Show your work or at least the set up for each problem. Correct answers without supporting work will receive minimal credit.

1. [6 points] Suppose \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \), \( A = \{1, 4, 7, 10, 11, 12\} \), and \( B = \{3, 5, 7, 9, 11\} \)

   a) Find: \( B' = \{1, 2, 4, 6, 8, 10, 12\} \)

   b) Find \( A \cup B' = \{1, 4, 7, 10, 11, 12\} \cup \{1, 2, 4, 6, 8, 10, 12\} = \{1, 2, 4, 6, 7, 8, 10, 11, 12\} \)

   c) Find \( (A \cap B)' = \left(\{1, 4, 7, 10, 11, 12\} \cap \{3, 5, 7, 9, 11\}\right)' = \{7, 11\}' = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12\} \)

2. [4 points] Indicate whether each of the following statements is true (T) or false (F).
   
   a) \( \{a, b, c\} = \{c, a, b\} \)

   b) \( \{c\} \subseteq \{a, b, c\} \)

   c) \( \{b, a\} \subseteq \{c, a, b\} \)

   d) \( \emptyset \subseteq \{a, b, c\} \)

   2a) \( T \)

   2b) \( F \)

   2c) \( T \)

   2d) \( T \)

3. [6 points] Let \( U \) denote the set of all employees at the Universal Life Insurance Company and let \( T = \{x \in U \mid x \text{ drinks tea}\} \) and \( C = \{x \in U \mid x \text{ drinks coffee}\} \). Describe in words the sets

   a) \( T' \) set of employees who don't drink tea

   b) \( T \cup C \) set of employees who drink coffee or tea or both

   c) \( T \cap C' \) set of employees who drink tea but not coffee
4. [6 points] If $A$ and $B$ are sets with $n(A) = 65$, $n(B) = 54$, $n(A \cup B) = 100$, find $n(A \cap B)$.

\[
\begin{align*}
\quad n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\
\quad 100 &= 65 + 54 - n(A \cap B) \\
\quad 100 &= 119 - n(A \cap B) \\
\quad 19 &= n(A \cap B)
\end{align*}
\]

5. [6 points] A secretary and a treasurer of a club are to be selected from Alice, Bill, Kesha, and Dave. Draw a tree diagram showing the different ways these two selections can be made.
6. [8 points] A survey of 100 students who spend time in the magazine section of the SMC Library revealed the following results:
   - 40 read *Time* magazine,
   - 30 read *Newsweek*,
   - 25 read *U.S. News and World Report*,
   - 15 read *Time* and *Newsweek*,
   - 12 read *Time* and *U.S. News and World Report*,
   - 10 read *Newsweek* and *U.S. News and World Report*, and
   - 4 read all 3 magazines.

Create a Venn Diagram to represent this information and then use your Venn Diagram to answer the questions below.

How many of the students surveyed read
a) none of these magazines? 38
b) *Time* but not the other two magazines? 17
c) *Newsweek* or *U.S. News and World Report*? 45
d) exactly two of these magazines? 25

\[ T = \text{readers of Time} \]
\[ N = \text{readers of Newsweek} \]
\[ U = \text{readers of US News and World} \]
7. [6 points] How many three-digit numbers can be formed using the digits \(\{2, 3, 4, 5\}\)
   a) if a digit may be repeated?
   \[4 \times 4 \times 4 = 64\]
   
b) if a digit may not be repeated?
   \[4 \times 3 \times 2 = 24\]

8. [6 points] Demonstrate how to calculate the following values using factorials and the definitions of
   \(P(n,k)\) and \(C(n,k)\). Then express your final answer as a product of integers. DO NOT EVALUATE.
   a) \(P(35, 6) = \frac{35!}{29!} = 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30\)
   
   b) \(C(55, 52) = \frac{55!}{(52-3)!} \cdot \frac{(52-3)!}{3!} = \frac{55 \cdot 54 \cdot 53 \cdot 52}{2 \cdot 1} = 55 \cdot 54 \cdot 53 \cdot 52 \cdot 3 \cdot 2 \cdot 1\)

9. [6 points] A quiz has 6 true-false questions followed by 4 multiple choice questions, each with 5 possible answers. In how many different ways can the answers to the 10 questions be given?

\[
\begin{align*}
6 \ T/F & \quad \quad \quad 4 \ M/C \\
6 \times 2 \times 2 \times 2 \times 2 \times 2 & \quad (6 \times 5 \times 5 \times 5) = 40,000
\end{align*}
\]
10. [6 points] The local Chamber of Commerce has 25 men and 32 women members. In how many ways can a Finance Committee consisting of a chair, a secretary and 4 members at large be selected?

\[ 25 + 32 = 57 \text{ members} \]

\[ P(57, 2) \cdot C(55, 4) = (3192)(341055) \]

\[ \uparrow \quad \uparrow \quad \uparrow \]

\[ = 1085547560 \]

\# ways to select 4 committee members from remaining 55 members.

\# ways to select chair & secretary from 57 members.

11. [6 points] How many different numbers can be formed by using all the digits of the number 212231?

\[
\begin{align*}
    2 & \, 1 & \, 3 \\
    2 & \, 1 & \\
    2 & \\
\end{align*}
\]

\[
\begin{align*}
\frac{6!}{3!2!} & = \frac{720}{6 \cdot 2} \\
& = 60
\end{align*}
\]
12. [6 points] The art department plans to display an arrangement of 7 pieces of student art work. The pieces for display will be selected from a collection of 8 oil paintings, 6 watercolors and 10 photos.

a) How many arrangements are possible with 3 oil paintings in the center, 2 watercolors on the left and 2 photos on the right?

\[ P(8,3) \cdot P(6,2) = 30 \cdot 336 \cdot 40 = 907,200 \]

b) How many arrangements are possible if no more than 2 of the pieces displayed are photos?

\[ P(10, 2) \cdot P(14, 5) + P(10, 1) \cdot P(14, 4) + P(10, 0) \cdot P(14, 7) = 90 \cdot 240 \cdot 240 + 10 \cdot 2162 \cdot 80 + 1 \cdot 17297 \cdot 280 = 69,540,480 \]

13. [6 points] An advertising agency assigns seven employees to work on an advertising campaign, two to work on copy, two to work on art, and three to work on production. How many ways can the assignment be made?

\[ \binom{7}{2} \cdot \binom{5}{2} \cdot \binom{3}{2} = \frac{7!}{2! \cdot 5!} \cdot \frac{5!}{3! \cdot 2!} \cdot \frac{3!}{0! \cdot 3!} = \frac{7!}{2! \cdot 3! \cdot 2!} \cdot \frac{5!}{2! \cdot 6!} = 210. \]