
6. \( \sum_{i=2}^{6} 3^{i-2} \)  
7. \( (x^2 - 5y)^4 \)

Solve the following equations/inequalities. Check when necessary.

8. \( x^\frac{1}{2} - 5x^\frac{1}{2} - 6 = 0 \)  
9. \( x^2 - 6x \leq -5 \)  
10. \( \sqrt{x+16} - \sqrt{x} = 2 \)

11. \( \log_2 (5 - 3x) - 2 \cdot \log_2 (7 + x) = 1 \)  
12. \( 2^{x^2-4} = 64 \)  
13. \( 2^{x+1} = 3^{2x-3} \)

14. \( \log_4 (x+3) - \log_4 x = 2 \)  
15. \( \frac{x + 5}{x^2 - 7x + 12} \leq 0 \)  
16. \( \begin{cases} \frac{1}{2} x - \frac{1}{6} y = -1 \\ \frac{1}{3} x + \frac{1}{2} y = \frac{33}{2} \end{cases} \)

Graph (on separate axes)

17. \( y = 2|x+3| - 4 \)  
18. \( \begin{cases} x, y \geq 0 \\ y \geq x^2 - 4 \\ y \leq x + 2 \end{cases} \)  
19. \( y = \begin{cases} \frac{x^2 - 4}{2-x}, \text{ if } x \neq 2 \\ 1, \text{ if } x = 2 \end{cases} \)

20. \( y = -\sqrt{x} - 1 \)  
21. \( y = 4 - e^{-x} \)  
22. \( y = 2 \ln(-x) + 1 \)

23. Given that \( f(x) = \sqrt{x+1} \) and \( g(x) = \frac{2}{x+1} \).
   a) Find \((f \circ g)\)  
   b) Determine the domain of \( f, g, \) and \((f \circ g)\).

24. The demand function of a product can be represented by \( p = -2x + 200 \), where \( p \) is the price (in dollars) and \( x \) is the quantity (in units). (assume \( 0 \leq x \leq 100 \))
   a) Give the revenue function.
   b) Determine the quantity that will maximize the revenue.
   c) What is the maximum revenue?
   d) What price gives maximum revenue?

25. For the quadratic function \( f(x) = -3x^2 + x + 2 \), determine the vertex, axis of symmetry, and the x- and y-intercepts.
26. The graph of a fourth degree polynomial function is given below. Write an equation for the function.

\[ f(x) = \frac{(x - 3)(2x^2 + 8x + 6)}{(x - 3)(x^2 - 2x)} \]

27. Using information on asymptotes, intercepts, end behavior, sign analysis, etc., sketch the graph of.

28. Given: \( f(x) = x^3(x + 1)^2(x - 2)(x - 4) \)
   a) Determine the zeros of \( f(x) \).
   b) Determine the sign in each of the intervals.
   c) Using the above information and plotting several crucial points, sketch the graph of \( f(x) \).

29. Given the function \( P(x) = 2x^4 + 3x^3 - 6x^2 - 5x + 6 \).
   a) Using the Theorem on Rational Zeros, list all the possible rational zeros of \( P(x) \).
   b) Use Descartes' Rules of Sign to determine the possibilities for the number of positive, negative, and complex roots.
   c) Using the information obtained from parts a and b, give all roots of the equation \( P(x) = 0 \).

30. Sketch the graph of \( f(x) = \log_2 x \). Then sketch the following on separate axes.
   a) \( f(x) = -\log_2 x \)
   b) \( f(x) = \log_2 (-x) \)
   c) \( f(x) = \log_2 x + 2 \)
   d) \( f(x) = \log_2 (x + 2) \)
   e) \( f(x) = \log_2 |x| \)
   f) \( f(x) = |\log_2 x| \)
   g) \( f(x) = \log_2 |x + 2| \)

31. Express \( \log \left( \frac{x}{y} \right)^3 + 3 \cdot \log y \) as the logarithm of a single term.

32. Express as a sum or difference of logs:

\[ \log \left[ \frac{x^3(x - 5)}{\sqrt[3]{x + 6}} \right] \]

33. An open box with a square base is made from a square piece of cardboard 20 inches on a side by cutting out a square from each corner and turning up the sides.
   a) Express the volume \( V \) of the box as a function of the length \( x \) of the side of the square cut from each corner.
   b) Express the surface area \( S \) of the box as a function of \( x \).
34. (Set up as a linear programming problem)
   You have $40,000 with which to invest in stocks and bonds. You want to invest at least $6,000 and no more
   than $22,000 in stocks. Furthermore, you don't want to invest more than $30,000 in bonds. Currently, the
   earnings on stocks and bonds are 8% and 7 1/2%, respectively.
   a) Determine the amount that should be invested in each in order that earnings are maximized.
   b) What are the maximum earnings?

35. The sixth term of a geometric sequence is 20 and its 10th term is 320. Find the second term.

36. A college graduate earns $39,000 during his first year on the job. Suppose that his salary increases 4% for the
   next five years. Find his total income for those five years.

37. Find the fifteenth term in the expansion of \((x^2 - \frac{1}{x})^{18}\).

38. Consider the expansion of \((x - 2)^{10}\).
   a) Determine the coefficient of the term that contains \(x^4\).
   b) Which term is it?
   c) Give that term.

39. How much should one invest now at 10% compounded monthly to have $5000 in two years?

40. Solve, using matrices:
   \[
   \begin{align*}
   x + 2y - 3z &= -12 \\
   2x - y + 4z &= 1 \\
   5y - z &= 2
   \end{align*}
   \]

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**Answers**

1. \(\frac{3}{5}\)  
2. 81  
3. \(\frac{3}{2}\)  
4. \(\frac{1}{2}\)  
5. 80  
6. 121

7. \(x^8 - 20x^6y + 150x^4y^2 - 500x^2y^3 + 625y^4\)  
8. \(-1\) and 216  
9. \(1 \leq x \leq 5\) or \([1,5]\)

10. 9  
11. -5  
12. \(\pm\sqrt{11}\)  
13. \(\frac{3\log 3 + \log 2}{2\log 3 - \log 2}\)  
14. 1 and 9  
15. \((-\infty, -5) \cup (3, 4)\)

16. \((\frac{3}{5}, 8)\)

17. [Graph of a line]
18. [Graph of a parabola]
19. [Graph of a line]
20. \[ \sqrt{\frac{x+3}{x+1}} \]

21. \[ \sqrt{2} \]

22. \[ (-e, 3) \]

23. a) \[ f(x) = \frac{x+3}{x+1} \]
   b) \[ D: f = [-1, \infty) \]
   c) \[ g(x) = -2x^2 + 200x \]
   d) \[ (-\infty, -1) \cup (-1, \infty) \]
   e) \[ (f \circ g)(x) = (-\infty, -3) \cup (-1, \infty) \]

24. a) \[ R(x) = -2x^2 + 200x \]
   b) 50 units
   c) $5000$
   d) $100$

25. vertex at \( \left( \frac{1}{4}, \frac{25}{16} \right) \); axis of symmetry at \( x = \frac{1}{4} \); x-intercepts at \( (-\frac{5}{2}, 0) \) and \( (0, 0) \); y-intercept at \( (0, 2) \)

26. \[ y = (x + 3)(x)(x - 4)^2 \]

27. \[ (-\frac{1}{2}, 2) \]

28. a) \[ -1, 0, 2, 4 \]
   b) \[ -1, 0, 2, 4 \]
   c) \[ (1, 12) \]
   d) \[ (3, 432) \]

29. \[ -2, -\frac{3}{2}, 1 \]

30. \[ y = \log_{\sqrt{2}} \]
   a) \[ y = -\log_2 x \]
   b) \[ y = \log_2 (-x) \]
   c) \[ y = \log_2 x + 2 \]
   d) \[ y = \log_2 (x + 2) \]
   e) \[ y = \log_2 |x| \]
   f) \[ y = |\log_2 x| \]
g) \( y = \log_2 |x + 2| \)

31. \( \log \left( \frac{1}{e} \right) \)
32. \( 2 \log x + \log(x - 5) - \frac{1}{2} \log(x + 6) \)
33. a) \( V(x) = 4x(10 - x)^2 \)    b) \( S(x) = 4(10 - x)(10 + x) \)
34. a) $22,000 in stocks; $18,000 in bonds    b) $3,110
35. \( \frac{5}{4} \)
36. Approx. $211,237
37. \( \frac{3060}{x^8} \)
38. a) 210    b) 7th term    c) 13,440x^4
39. $4097.05
40. \((-5,1,3)\)

Part II: Multiple Choice

1. If \( f(x) = x^2 + 7 \), then \( \frac{f(x + h) - f(x)}{h} = \)
   (A) \( x + h + 7 \)    (B) \( (x + h)^2 + 7x \)    (C) \( x + 2h + 7 \)    (D) \( 2x + h \)    (E) none of these

2. The domain of \( f(x) = \frac{\sqrt{x + 9}}{x - 6} \) is
   (A) \( (-\infty, 6) \cup (6, +\infty) \)    (B) \( [-9, 6) \cup (6, +\infty) \)    (C) \( (-\infty, -9) \cup (6, +\infty) \)
   (D) \( (-\infty, +\infty) \)    (E) none of these

3. A product has a total cost function \( C(x) = 100x + 1250 \) and total revenue function \( R(x) = 250x \). The equation for its profit function \( P(x) \) is
   (A) \( P(x) = -150x + 1250 \)    (B) \( P(x) = 150x - 1250 \)    (C) \( P(x) = 150x \)
   (D) \( P(x) = 350x + 1250 \)    (E) none of these

4. If \( f(x) = \frac{x + 1}{x + 2} \) and \( g(x) = \frac{1}{x - 3} \), then \( (f \circ g)(x) = \)
   (A) \( \frac{2x + 5}{x - 3} \)    (B) \( \frac{x + 1}{x - 1} \)    (C) \( \frac{x - 2}{2x - 5} \)
   (D) \( \frac{x - 2}{2x + 5} \)    (E) none of these

5. The domain of \( (f \circ g)(x) \) in the above problem is
   (A) all reals    (B) \( (-\infty, \frac{3}{2}) \cup (\frac{3}{2}, 3) \cup (3, +\infty) \)    (C) \( (-\infty, -2) \cup (-2, 3) \cup (3, +\infty) \)
   (D) \( (-\infty, -1) \cup (-1, \frac{3}{2}) \cup (\frac{3}{2}, +\infty) \)    (E) none of these

6. If \( g(x) = \sqrt{100 - x^2} \) and \( -10 \leq x \leq 0 \), then \( g^{-1}(x) = \)
   (A) \( -\sqrt{10 - x} \)    (B) \( \sqrt{10 - x} \)    (C) \( \sqrt{100 - x^2} \)
   (D) \( -\sqrt{100 - x^2} \)    (E) none of these
7. Which of the following are asymptotes of \( f(x) = \frac{x+10}{x^2-5x-6} \)?
   I. \( x = -1 \)    
   II. \( x = 6 \)    
   III. \( x = 2 \)    
   IV. \( x = -3 \)    
   V. \( y = 0 \)
   (A) I & II only    
   (B) III & IV only    
   (C) I, II, & V    
   (D) III, IV, & V    
   (E) none of these

8. The oblique (slant) asymptote, if any exist, for the function \( f(x) = \frac{4x^4-2x^3-7}{x^3} \) is
   (A) \( y = 4x - 2 \)    
   (B) \( y = -2x - 7 \)    
   (C) \( y = 4 \)    
   (D) none exist    
   (E) none of these

9. If \( x - 2 \) is a factor of \( x^2 + kx^2 + 12x - 8 \), then \( k = \)
   (A) -6    
   (B) -3    
   (C) 2    
   (D) 6    
   (E) none of these

10. The maximum number of positive real zeros of \( P(x) = 2x^4 + 2x^3 + x^2 + 10 \) is
   (A) 3    
   (B) 2    
   (C) 1    
   (D) 0    
   (E) none of these

11. By the Theorem on Rational Zeros, the possible rational zeros of \( P(x) = 2x^3 - 3x^2 + 7x + 5 \) are
   (A) \( \pm 1, \pm \frac{1}{2}, \pm 1, \pm 5 \)    
   (B) \( \pm 1, \pm \frac{1}{2}, \pm 1, \pm 2 \)    
   (C) \( \pm 1, \pm \frac{1}{2}, \pm 1 \)    
   (D) \( \pm 1, \pm \frac{1}{2}, \pm 1, \pm 5 \)    
   (E) none of these

12. Using synthetic division, the quotient and the remainder when \( x^4 - 7x^2 + 2x - 6 \) is divided by \( (x+3) \) is
   (A) \( x^3 - 10x^2 + 32x; \ -102 \)    
   (B) \( x^3 - 4x^2 - 10x; \ -36 \)    
   (C) \( x^3 + 3x^2 - 2x + 8; \ 18 \)    
   (D) \( x^3 - 3x^2 + 2x - 4; \ 6 \)    
   (E) none of these

13. A product with a profit function given by \( P(x) = -4x^2 + 400x - 3600 \) will have maximum profit at
   (A) 20 units    
   (B) 30 units    
   (C) 40 units    
   (D) 50 units    
   (E) none of these

14. The maximum profit of the above problem is
   (A) $2,000    
   (B) $4,300    
   (C) $6,400    
   (D) $10,400    
   (E) none of these

15. Which of the following functions have inverses, which are not functions?
   I. \( y = x + 1 \)    
   II. \( y = x^2 + 3 \)    
   III. \( y = \left| x + 1 \right| \)
   (A) I only    
   (B) II only    
   (C) III only    
   (D) I & II    
   (E) II & III

16. Which of the graphs of the following functions have symmetry with respect to the \( y \)-axis?
   I. \( y = 2^{|x|} \)    
   II. \( y = x + 2 \)    
   III. \( y = x^2 + x \)    
   IV. \( y = x^2 \)
   (A) I only    
   (B) I & III    
   (C) II only    
   (D) I & IV    
   (E) none of these

17. If \( \frac{1}{3} \log_3(x-2) = 2 \log_3 2 - \frac{1}{3} \log_3 (x-2) \), then \( x = \)
   (A) 0    
   (B) 4    
   (C) 0 and 4    
   (D) 0 and -4    
   (E) none of these
18. The domain of \( \log_{\sqrt{x}}(x^2 - 2x - 15) \) is
   (A) \((-\infty, -3) \cup (5, +\infty)\)  \hspace{1cm} (B) \((-\infty, -3) \cup [5, +\infty)\)  \hspace{1cm} (C) \((-\infty, -5) \cup (3, +\infty)\)
   (D) all reals  \hspace{1cm} (E) none of these

19. Which of the following are intercepts of \( y = \ln \left( \frac{x-1}{2x-3} \right) \)?
   I. \((2, 0)\)  \hspace{1cm} II. \((0, \frac{1}{2})\)  \hspace{1cm} III. \((0, 2)\)  \hspace{1cm} IV. \((0, \ln \frac{1}{3})\)
   (A) I only  \hspace{1cm} (B) I and II  \hspace{1cm} (C) I and IV  \hspace{1cm} (D) III only  \hspace{1cm} (E) IV only

20. Which one of the following expressions is equal to 0?
   (A) \(\ln\left(\frac{1}{2}\right) + \ln\left(\frac{1}{4}\right)\)  \hspace{1cm} (B) \(\ln\left(\frac{1}{2}\right) + \ln\left(\frac{1}{4}\right)\)
   (C) \(\ln\left(\frac{1}{2}\right) - \ln\left(\frac{1}{4}\right)\)  \hspace{1cm} (D) \(\ln(2e) - \ln 2\)
   (E) none of these

21. In sigma (\(\sum\)) notation, the sum \(\frac{2}{5} + \frac{5}{6} + \frac{10}{9} + \frac{17}{12}\) can be written as
   (A) \(\sum_{k=1}^{4} \frac{2k+1}{3k}\)  \hspace{1cm} (B) \(\sum_{k=1}^{4} \frac{2k^2-1}{3k}\)
   (C) \(\sum_{k=1}^{4} \frac{2k+1}{3k}\)  \hspace{1cm} (D) \(\sum_{k=1}^{4} \frac{k^2+k-1}{3k}\)
   (E) none of these

22. In sigma (\(\sum\)) notation, the sum \(2 - 2^2 - 2^3 + ... + 2^{15}\) can be written as
   (A) \(\sum_{k=1}^{15} (-1)^k \cdot 2^k\)  \hspace{1cm} (B) \(\sum_{k=1}^{15} (-1)^{k+1} \cdot 2^k\)
   (C) \(\sum_{k=1}^{15} (-1)^{k+1} \cdot 2^{k+1}\)  \hspace{1cm} (D) \(\sum_{k=1}^{15} (-1)^{k+1} \cdot 2^{k+1}\)
   (F) none of these

23. \(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... = \)
   (A) \(\frac{3}{2}\)  \hspace{1cm} (B) \(\frac{2}{3}\)  \hspace{1cm} (C) \(\frac{40}{27}\)  \hspace{1cm} (D) \(\frac{52}{39}\)  \hspace{1cm} (E) none of these

24. In simplest form, \(\sum_{k=1}^{5} \log \left( \frac{k}{k+1} \right) = \)
   (A) \(\log \frac{1}{2} + \log \frac{1}{3}\)  \hspace{1cm} (B) \(-\log 5\)  \hspace{1cm} (C) \(-\log 6\)  \hspace{1cm} (D) \(\log 5\)  \hspace{1cm} (E) none of these

25. The number of years it would take to triple the amount of principle invested at 8.8% compounded continuously is
   (A) 3.5 years  \hspace{1cm} (B) 5.4 years  \hspace{1cm} (C) 9.7 years  \hspace{1cm} (D) 12.5 years  \hspace{1cm} (E) none of these

26. The constraints of a linear programming problem are given by \(x, y \geq 0\), \(x + 3y \leq 20\), and \(5x + 2y \leq 35\). The corner points of its feasibility (shaded) region are
   (A) \((0, 0), (0, 6 \frac{2}{3}), \) and \((7, 0)\)  \hspace{1cm} (B) \((5, 5), (7, 0), \) and \((20, 0)\)
   (C) \((0, 0), (5, 5), \) and \((7, 0)\)  \hspace{1cm} (D) \((0, 0), (0, 6 \frac{2}{3}), (5, 5), \) and \((7, 0)\)
   (E) none of these
27. If \( A = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \) and \( B = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} \), then \( A \cdot B = \)

\( \text{(A) } \begin{bmatrix} 2 & -1 \\ 11 & 18 \end{bmatrix} \quad \text{(B) } \begin{bmatrix} -8 & 7 \\ -5 & -10 \end{bmatrix} \quad \text{(C) } \begin{bmatrix} 0 & 1 \\ 11 & -18 \end{bmatrix} \quad \text{(D) none of these} \)

28. The twenty-seventh term of the arithmetic sequence \( 2x - 3y, 0, -2x + 3y, \ldots \) is

\( \text{(A) } -52x + 78y \quad \text{(B) } -50x + 75y \quad \text{(C) } -50x - 84y \quad \text{(D) } 54x + 81y \quad \text{(E) none of these} \)

29. The recursive definition of a sequence is given by \( a_1 = -1 \) and \( a_n = 2a_{n-1} + 3 \). Its fourth term is

\( \text{(A) } 1 \quad \text{(B) } 5 \quad \text{(C) } 13 \quad \text{(D) } 29 \quad \text{(E) none of these} \)

30. The coefficient of the middle term in the expansion of \((3x^2 - 2y)^{50}\) can be found by

\( \text{(A) } \binom{50}{25} \quad \text{(B) } \binom{50}{25}(3x^2)(-2y) \quad \text{(C) } \binom{50}{25}(3x^2)^{25}(2y)^{25} \quad \text{(D) } \binom{50}{25}(3x^2)^{25}(-2y)^{25} \quad \text{(E) none of these} \)

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(Answers)

1. D  
2. B  
3. B  
4. C  
5. B  
6. D  
7. C  
8. A  
9. A  
10. D  
11. A  
12. D  
13. D  
14. C  
15. F  
16. D  
17. B  
18. A  
19. C  
20. B  
21. D  
22. B  
23. A  
24. C  
25. D  
26. D  
27. C  
28. B  
29. C  
30. D