section 6.5  Combinations

Notation: \( P(n, r) = \# \) of ways to arrange \( r \) distinct items in order.

\[ C(n, r) = \# \) of ways to select \( r \) distinct items without order. \]

Thm  \( P(n, r) = (n) (n-1)(n-2)(n-3)\ldots (n-(r-1)) \)  
Def  \( n! = (n) (n-1)(n-2)(n-3)\ldots(2)(1) \) (where \( n \in \mathbb{N} \))

Thm  \[ C(n, r) = \frac{n!}{r!(n-r)!} = \frac{(n-l)(n-2)(n-3)\ldots(n-r+1)}{r!} \]  (where \( n, r \in \mathbb{N}, \ r \leq n \))

Ex1,  An SMC Club has 6 students. Find the number of ways to select  
(a) 1 President and 1 Vice President. \( P(6, 2) \)
(b) 2 representatives. \( C(6, 2) \)
(c) 1 President, 1 VP, and 1 Treasurer. \( P(6, 3) \)
(d) 3 representatives. \( C(6, 3) \)

Ex2,  At a restaurant, there are 5 veg, 4 fruits, and 3 meats.  
How many ways to select  
(a) any 2 (distinct) items

(b) any 2 items from fruits or meats.

(c) 3 items, 1 veg, 1 fruit, 1 meat.

(d) 4 items, 2 veg, 1 fruit, and 1 meat.

(e) 4 items, any 2 items from veg or fruits and 2 meats.
Ex3, Ice cream store is 4 blocks east and 3 blocks north of your house. Assuming that there are streets at every block, how many ways are there from your house to the ice cream store?

Pascal's △

Binomial Theorem

\[(A+B)^n = \binom{n}{0}A^n + \binom{n}{1}A^{n-1}B + \binom{n}{2}A^{n-2}B^2 + \binom{n}{3}A^{n-3}B^3 + \ldots + \binom{n}{n-1}AB^{n-1} + \binom{n}{n}B^n\]

Ex4, Expand \((2x-3y)^5\)

Ex5, Find the coefficient of \(x^7y^3\) in the expansion of \((x-2y)^{10}\)