1. Suppose the $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 3, 5, 7\}$ and $B = \{1, 3, 5, 7, 9\}$.
   a) Find $B' = \{\text{set of elements in } U \text{ but not in } B\}$
      $B' = \{0, 1, 4, 6, 8\}$
   b) Find $A \cap B = \{\text{set of elements that belong to both } A \text{ and } B\}$
      $A \cap B = \{2, 3, 5, 7\} \cap \{1, 3, 5, 7, 9\}$
      $= \{3, 5, 7\}$
   c) True or False: $2 \in A \cup B$. Explain how you know. $A \cup B = \{\text{set of elements in } A \text{ or in } B \text{ or in both}\}$
      Since $2 \in A$, $2 \in A \cup B$.
   d) Find a set $C$ such that $C \subseteq A$. $C$ could be any set consisting of elements of $A$ but not all of $A$. Some examples:
      $\{2, 3\}$, $\{2, 3, 5\}$, $\{3\}$, - there are others!
2. Among the first 41 presidents of the United States, the following facts are known.
   8 held cabinet posts.
   14 served as vice-president.
   15 served in the United States Senate.
   2 served in cabinet posts and as vice-president.
   4 served in cabinet posts and in the United States Senate.
   6 served in the United States Senate and as vice-president.
   1 served in all three of these positions.

Create a Venn diagram to represent this information. Then answer the following questions.

a) How many U.S. presidents served in none of the three positions mentioned? 16

b) How many U.S. presidents served only in the United States Senate? 6

Look at $n(C'V'S)$

c) How many U.S. presidents served in at least 2 of these positions? 10

This means those who served in 2 or 3 of these positions. Add $1 + 3 + 5 + 1 = 10$

d) How many U.S. presidents served in exactly one of these positions? 16

Look at $n(C'V'S') + n(CV'S') + n(C'V'S) = 6 + 7 + 3$

Let $C$ = set who held cabinet post
$V$ = set who served as vice-president
$S = \text{set who served in US Senate}$

See next page for detailed solution.
One president held all 3 positions so
\[ n(CV\cap CS) = 1 \]

6 presidents served in the US Senate and as vice president, so \( n(SV\cup V) = 6 \). Note that one of the 6 also served in a cabinet post. The remaining 5 go in the region labelled \( CV\cap S \).

Similarly, \( n(CV\cap S') = 2 - 1 = 1 \) and \( n(C\cap V) = 4 - 1 = 3 \)

We know that 15 presidents served in the US Senate, so that total in the S circle is 15. Calculate number who were senators only by subtracting 15 - (3 + 1 + 3) = 6

Likewise, find the number who were vice-pres only by subtracting what's already in the V circle from 14 and those who held cabinet posts only by subtracting the total in the C circle from 8

Finally, determine the number who held none of the posts by subtracting the total inside the Venn diagram from the number of presidents:
\[ 41 - (3 + 1 + 7 + 3 + 1 + 5 + 6) = 15 \]