

1.1 EXERCISE SET

Practice Exercises
In Exercises 1–14, evaluate each expression for \( x = 4 \).

1. \( x + 8 \) 12
2. \( x + 10 \) 14
3. \( 12 - x \) 8
4. \( 16 - x \) 12
5. \( 5x \) 20
6. \( 6x \) 24
7. \( \frac{28}{x} \) 7
8. \( \frac{36}{x} \) 9
9. \( 5 + 3x \) 17
10. \( 3 + 5x \) 23
11. \( 2(x + 5) \) 18
12. \( 5(x + 3) \) 35
13. \( \frac{12x - 8}{2x} \) 5
14. \( \frac{5x + 52}{3x} \) 6

In Exercises 15–24, evaluate each expression for \( x = 7 \) and \( y = 5 \).

15. \( 2x + y \) 19
16. \( 3x + y \) 26
17. \( 2(x + y) \) 24
18. \( 3(x + y) \) 36
19. \( 4x - 3y \) 13
20. \( 5x - 4y \) 15
21. \( \frac{21}{x} + \frac{55}{y} \) 10
22. \( \frac{50}{y} - \frac{14}{x} \) 8
23. \( \frac{2x - y + 6}{2y - x} \) 5
24. \( \frac{2y - x + 24}{2x - y} \) 3

In Exercises 25–42, write each English phrase as an algebraic expression. Let the variable \( x \) represent the number.

25. four more than a number \( x + 4 \)
26. six more than a number \( x + 6 \)
27. four less than a number \( x - 4 \)
28. six less than a number \( x - 6 \)
29. the sum of a number and 4 \( x + 4 \)
30. the sum of a number and 6 \( x + 6 \)
31. nine subtracted from a number \( x - 9 \)
32. three subtracted from a number \( x - 3 \)
33. nine decreased by a number \( 9 - x \)
34. three decreased by a number \( 3 - x \)
35. three times a number, decreased by 5 \( 3x - 5 \)
36. five times a number, decreased by 3 \( 5x - 3 \)
37. one less than the product of 12 and a number \( 12x - 1 \)
38. three less than the product of 13 and a number \( 13x - 3 \)
39. the sum of 10 divided by a number and that number divided by 10 \( \frac{10}{x} + \frac{x}{10} \)
40. the sum of 20 divided by a number and that number divided by 20 \( \frac{20}{x} + \frac{x}{20} \)
41. six more than the quotient of a number and 30 \( \frac{x}{30} + 6 \)
42. four more than the quotient of 30 and a number \( \frac{30}{x} + 4 \)

In Exercises 43–58, determine whether the given number is a solution of the equation.

43. \( x + 14 = 20 \) solution
44. \( x + 17 = 22 \) solution
45. \( 30 - y = 10 \) solution
46. \( 50 - y = 20 \) solution
47. \( 4x = 20; 10 \) not a solution
48. $5x = 30; 8$ not a solution
49. $\frac{r}{4} = 8; 48$ solution
50. $\frac{r}{9} = 7; 63$ solution
51. $4m + 3 = 23; 6$ not a solution
52. $3m + 4 = 19; 6$ not a solution
53. $5a - 4 = 2a + 5; 3$ solution
54. $5a - 3 = 2a + 6; 3$ solution
55. $6(p - 4) = 3p; 8$ solution
56. $4(p + 3) = 6p; 6$ solution
57. $2(w + 1) = 3(w - 1); 7$ not a solution
58. $3(w + 2) = 4(w - 3); 10$ not a solution

In Exercises 59–74, write each sentence as an equation. Let the variable $x$ represent the number.

59. Four times a number is 28. $4x = 28$
60. Five times a number is 35. $5x = 35$
61. The quotient of 14 and a number is $\frac{1}{2}$. $\frac{14}{x} = \frac{1}{2}$
62. The quotient of a number and 8 is $\frac{1}{4}$. $\frac{x}{8} = \frac{1}{4}$
63. The difference between 20 and a number is 5. $20 - x = 5$
64. The difference between 40 and a number is 10. $40 - x = 10$
65. The sum of twice a number and 6 is 16. $2x + 6 = 16$
66. The sum of twice a number and 9 is 29. $2x + 9 = 29$
67. Five less than 3 times a number gives 7. $3x - 5 = 7$
68. Three less than 4 times a number gives 29. $4x - 3 = 29$
69. The product of 4 and a number, increased by 5, is 33. $4x + 5 = 33$
70. The product of 6 and a number, increased by 3, is 33. $6x + 3 = 33$
71. The product of 4 and a number increased by 5 is 33. $4(x + 5) = 33$
72. The product of 6 and a number increased by 3 is 33. $6(x + 3) = 33$
73. Five times a number is equal to 24 decreased by the number. $5x = 24 - x$
74. Four times a number is equal to 25 decreased by the number. $4x = 25 - x$

Practice PLUS

75. Evaluate $\frac{x - y}{4}$ when $x$ is 2 more than 7 times $y$ and $y = 5$. 8
76. Evaluate $\frac{x - y}{3}$ when $x$ is 2 more than 5 times $y$ and $y = 4$. 6
77. Evaluate $4x + 3(y + 5)$ when $x$ is 1 less than the quotient of $y$ and 4 and $y = 12$. 39
78. Evaluate $3x + 4(y + 6)$ when $x$ is 1 less than the quotient of $y$ and 3, and $y = 15$. 96
79. Evaluate $2(x + 3y)$ for $x = 4$ and $y = 1$. 14
   a. Is the number you obtained in part (a) a solution of $5z - 30 = 40$? yes
   b. Is the number you obtained in part (a) a solution of $4z - 30 = 54$? yes
80. a. Evaluate $3(2x + y)$ for $x = 3$ and $y = 6$. 6
   b. Is the number you obtained in part (a) a solution of $3w = 45 - 2w$? no
81. a. Evaluate $6x - 2y$ for $x = 3$ and $y = 6$. 6
   b. Is the number you obtained in part (a) a solution of $7w = 45 - 2w$? no

82. a. Evaluate $5x - 14y$ for $x = 3$ and $y = 1$. 8
   b. Is the number you obtained in part (a) a solution of $4w = 54 - 5w$? no

Application Exercises

In 2006, 60 years after Jackie Robinson broke the sports color barrier, only 8% of Major League Baseball players were African American. The bar graph shows the decline in the percentage of African-American Major League Baseball players from 1997 through 2006.

![Bar Graph](image)

Source: University of Central Florida's Institute for Diversity and Ethics in Sports (Data for 2005 unavailable.)

Here is a mathematical model that approximates the data displayed by the bar graph:

$$p = 16 - n$$

Here is a mathematical model that approximates the data displayed by the bar graph:

Use this formula to solve Exercises 83–84.

83. a. Use the formula to find the percentage of African-American players 5 years after 1997, or in 2002. Does the mathematical model underestimate or overestimate the actual percent shown by the bar graph for 2002? By how much? 11%; overestimates by 1%
b. Use the formula to find the percentage of African-American players in 2004. How well does the model describe the percent in the bar graph for this year? 9%; perfectly well: It gives the percent displayed for 2004.

84. a. Use the formula to find the percentage of African-American players 8 years after 1997, or in 2005. Does the mathematical model underestimate or overestimate the actual percent shown by the bar graph for 2005? By how much? 8%; underestimates by 1%
b. Use the formula to find the percentage of African-American players in 2000. How well does the model describe the percent in the bar graph for this year? 13%; perfectly well: It gives the percent displayed for 2000.
Although lack of sleep affects the way you feel and perform, fewer Americans are getting enough sleep. The graph compares hours of sleep in 1998 and 2005. For each year, it shows the number of hours of sleep per night on weekdays and the percentage of U.S. adults sleeping for this number of hours.

![Graph showing number of hours of sleep per night on weekdays for U.S. adults in 1998 and 2005.](image)

Source: National Sleep Foundation poll of 1506 adults (percent do not add up to 100% due to rounding.)

Here are two mathematical models that approximate the data displayed by the graph:

1998: \( p = 7.4h + 6 \)

2005: \( p = 3.7h + 15 \)

A bowler’s handicap, \( H \), is often found using the following formula:

\[
H = 0.8(200 - A).
\]

A bowler’s final score for a game is the score for that game increased by the handicap.

Use this information to solve Exercises 87–88.

87. a. If your average bowling score is 145, what is your handicap? 44.

b. What would your final score be if you bowled 120 in a game? 164

88. a. If your average bowling score is 165, what is your handicap? 28

b. What would your final score be if you bowled 140 in a game? 168

Writing in Mathematics

Writing about mathematics will help you to learn mathematics. For all writing exercises in this book, use complete sentences to respond to the questions. Some writing exercises can be answered in a sentence; others require a paragraph or two. You can decide how much you need to write as long as your writing clearly and directly answers the question in the exercise. Standard references such as a dictionary and a thesaurus should be helpful.

89. What is a variable? 89–97. Answers will vary.

90. What is an algebraic expression?

91. Explain how to evaluate \( 2 + 5x \) for \( x = 3 \).

92. If \( x \) represents the number, explain the difference between translating the following phrases:

- a number decreased by 5
- a number subtracted from 5

93. What is an equation?

94. How do you tell the difference between an algebraic expression and an equation?

95. How do you determine whether a given number is a solution of an equation?

96. What is a mathematical model?

97. The bar graph for Exercises 83–84 shows a decline in the percentage of African-American Major League Baseball players from 1997 through 2006. What explanations can you offer for this trend?

98. In Exercises 87–88, we used the formula \( H = 0.8(200 - A) \) to find a bowler’s handicap, \( H \), where the variable \( A \) represents the bowler’s average score. Describe what happens to the handicap when the average score is 200. There is no handicap.
Critical Thinking Exercises

Make Sense? In Exercises 99–102, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning. Explanations will vary. These exercises can also be used to start group discussions.

99. As I read this book, I write questions in the margins that I might ask in class. makes sense

100. I'm solving a problem that requires me to determine if 5 is a solution of $4x + 7$. does not make sense

101. The model $P = 16 - n$ describes the percentage of African-American Major League Baseball players $n$ years after 1997, so I can use it to estimate the percentage of African-American players in 1997. makes sense

102. Because there are four quarters in a dollar, I can use the formula $q = 4d$ to determine the number of quarters, $q$, in $d$ dollars. makes sense

In Exercises 103–106, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

103. The algebraic expression for "3 less than a number" is the same as the algebraic expression for "a number decreased by 3." true

104. Some algebraic expressions contain the equality symbol, =. false

105. The algebraic expressions $3 + 2x$ and $(3 + 2)x$ do not mean the same thing. true

106. The algebraic expression for "the quotient of a number and 6" is the same as the algebraic expression for "the quotient of 6 and a number." false

In Exercises 107–108, define variables and write a formula that describes the relationship in each table. 107–108. Choices of variables may vary. Examples are given.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$60</td>
</tr>
<tr>
<td>4</td>
<td>$80</td>
</tr>
<tr>
<td>5</td>
<td>$100</td>
</tr>
<tr>
<td>6</td>
<td>$120</td>
</tr>
</tbody>
</table>

$h = \text{hours worked}, s = \text{salary}; s = 20h$

108. Number of Workers | Number of Televisions Built
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
</tr>
</tbody>
</table>

$w = \text{number of workers}, t = \text{number of televisions built}; t = 10w$

Technology Exercise

109. In 2006, a month into their new jobs, Entertainment Weekly magazine wanted to know which of three TV ladies was the "best buy." The data:

<table>
<thead>
<tr>
<th></th>
<th>Annual Salary</th>
<th>Average Number of People Watching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosie O'Donnell</td>
<td>$2 million</td>
<td>3.1 million</td>
</tr>
<tr>
<td>Meredith Vieira</td>
<td>$10 million</td>
<td>5.6 million</td>
</tr>
<tr>
<td>Katie Couric</td>
<td>$15 million</td>
<td>7.3 million</td>
</tr>
</tbody>
</table>

Source: Entertainment Weekly, November 3, 2006

Here's the formula used by Entertainment Weekly:

$$ p = \frac{s}{w} $$

In the words of the magazine, "the lower the price per viewer, the better the bargain."

a. Use a calculator to find the price per viewer, correct to the nearest cent, for each TV lady. O'Donnell: $0.65; Vieira: $1.79; Couric: $2.03

b. Using the magazine's criterion, which of the three women was the best buy? O'Donnell

Preview Exercises

Exercises 110–112 will help you prepare for the material covered in the next section. In each exercise, use the given formula to perform the indicated operation with the two fractions.

110. $\frac{a \cdot c}{b \cdot d} = \frac{a \cdot d}{b \cdot c} = \frac{3 \cdot 2}{7 \cdot 5} = \frac{6}{35}$

111. $\frac{a + c}{b + d} = \frac{a - c}{b - d} = \frac{2 + 3}{7 - 5} = \frac{10}{2}$

112. $\frac{a - c}{b - d} = \frac{9 - 5}{17 - 17} = \frac{4}{4}$
Practice Exercises

In Exercises 1–6, convert each mixed number to an improper fraction.

1. \( \frac{3}{4} \) 2. \( \frac{7}{9} \) 3. \( \frac{7}{5} \) 4. \( \frac{5}{8} \) 5. \( \frac{7}{16} \) 6. \( \frac{9}{16} \)

In Exercises 7–12, convert each improper fraction to a mixed number.

7. \( \frac{23}{5} \) 8. \( \frac{47}{8} \) 9. \( \frac{76}{9} \) 10. \( \frac{59}{9} \) 11. \( \frac{711}{20} \) 12. \( \frac{788}{25} \)

In Exercises 13–28, identify each natural number as prime or composite. If the number is composite, find its prime factorization.


In Exercises 29–40, simplify each fraction by reducing it to its lowest terms.

29. \( \frac{10}{16} \) 30. \( \frac{8}{14} \) 31. \( \frac{15}{18} \) 32. \( \frac{18}{22} \) 33. \( \frac{35}{70} \) 34. \( \frac{45}{90} \) 35. \( \frac{32}{80} \) 36. \( \frac{75}{80} \) 37. \( \frac{44}{50} \) 38. \( \frac{38}{50} \) 39. \( \frac{120}{140} \) 40. \( \frac{86}{116} \)

In Exercises 41–50, perform the indicated operation. Where possible, reduce the answer to its lowest terms.

41. \( \frac{2}{5} \cdot \frac{1}{2} \) 42. \( \frac{3}{4} + \frac{1}{4} \) 43. \( \frac{3}{4} \cdot \frac{7}{11} \) 44. \( \frac{5}{4} \cdot \frac{3}{15} \) 45. \( \frac{5}{4} \cdot \frac{3}{15} \) 46. \( \frac{8}{4} \cdot \frac{3}{7} \) 47. \( \frac{1}{10} \cdot \frac{5}{6} \) 48. \( \frac{1}{8} \cdot \frac{1}{12} \) 49. \( \frac{5}{13} \cdot \frac{5}{4} \) 50. \( \frac{7}{14} \cdot \frac{6}{11} \) 51. \( \frac{3}{4} \cdot \frac{7}{5} \) 52. \( \frac{2}{5} \cdot \frac{1}{4} \) 53. \( \frac{5}{4} + \frac{3}{5} \) 54. \( \frac{7}{8} \cdot \frac{2}{15} \) 55. \( \frac{18}{5} + \frac{2}{9} \) 56. \( \frac{12}{7} + \frac{3}{4} \) 57. \( \frac{2}{1} + \frac{18}{5} \) 58. \( \frac{3}{4} + \frac{1}{4} \) 59. \( \frac{3}{4} + \frac{1}{4} \) 60. \( \frac{3}{7} + \frac{1}{7} \) 61. \( \frac{7}{8} + \frac{1}{10} \) 62. \( \frac{7}{4} + \frac{3}{8} \) 63. \( \frac{1}{14} + \frac{1}{7} \) 64. \( \frac{1}{8} + \frac{1}{4} \) 65. \( \frac{3}{5} + \frac{1}{10} \)

In Exercises 51–56, determine whether the given equation is a solution to the equation.

51. \( \frac{2}{7} \cdot \frac{1}{2} = 28 \) 52. \( \frac{5}{3} \cdot \frac{1}{2} = 30 \) 53. \( \frac{2}{3} = 18 \)

In Exercises 57–62, determine whether the given equation is a solution to the equation.

57. \( \frac{2}{3} = \frac{1}{4} \) 58. \( \frac{3}{4} + \frac{1}{4} \) 59. \( \frac{3}{4} + \frac{1}{4} \) 60. \( \frac{3}{7} + \frac{1}{7} \) 61. \( \frac{7}{8} + \frac{1}{10} \) 62. \( \frac{7}{4} + \frac{3}{8} \)

In Exercises 63–72, translate from English to an algebraic expression or equation, whichever is appropriate. Let the variable \( x \) represent the number.

63. \( \frac{1}{4} \) of a number \( \frac{1}{2} x \) 64. \( \frac{1}{8} \) of a number \( \frac{3}{4} x \) 65. \( \frac{1}{4} \) of a number \( \frac{1}{2} x \) 66. \( \frac{3}{4} + \frac{5}{2} \) 67. \( \frac{2}{11} + \frac{4}{11} \) 68. \( \frac{5}{13} + \frac{2}{13} \) 69. \( \frac{7}{12} + \frac{1}{12} \) 70. \( \frac{5}{16} + \frac{1}{16} \) 71. \( \frac{5}{8} + \frac{5}{8} \) 72. \( \frac{3}{8} + \frac{3}{8} \) 73. \( \frac{7}{12} + \frac{5}{12} \) 74. \( \frac{13}{18} - \frac{5}{18} \) 75. \( \frac{16}{7} - \frac{2}{7} \) 76. \( \frac{17}{5} - \frac{2}{5} \) 77. \( \frac{1}{2} + \frac{1}{5} \) 78. \( \frac{1}{3} + \frac{1}{5} \) 79. \( \frac{3}{4} + \frac{20}{30} \) 80. \( \frac{2}{5} + \frac{1}{15} \) 81. \( \frac{3}{8} + \frac{19}{12} \) 82. \( \frac{3}{10} + \frac{13}{15} \) 83. \( \frac{11}{18} - \frac{2}{9} \) 84. \( \frac{17}{8} + \frac{1}{9} \) 85. \( \frac{3}{4} + \frac{3}{7} \) 86. \( \frac{2}{3} + \frac{2}{3} \) 87. \( \frac{7}{10} - \frac{3}{16} \) 88. \( \frac{7}{30} - \frac{5}{24} \) 89. \( \frac{3}{4} - \frac{1}{3} \) 90. \( \frac{3}{2} - \frac{1}{2} \) 91. \( \frac{7}{2} \cdot \frac{1}{2} = 28 \) 92. \( \frac{5}{3} \cdot \frac{1}{2} = 30 \) 93. \( \frac{2}{3} = 18 \) 94. \( \frac{3}{4} + \frac{1}{4} \) 95. \( \frac{1}{3} + \frac{1}{2} \) 96. \( \frac{1}{4} + \frac{1}{2} \) 97. \( \frac{3}{9} + \frac{2}{3} = \frac{3}{27} \) 98. \( \frac{2}{9} + \frac{5}{6} = \frac{2}{13} \) 99. \( \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6} \) 100. \( \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8} \) 101. \( \frac{1}{2} \cdot \frac{3}{4} = \frac{1}{8} \) 102. \( \frac{2}{3} + \frac{1}{3} = \frac{2}{6} \) 103. \( \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \) 104. \( \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \) 105. A number decreased by \( \frac{1}{4} \) of itself \( x - \frac{1}{4} x \) 106. A number decreased by \( \frac{1}{3} \) of itself \( x - \frac{1}{3} x \)
A number decreased by \( \frac{1}{4} \) is half of that number. 
\[ x - \frac{1}{4} = \frac{1}{2}x \]

A number decreased by \( \frac{1}{3} \) is half of that number. 
\[ x - \frac{1}{3} = \frac{1}{2}x \]

The sum of \( \frac{1}{2} \) of a number and \( \frac{1}{5} \) of that number gives 12. 
\[ \frac{1}{2}x + \frac{1}{5}x = 12 \]

The sum of \( \frac{1}{2} \) of a number and \( \frac{1}{10} \) of that number gives 15. 
\[ \frac{1}{2}x + \frac{1}{10}x = 15 \]

The product of \( \frac{2}{3} \) and a number increased by 6 \( \frac{2}{3}(x + 6) \)

The product of \( \frac{3}{4} \) and a number increased by 9 \( \frac{3}{4}(x + 9) \)

The product of \( \frac{3}{4} \) and a number, increased by 6, is 3 less than the number. \( \frac{3}{4}x + 6 = x - 3 \)

The product of \( \frac{3}{4} \) and a number, increased by 9, is 2 less than the number. \( \frac{3}{4}x + 9 = x - 2 \)

**Practice PLUS**

In Exercises 115–118, perform the indicated operation. Write the answer as an algebraic expression.

\[
\begin{align*}
115. & \quad \frac{3}{4} + \frac{a}{5} + \frac{3a}{20} \\
116. & \quad \frac{2}{3} + \frac{a}{7} + \frac{14}{3a}
\end{align*}
\]

\[
\begin{align*}
117. & \quad \frac{11}{x} + \frac{9}{20} \\
118. & \quad \frac{10}{y} - \frac{6}{4}
\end{align*}
\]

In Exercises 119–120, perform the indicated operations. Begin by performing operations in parentheses.

\[
\begin{align*}
119. & \quad \left( \frac{1}{2} - \frac{1}{3} \right) + \frac{5}{8} + \frac{4}{15} \\
120. & \quad \left( \frac{1}{2} + \frac{1}{4} \right) + \left( \frac{1}{2} + \frac{1}{3} \right) \cdot \frac{9}{10}
\end{align*}
\]

In Exercises 121–122, determine whether the given number is a solution of the equation.

\[
\begin{align*}
121. & \quad \frac{1}{5}(x + 2) = \frac{1}{2}(x - \frac{1}{5}) \cdot \frac{5}{8} \quad \text{not a solution} \\
122. & \quad 12 - 3(x - 2) = 4x - (x + 3); \frac{3}{2} \quad \text{solution}
\end{align*}
\]

**Application Exercises**

The formula

\[ C = \frac{5}{9}(F - 32) \]

expresses the relationship between Fahrenheit temperature, \( F \), and Celsius temperature, \( C \). In Exercises 123–124, use the formula to convert the given Fahrenheit temperature to its equivalent temperature on the Celsius scale.

\[
\begin{align*}
123. & \quad 88^\circ F \quad 124. & \quad 41^\circ F
\end{align*}
\]

The maximum heart rate, in beats per minute, that you should achieve during exercise is 220 minus your age:

\[ 220 - a. \]

This algebraic expression gives maximum heart rate in terms of age, \( a \).

The bar graph at the top of the next column shows the target heart rate ranges for four types of exercise goals. The lower and upper limits of these ranges are fractions of the maximum heart rate, 220 – \( a \). Exercises 125–128 are based on the information in the graph.

<table>
<thead>
<tr>
<th>Exercise Goal</th>
<th>Target Heart Rate Ranges for Exercise Goals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boost performance as a competitive athlete</td>
<td>Improve cardiovascular conditioning</td>
</tr>
<tr>
<td>Improve overall health and reduce risk of heart attack</td>
<td>Lose weight</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
2 & \quad \frac{5}{10} \quad 3 & \quad \frac{7}{10} \quad 4 & \quad \frac{9}{10} \quad 5 & \quad 1
\end{align*}
\]

Fraction of Maximum Heart Rate, 220 – \( a \)

125. If your exercise goal is to improve cardiovascular conditioning, the graph shows the following range for target heart rate, \( H \), in beats per minute:

\[ \frac{7}{10} (220 - a) \]

126. If your exercise goal is to improve overall health, the graph shows the following range for target heart rate, \( H \), in beats per minute:

\[ \frac{7}{10} (220 - a) \]

127. a. Write a formula that models the heart rate, \( H \), in beats per minute, for a person who is \( a \) years old and would like to achieve \( \frac{9}{10} \) of maximum heart rate during exercise.

\[ H = \frac{9}{10} (220 - a) \]

b. Use your formula from part (a) to find the heart rate during exercise for a 40-year-old with this goal.

162 beats per minute

128. a. Write a formula that models the heart rate, \( H \), in beats per minute, for a person who is \( a \) years old and would like to achieve \( \frac{7}{8} \) of maximum heart rate during exercise.

\[ H = \frac{7}{8} (220 - a) \]

b. Use your formula from part (a) to find the heart rate during exercise for a 20-year-old with this goal.

175 beats per minute
The graph shows the number of cigarettes smoked, in billions, in the United States for selected years from 1980 through 2005.

**Critical Thinking Exercises**

**Make Sense?** In Exercises 141–144, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning. Explanations will vary. These exercises can also be used to start group discussions.

141. I find it easier to multiply \( \frac{3}{4} \) and \( \frac{5}{3} \) than to add them. makes sense

142. Fractions frustrated me in arithmetic, so I'm glad I won't have to use them in algebra. does not make sense

143. I need to be able to perform operations with fractions to determine whether \( \frac{3}{4} \) is a solution of \( 8x = 12 \left( x - \frac{1}{2} \right) \). makes sense

144. I saved money by buying a computer for \( \frac{3}{4} \) of its original price. does not make sense

In Exercises 145–148, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

145. \( \frac{1}{2} + \frac{1}{5} = \frac{2}{7} \) false

146. \( \frac{1}{2} + 4 = 2 \) false

147. Every fraction has infinitely many equivalent fractions. true

148. \( \frac{3 + 7}{30} = \frac{10}{30} = \frac{1}{3} \) false

149. Shown below is a short excerpt from “The Star-Spangled Banner.” The time is \( \frac{3}{4} \), which means that each measure must contain notes that add up to \( \frac{3}{4} \). The values of the different notes tell musicians how long to hold each note.

\[ \begin{align*}
\text{Note} & \quad \text{Value} \\
\text{\#1} & \quad \text{\#2} \\
\hline
\ast & \quad \frac{1}{2} \\
\ddot{\text{J}} & \quad \frac{1}{4} \\
\dddot{\text{J}} & \quad \frac{1}{8}
\end{align*} \]

Use vertical lines to divide this line of “The Star-Spangled Banner” into measures.

**Writing in Mathematics** 131–140. Answers will vary.

131. Explain how to convert a mixed number to an improper fraction and give an example.

132. Explain how to convert an improper fraction to a mixed number and give an example.

133. Describe the difference between a prime number and a composite number.

134. What is meant by the prime factorization of a composite number?

135. What is the Fundamental Principle of Fractions?

136. Explain how to reduce a fraction to its lowest terms. Give an example with your explanation.

137. Explain how to multiply fractions and give an example.

138. Explain how to divide fractions and give an example.

139. Describe how to add or subtract fractions with identical denominators. Provide an example with your description.

140. Explain how to add fractions with different denominators. Use \( \frac{3}{4} + \frac{1}{2} \) as an example.

**Preview Exercises**

Exercises 150–152 will help you prepare for the material covered in the next section. Consider the following “infinite ruler” that shows numbers that lie to the left and to the right of zero.

<table>
<thead>
<tr>
<th>(c)</th>
<th>(b)</th>
<th>(a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-4</td>
<td>-3</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

150. What number is represented by point (a)? 5

151. What number is represented by point (b)? Express the number as an improper fraction. \( \frac{5}{2} \)

152. What number is represented by point (c)? \(-4\)
EXAMPLE 8 Finding Absolute Value

Find the absolute value:

a. \(|-3|\)  
b. \(|5|\)  
c. \(|0|\).

**Solution** The solution is illustrated in Figure 1.10.

- a. \(|-3| = 3\) The absolute value of \(-3\) is 3 because \(-3\) is 3 units from 0.
- b. \(|5| = 5\) 5 is 5 units from 0.
- c. \(|0| = 0\) 0 is 0 unit from itself.

Example 8 illustrates that the absolute value of a positive real number or 0 is the number itself. The absolute value of a negative real number, such as \(-3\), is the number without the negative sign. Zero is the only real number whose absolute value is 0: \(|0| = 0\). The absolute value of any real number other than 0 is always positive.

**CHECK POINT 8** Find the absolute value:

a. \(|-4|\)  
b. \(|6|\)  
c. \(|-\sqrt{2}|\).

### 1.3 EXERCISE SET

**Practice Exercises**

In Exercises 1–8, write a positive or negative integer that describes each situation.

1. Meteorology: 20° below zero  
2. Navigation: 65 feet above sea level  
3. Health: A gain of 8 pounds  
4. Economics: A loss of $12,500.00  
5. Banking: A withdrawal of $3000.00  
6. Physics: An automobile slowing down at a rate of 3 meters per second each second  
7. Economics: A budget deficit of 4 billion dollars  
8. Football: A 14-yard loss

In Exercises 9–20, start by drawing a number line that shows integers from \(-5\) to 5. Then graph each real number on your number line. 9–20. See graphing answer section.

9. \(2\) 10. \(5\) 11. \(-5\) 12. \(-2\) 13. \(\frac{1}{2}\) 14. \(\frac{1}{4}\) 15. \(\frac{11}{3}\) 16. \(\frac{7}{3}\) 17. \(-\frac{1}{2}\) 18. \(-\frac{3}{2}\) 19. \(-\frac{16}{5}\) 20. \(-\frac{11}{5}\)

In Exercises 21–32, express each rational number as a decimal.

21. \(\frac{3}{4}\) 0.75 22. \(\frac{3}{5}\) 0.6 23. \(\frac{7}{20}\) 0.35 24. \(\frac{3}{20}\) 0.15 25. \(\frac{7}{8}\) 0.875 26. \(\frac{5}{16}\) 0.3125 27. \(\frac{9}{11}\) 0.81 28. \(\frac{3}{11}\) 0.27 29. \(-\frac{1}{2}\) -0.5 30. \(-\frac{1}{4}\) -0.25 31. \(-\frac{5}{6}\) -0.83 32. \(-\frac{7}{6}\) -1.16

In Exercises 33–36, list all numbers from the given set that are:

- a. natural numbers  
- b. whole numbers  
- c. integers  
- d. rational numbers  
- e. irrational numbers  
- f. real numbers.

33. \([-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}]\)  
   a. \(\sqrt{100}\)  
   b. 0, \(\sqrt{100}\)  
   c. \(-9, 0, \sqrt{100}\)  
   d. \(-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}\)  
   e. \(\sqrt{3}\)  
   f. \(-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}\)

34. \([-7, -0.6, 0, \sqrt{49}, \sqrt{50}]\)  
   a. \(\sqrt{49}\)  
   b. 0, \(\sqrt{49}\)  
   c. \(-7, 0, \sqrt{49}\)  
   d. \(-7, -0.6, 0, \sqrt{49}\)  
   e. \(\sqrt{50}\)  
   f. \(-7, -0.6, 0, \sqrt{49}, \sqrt{50}\)

35. \([-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}]\)  
   a. \(\sqrt{64}\)  
   b. 0, \(\sqrt{64}\)  
   c. \(-11, 0, \sqrt{64}\)  
   d. \(-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}\)  
   e. \(\sqrt{5}, \pi\)  
   f. \(-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}\)

36. \([-5, -0.3, 0, \sqrt{2}, \sqrt{4}]\)  
   a. \(\sqrt{4}\)  
   b. 0, \(\sqrt{4}\)  
   c. \(-5, 0, \sqrt{4}\)  
   d. \(-5, -0.3, 0, \sqrt{4}\)  
   e. \(\sqrt{2}\)  
   f. \(-5, -0.3, 0, \sqrt{2}, \sqrt{4}\)

37. Give an example of a whole number that is not a natural number.

38. Give an example of a number that is not a whole number. Answers will vary; example is \(\frac{1}{2}\).

39. Give an example of a rational number that is not an integer. Answers will vary; example is \(\frac{1}{2}\).

40. Give an example of a rational number that is not a natural number. Answers will vary; example is \(-5\).

41. Give an example of a number that is an integer, a whole number, and a natural number. Answers will vary; example is 6.

42. Give an example of a number that is a rational number, an integer, and a real number. Answers will vary; example is 5.
43. Give an example of a number that is an irrational number and a real number. Answers will vary; example is \( \pi \).

44. Give an example of a number that is a real number, but not an irrational number. Answers will vary; example is \( \sqrt{4} \).

In Exercises 45–62, insert either < or > in the shaded area between each pair of numbers to make a true statement.

45. \( \frac{1}{2} \text{ or } 2 \) < 46. \( 4 \text{ or } -3 \) >

47. 3 5 < 48. 3 2 >

49. -4 -6 < 50. -5 -5 <

51. -2.5 1.5 < 52. -1.25 -0.5 <

53. -3 -5 < 54. 0 -1 2 >

55. -4.5 3 < 56. -5.5 2.5 <

57. \( \sqrt{2} \text{ or } 1.5 \) < 58. \( \sqrt{3} \text{ or } 2 \) <

59. 0.3 0.3 > 60. 0.6 0.6 <

61. -\( \pi \) -3.5 > 62. -\( \pi \) -2.3 >

In Exercises 63–70, determine whether each inequality is true or false.

63. -5 \( \geq \) -13 true 64. -5 \( \leq \) -8 false

65. -9 \( \geq \) -9 true 66. -14 \( \leq \) -14 true

67. 0 \( \geq \) -6 true 68. 0 \( \geq \) -13 true

69. -17 \( \leq \) 6 false 70. -14 \( \leq \) 8 false

In Exercises 71–78, find each absolute value.

71. |6| = 6 72. |3| = 3

73. |-7| = 7 74. |-9| = 9

75. |5| = 5 76. |4| = 4

77. \( |-\sqrt{11}| = \sqrt{11} \) 78. \( |\sqrt{29}| = \sqrt{29} \)

Practice PLUS

In Exercises 79–86, insert either <, >, or = in the shaded area to make a true statement.

79. |6| = |-3| > 80. |-20| = |-50| <

81. \( \frac{3}{4} \text{ or } -0.6 \) = 82. \( \frac{5}{2} \text{ or } -2.5 \) =

83. \( \frac{3}{4} \text{ or } \frac{14}{15} \) = 84. \( \frac{17}{18} \text{ or } \frac{50}{5} \)

85. \( \frac{8}{13} \text{ or } -1 \) = 86. \( \frac{4}{17} \text{ or } \frac{4}{17} \)

Application Exercises

In Exercises 87–94, determine whether natural numbers, whole numbers, integers, rational numbers, or all real numbers are appropriate for each situation.

87. Shoe sizes of students on campus rational numbers

88. Heights of students on campus rational numbers

89. Temperatures in weather reports integers

90. Class sizes of algebra courses natural numbers

91. Values of d given by the formula \( d = \sqrt{1.5h} \), where d is the distance, in miles, that you can see to the horizon from a height of h feet all real numbers

92. Values of C given by the formula \( C = 2\pi r \), where C is the circumference of a circle with radius r all real numbers

93. The number of pets a person has whole numbers

94. The number of siblings a person has whole numbers

95. The table shows the record low temperatures for five U.S. states.

<table>
<thead>
<tr>
<th>State</th>
<th>Record Low (°F)</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Florida</td>
<td>-2</td>
<td>Feb. 13, 1899</td>
</tr>
<tr>
<td>Georgia</td>
<td>-17</td>
<td>Jan. 27, 1940</td>
</tr>
<tr>
<td>Hawaii</td>
<td>12</td>
<td>May 17, 1979</td>
</tr>
<tr>
<td>Louisiana</td>
<td>-16</td>
<td>Feb. 13, 1899</td>
</tr>
<tr>
<td>Rhode Island</td>
<td>-25</td>
<td>Feb. 5, 1996</td>
</tr>
</tbody>
</table>

Source: National Climatic Data Center

96. The table shows the record low temperatures for five U.S. states.

<table>
<thead>
<tr>
<th>State</th>
<th>Record Low (°F)</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virginia</td>
<td>-30</td>
<td>Jan. 22, 1985</td>
</tr>
<tr>
<td>West Virginia</td>
<td>-37</td>
<td>Dec. 30, 1917</td>
</tr>
<tr>
<td>Wisconsin</td>
<td>-55</td>
<td>Feb. 4, 1996</td>
</tr>
<tr>
<td>Wyoming</td>
<td>-66</td>
<td>Feb. 9, 1933</td>
</tr>
</tbody>
</table>

Source: National Climatic Data Center


98. What are the natural numbers?

99. What are the whole numbers?

100. What are the integers?

101. How does the set of integers differ from the set of whole numbers?

102. Describe how to graph a number on the number line.

103. What is a rational number?

104. Explain how to express \( \frac{3}{4} \) as a decimal.

105. Describe the difference between a rational number and an irrational number.

106. If you are given two different real numbers, explain how to determine which one is the lesser.

107. Describe what is meant by the absolute value of a number. Give an example with your explanation.
Critical Thinking Exercises

Make Sense? In Exercises 108–111, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning. Explanations will vary. These exercises can also be used to start group discussions.

108. The humor in this joke is based on the fact that the football will never be kicked. makes sense

109. Titanic came to rest 12,500 feet below sea level and Bismarck came to rest 15,617 feet below sea level, so Bismarck’s resting place is higher than Titanic’s. does not make sense

110. I expressed a rational number as a decimal and the decimal neither terminated nor repeated. does not make sense

111. I evaluated the formula \( d = \sqrt{1.5h} \) for a value of \( h \) that resulted in a rational number for \( d \). makes sense

In Exercises 112–117, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

112. Every rational number is an integer. false

113. Some whole numbers are not integers. false

SECTION 1.4 Basic Rules of Algebra

Objectives

1. Understand and use the vocabulary of algebraic expressions.
2. Use commutative properties.
3. Use associative properties.
4. Use distributive properties.
5. Combine like terms.
6. Simplify algebraic expressions.

Starting as a link among U.S. research scientists, more than one billion people worldwide now use the Internet. Some random Internet factoids:

- Number of searches performed per day on Google: 200 million
- Peak time for sex-related searches: 11 P.M.
- Fraction of people who use the word “password” as their password: \( \frac{1}{8} \)
1.4 EXERCISE SET

Practice Exercises

In Exercises 1–6, an algebraic expression is given. Use each expression to answer the following questions.

a. How many terms are there in the algebraic expression?

b. What is the numerical coefficient of the first term?

c. What is the constant term?

d. Does the algebraic expression contain like terms? If so, what are the like terms?

<table>
<thead>
<tr>
<th>Expression</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 3x + 5</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>no</td>
</tr>
<tr>
<td>2. 9x + 4</td>
<td>2</td>
<td>9</td>
<td>4</td>
<td>no</td>
</tr>
<tr>
<td>3. x + 2 + 5x</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>yes, x and 5x</td>
</tr>
<tr>
<td>4. x + 6 + 7x</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>yes, x and 7x</td>
</tr>
<tr>
<td>5. 4y + 1 + 3x</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>no</td>
</tr>
<tr>
<td>6. 8y + 1 + 10x</td>
<td>3</td>
<td>8</td>
<td>1</td>
<td>no</td>
</tr>
</tbody>
</table>

In Exercises 7–14, use the commutative property of addition to write an equivalent algebraic expression.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Added Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. y + 4</td>
<td>4 + y</td>
</tr>
<tr>
<td>8. x + 7</td>
<td>7 + x</td>
</tr>
<tr>
<td>9. 5 + 3x</td>
<td>3x + 5</td>
</tr>
<tr>
<td>10. 4 + 9x</td>
<td>9x + 4</td>
</tr>
<tr>
<td>11. 4x + 5y</td>
<td>5y + 4x</td>
</tr>
<tr>
<td>12. 10x + 9y</td>
<td>9y + 10x</td>
</tr>
<tr>
<td>13. 5(x + 3)</td>
<td>5(x + 3)</td>
</tr>
<tr>
<td>14. 6(x + 4)</td>
<td>6(4 + x)</td>
</tr>
</tbody>
</table>

In Exercises 15–22, use the commutative property of multiplication to write an equivalent algebraic expression.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Multiplied Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>15. 9x × 9</td>
<td>9x × 9</td>
</tr>
<tr>
<td>16. 8x × 8</td>
<td>8x × 8</td>
</tr>
<tr>
<td>17. x + y</td>
<td>x + y</td>
</tr>
<tr>
<td>18. x + y</td>
<td>x + y</td>
</tr>
<tr>
<td>19. 7x + 23</td>
<td>x7 + 23</td>
</tr>
<tr>
<td>20. 13x + 11</td>
<td>11x + 13</td>
</tr>
<tr>
<td>21. 5(x + 3)</td>
<td>(x + 3)5</td>
</tr>
<tr>
<td>22. 6(x + 4)</td>
<td>(x + 4)6</td>
</tr>
</tbody>
</table>

In Exercises 23–26, use an associative property to rewrite each algebraic expression. Once the grouping has been changed, simplify the resulting algebraic expression.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Grouped Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>23. 7 + (5 + x)</td>
<td>(7 + 5) + x = 12 + x</td>
</tr>
<tr>
<td>24. 9 + (3 + x)</td>
<td>(9 + 3) + x = 12 + x</td>
</tr>
<tr>
<td>25. (7 + 4)x = (7 × 4)x = 28x</td>
<td></td>
</tr>
<tr>
<td>26. (8 + 5) + 30x = 40x</td>
<td></td>
</tr>
</tbody>
</table>

In Exercises 27–46, use a form of the distributive property to rewrite each algebraic expression without parentheses.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Distributed Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>27. 3(x + 5)</td>
<td>3x + 15</td>
</tr>
<tr>
<td>28. 4(x + 6)</td>
<td>4x + 24</td>
</tr>
<tr>
<td>29. 8(2x + 3)</td>
<td>16x + 24</td>
</tr>
<tr>
<td>30. 9(2x + 5)</td>
<td>18x + 45</td>
</tr>
<tr>
<td>31. (\frac{1}{3}(12 + 6r))</td>
<td>4 + 2r</td>
</tr>
<tr>
<td>32. (\frac{1}{4}(12 + 8r))</td>
<td>3 + 2r</td>
</tr>
<tr>
<td>33. 5(x + y)</td>
<td>5x + 5y</td>
</tr>
<tr>
<td>34. 7(x + y)</td>
<td>7x + 7y</td>
</tr>
<tr>
<td>35. 3(x - 2)</td>
<td>3x - 6</td>
</tr>
<tr>
<td>36. 4(x - 5)</td>
<td>4x - 20</td>
</tr>
<tr>
<td>37. 2(4x - 5)</td>
<td>8x - 10</td>
</tr>
<tr>
<td>38. 6(3x - 2)</td>
<td>18x - 12</td>
</tr>
<tr>
<td>39. (\frac{1}{2}(5x - 12))</td>
<td>(\frac{5}{2}x - 6)</td>
</tr>
<tr>
<td>40. (\frac{1}{3}(7x - 21))</td>
<td>(\frac{7}{3}x - 7)</td>
</tr>
<tr>
<td>41. (2x + 7)4</td>
<td>8x + 28</td>
</tr>
<tr>
<td>42. (5x + 3)6</td>
<td>30x + 18</td>
</tr>
<tr>
<td>43. 6(x + 3 + 2y)</td>
<td>6x + 18 + 12y</td>
</tr>
<tr>
<td>44. 7(2x + 4 + y)</td>
<td>14x + 28 + 7y</td>
</tr>
<tr>
<td>45. 5(3x - 2 + 4y)</td>
<td>15x - 10 + 20y</td>
</tr>
<tr>
<td>46. 4(5x - 3 + 7y)</td>
<td>20x - 12 + 28y</td>
</tr>
</tbody>
</table>

In Exercises 47–64, simplify each algebraic expression.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Simplified Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>47. 7x + 10x</td>
<td>17x</td>
</tr>
<tr>
<td>48. 5x + 13x</td>
<td>18x</td>
</tr>
<tr>
<td>49. 11a - 3a</td>
<td>8a</td>
</tr>
<tr>
<td>50. 14b - 5b</td>
<td>9b</td>
</tr>
<tr>
<td>51. 3 + (x + 11)</td>
<td>14 + x</td>
</tr>
<tr>
<td>52. 7 + (x + 10)</td>
<td>17 + x</td>
</tr>
<tr>
<td>53. 5y + 3 + 6y</td>
<td>11y + 3</td>
</tr>
<tr>
<td>54. 8y + 7 + 10y</td>
<td>18y + 7</td>
</tr>
<tr>
<td>55. 2x + 5 + 7x - 4</td>
<td>9x + 1</td>
</tr>
<tr>
<td>56. 7x + 8 + 2x - 3</td>
<td>9x + 5</td>
</tr>
<tr>
<td>57. 11a + 12 + 3a + 2</td>
<td>14a + 14</td>
</tr>
<tr>
<td>58. 13a + 15 + 2a + 11</td>
<td>15a + 26</td>
</tr>
<tr>
<td>59. 5(3x + 2) - 4</td>
<td>15x + 6</td>
</tr>
<tr>
<td>60. 2(5x + 4) - 3</td>
<td>10x + 5</td>
</tr>
<tr>
<td>61. 12 + 5(3x - 2)</td>
<td>15x + 2</td>
</tr>
<tr>
<td>62. 14 + 2(5x - 1)</td>
<td>10x + 14</td>
</tr>
<tr>
<td>63. 7(3x + 2b) + 5(4a + 2b)</td>
<td>41a + 24b</td>
</tr>
<tr>
<td>64. 11(6a + 3b) + 4(12a + 5b)</td>
<td>114a + 53b</td>
</tr>
</tbody>
</table>
Practice PLUS
In Exercises 65–66, name the property used to go from step to step each time that "(why?)" occurs.

65. \( 7 + 2(x + 9) \)
   \( = 7 + (2x + 18) \) (why?) Distributive property
   \( = 7 + (18 + 2x) \) (why?) Commutative property of addition
   \( = (7 + 18) + 2x \) (why?) Associative property of addition
   \( = 25 + 2x \)
   \( = 2x + 25 \) (why?) Commutative property of addition

66. \( 5(x + 4) + 3x \)
   \( = (5x + 20) + 3x \) (why?) Distributive property
   \( = (20 + 5x) + 3x \) (why?) Commutative property of addition
   \( = 20 + (5x + 3x) \) (why?) Associative property of addition
   \( = 20 + (5 + 3)x \) (why?) Distributive property
   \( = 20 + 8x \)
   \( = 8x + 20 \) (why?) Commutative property of addition

In Exercises 67–76, write each English phrase as an algebraic expression. Then simplify the expression. Let \( x \) represent the number.

67. the sum of 7 times a number and twice the number
   \( 7x + 2x; 9x \)

68. the sum of 8 times a number and twice the number
   \( 8x + 2x; 10x \)

69. the product of 3 and a number, which is then subtracted from the product of 12 and a number
   \( 12x - 3x; 9x \)

70. the product of 5 and a number, which is then subtracted from the product of 11 and a number
   \( 11x - 5x; 6x \)

71. six times the product of 4 and a number
   \( 6(4x); 24x \)

72. nine times the product of 3 and a number
   \( 9(3x); 27x \)

73. six times the sum of 4 and a number
   \( 6(4 + x); 24 + 6x \)

74. nine times the sum of 3 and a number
   \( 9(3 + x); 27 + 9x \)

75. eight increased by the product of 5 and one less than a number
   \( 8 + 5(x - 1); 5x + 3 \)

76. nine increased by the product of 3 and 2 less than a number
   \( 9 + 3(x - 2); 3x + 3 \)

Application Exercises
The graph shows the percentage of U.S. men and women who used the Internet for four selected years. Exercises 77–78 involve mathematical models for the data.

77. The percentage of U.S. men, \( M \), who used the Internet \( n \) years after 2000 can be modeled by the formula
   \[ M = 22n + 25 \]
   a. Simplify the formula. \( M = 4.5n + 51 \)
   b. Use the simplified form of the mathematical model to find the percentage of U.S. men who used the Internet in 2005. Does the model underestimate or overestimate the actual percent shown by the bar graph for 2005? By how much? 73.5%; underestimates by 1.5%

78. The percentage of U.S. women, \( W \), who used the Internet \( n \) years after 2000 can be modeled by the formula
   \[ W = 22n + 20 \]
   a. Simplify the formula. \( W = 4.3n + 46 \)
   b. Use the simplified form of the mathematical model to find the percentage of U.S. women who used the Internet in 2005. Does the model underestimate or overestimate the actual percent shown by the bar graph for 2005? By how much? 67.5%; underestimates by 1.5%

Writing in Mathematics 79–88. Answers will vary.

79. What is a term? Provide an example with your description.
80. What are like terms? Provide an example with your description.

81. What are equivalent algebraic expressions?
82. State a commutative property and give an example.
83. State an associative property and give an example.
84. State a distributive property and give an example.
85. Explain how to add like terms. Give an example.
86. What does it mean to simplify an algebraic expression?
87. An algebra student incorrectly used the distributive property and wrote \( 3(5x + 7) = 15x + 7 \). If you were that student's teacher, what would you say to help the student avoid this kind of error?
88. You can transpose the letters in the word "conversation" to form the phrase "voices rant on." From "total abstainers" we can form "sit not at ale bars." What two algebraic properties do each of these transpositions (called anagrams) remind you of? Explain your answer.

Critical Thinking Exercises
Make Sense? In Exercises 89–92, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning. Explanations will vary. These exercises can also be used to start group discussions.

89. I applied the commutative property and rewrote \( x - 4 \) as \( 4 - x \). does not make sense
90. Just as the commutative properties change groupings, the associative properties change order. does not make sense
91. I did not use the distributive property to simplify \( 3(2x + 5x) \). makes sense
92. The commutative, associative, and distributive properties remind me of the rules of a game. makes sense
In Exercises 93–96, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

93. \( (24 ÷ 6) ÷ 2 = 24 ÷ (6 ÷ 2) \)  false
94. \( 2x + 5 = 5x + 2 \)  false
95. \( a + (bc) = (a + b)(a + c) \): In words, addition can be distributed over multiplication.  false
96. Like terms contain the same coefficients.  false

### Preview Exercises

Exercises 97–99 will help you prepare for the material covered in the next section. In each exercise, write an integer that is the result of the given situation.

97. You earn $150, but then you misplace $90.  60
98. You lose $30 and then you misplace $10.  −40
99. The temperature is 30 degrees, and then it drops by 35 degrees.  −5

### Section 1.1–Section 1.4

**What You Know:** Algebra uses variables, or letters that represent a variety of different numbers. These variables appear in algebraic expressions, equations, and formulas. Mathematical models use variables to describe real-world phenomena. We reviewed operations with fractions and saw how fractions are used in algebra. We defined the real numbers and represented them as points on a number line. Finally, we introduced some basic rules of algebra and used the commutative, associative, and distributive properties to simplify algebraic expressions.

1. Evaluate for \( x = 6 \): \( 2 + 10x \).
2. Evaluate for \( x = \frac{3}{2} \): \( 10x - 4 \).
3. Evaluate for \( x = 3 \) and \( y = 10 \):
   \[
   \frac{xy}{2} + 4(y - x)
   \]
4. Two less than \( \frac{1}{2} \) of a number: \( \frac{1}{2}x - 2 \)
5. Five more than the quotient of a number and 6 gives 19: \( \frac{y}{6} + 5 = 19 \)

### Graph

**Violent Crime in the United States**

<table>
<thead>
<tr>
<th>Year</th>
<th>Violent Crimes (per 100,000 Inhabitants)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>730</td>
</tr>
<tr>
<td>1995</td>
<td>685</td>
</tr>
<tr>
<td>2000</td>
<td>507</td>
</tr>
<tr>
<td>2004</td>
<td>466</td>
</tr>
</tbody>
</table>

*Source: Federal Bureau of Investigation*

Here is a mathematical model that approximates the data displayed by the bar graph:

\[ V = 747 - 21n. \]

\( n \) is the number of years after 1990.

- **Number of violent crimes per 100,000 inhabitants**
  - **a.** Use the formula to find the number of violent crimes per 100,000 inhabitants in 2000. Does the mathematical model underestimate or overestimate the actual number shown by the bar graph for 2000? By how much?
    - Overestimates by 37 per 100,000 inhabitants
  - **b.** If trends from 1990 through 2004 continue, use the formula to project the number of violent crimes per 100,000 inhabitants in 2010.
    - 327 per 100,000 inhabitants
1.5 EXERCISE SET

Practice Exercises

In Exercises 1–8, find each sum using a number line.
1. \(7 + (-3)\) 4
2. \(7 + (-2)\) 5
3. \(-2 + (-5)\) -7
4. \(-1 + (-5)\) -6
5. \(-6 + 2\) -4
6. \(-8 + 3\) -5
7. \(3 + (-3)\) 0
8. \(5 + (-5)\) 0

In Exercises 9–46, find each sum without the use of a number line.
9. \(-7 + 0\) -7
10. \(-5 + 0\) -5
11. \(30 + (-30)\) 0
12. \(15 + (-15)\) 0
13. \(-30 + (-30)\) -60
14. \(-15 + (-15)\) -30
15. \(-8 + (-10)\) -18
16. \(-4 + (-6)\) -10
17. \(-0.4 + (-0.9)\) -1.3
18. \(-1.5 + (-5.3)\) -6.8
19. \(-\frac{7}{10} + \left(-\frac{3}{10}\right)\) -1
20. \(-\frac{7}{8} + \left(-\frac{1}{8}\right)\) -1
21. \(-9 + 4\) -5
22. \(-7 + 3\) -4
23. \(12 + (-8)\) 4
24. \(13 + (-5)\) 8
25. \(6 + (-9)\) -3
26. \(3 + (-11)\) -8
27. \(-3.6 + 2.1\) -1.5
28. \(-6.3 + 5.2\) -1.1
29. \(-3.6 + (-2.1)\) -5.7
30. \(-6.3 + (-5.2)\) -11.5
31. \(\frac{9}{10} + \left(-\frac{3}{5}\right)\) 3
32. \(\frac{7}{10} + \left(-\frac{2}{5}\right)\) 3
33. \(-\frac{5}{8} + \frac{3}{4}\) 1
34. \(-\frac{5}{6} + \frac{1}{3}\) -\frac{1}{2}
35. \(-\frac{3}{7} + \left(-\frac{4}{5}\right)\) -\frac{43}{35}
36. \(-\frac{3}{8} + \left(-\frac{2}{3}\right)\) \frac{25}{24}
37. \(4 + (-7) + (-5)\) -8
38. \(10 + (-3) + (-8)\) -1
39. \(85 + (-15) + (-20) + 12\) 62
40. \(60 + (-50) + (-30) + 25\) 5
41. \(17 + (-4) + 2 + 3 + (-10)\) 8
42. \(19 + (-5) + 1 + 8 + (-13)\) 10
43. \(-45 + \left(\frac{3}{7}\right) + 25 + \left(-\frac{4}{7}\right)\) -21
44. \(-50 + \left(\frac{7}{9}\right) + 35 + \left(-\frac{11}{9}\right)\) -17
45. \(3.5 + (-45) + (-8.4) + 72\) 22.1
46. \(6.4 + (-35) + (-2.6) + 14\) -17.2

In Exercises 47–60, simplify each algebraic expression.
47. \(-10x + 2x\) -8x
48. \(-19x + 10x\) -9x
49. \(25y + (-12y)\) 13y
50. \(26y + (-14y)\) 12y
51. \(-8a + (-15a)\) -23a
52. \(-9a + (-13a)\) -22a
53. \(4y + (-13z) + (-10y) + 17z\) -6y + 4z
54. \(5y + (-11z) + (-15y) + 20z\) -10y + 9z
55. \(-7b + 10 + (-b) + (-6)\) -8b + 4
56. \(-10b + 13 + (-\ell) + (-4)\) -11b + 9
57. \(7x + (-5y) + (-9x) + 19y\) -2x + 14y
58. \(13x + (-9y) + (-17x) + 20y\) -4x + 11y
59. \(8(4y + 3) + (-35y)\) -3y + 24
60. \(7(3y + 5) + (-25y)\) -4y + 35

Practice PLUS

In Exercises 61–64, find each sum.
61. \(-3 + (-5)\) + |2 + (-6)| 12
62. \(4 + (-11)\) + |3 + (-4)| 14
63. \(-20 + \left[-15 + (-25)\right]\) -30
64. \(-25 + \left[-18 + (-26)\right]\) -33

In Exercises 65–66, insert either <, >, or = in the shaded area to make a true statement.
65. \(6 + [2 + (-13)]\) 6 + [4 + (-8)] >
66. [(8) + (-6)] + 10 = -8 + [9 + (-2)] <

In Exercises 67–70, write each English phrase as an algebraic expression. Then simplify the expression. Let x represent the number.
67. The product of -6 and a number, which is then increased by the product of -13 and a number \(-6x + (-13x); -19x\)
68. The product of -9 and a number, which is then increased by the product of -11 and a number \(-9x + (-11x); -20x\)
69. The quotient of -20 and a number increased by the quotient of 3 and a number \(-\frac{210}{x} + \frac{3}{x}; \frac{17}{x}\)
70. The quotient of -15 and a number increased by the quotient of 4 and a number \(-\frac{15}{x} + \frac{4}{x}; \frac{11}{x}\)

Application Exercises

Solve Exercises 71–78 by writing a sum of signed numbers and adding.
71. The greatest temperature variation recorded in a day is 100 degrees in Browning, Montana, on January 23, 1916. The low temperature was -56°F. What was the high temperature? 44°
72. In Spearfish, South Dakota, on January 22, 1943, the temperature rose 49 degrees in two minutes. If the initial temperature was -4°F, what was the temperature two minutes later? 45°
73. The Dead Sea is the lowest elevation on earth, 1312 feet below sea level. What is the elevation of a person standing 712 feet above the Dead Sea? 600 feet below sea level
74. Lake Assal in Africa is 512 feet below sea level. What is the elevation of a person standing 642 feet above Lake Assal? 130 feet above sea level
75. The temperature at 8:00 A.M. was -7°F. By noon it had risen 15°F, but by 4:00 P.M. it had fallen 5°F. What was the temperature at 4:00 P.M.? 3°F
76. On three successive plays, a football team lost 15 yards, gained 13 yards, and then lost 4 yards. What was the team’s total gain or loss for the three plays? loss of 6 yards

77. A football team started with the football at the 27-yard line, advancing toward the center of the field (the 50-yard line). Four successive plays resulted in a 4-yard gain, a 2-yard loss, an 8-yard gain, and a 12-yard loss. What was the location of the football at the end of the fourth play? the 25-yard line

78. The water level of a reservoir is measured over a five-month period. At the beginning, the level is 20 feet. During this time, the level rose 3 feet, then fell 2 feet, then fell 1 foot, then fell 4 feet, and then rose 2 feet. What is the reservoir’s water level at the end of the five months? 18 feet

The bar graph shows that in 2000 and 2001, the U.S. government collected more in taxes than it spent, so there was a budget surplus for each of these years. By contrast, in 2002 through 2005, the government spent more than it collected, resulting in budget deficits. Exercises 79–80 involve these deficits.

United States Government Budget Surplus/Deficit

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>236</td>
<td>128</td>
<td>228</td>
<td>200</td>
<td>300</td>
<td>300</td>
<td></td>
</tr>
</tbody>
</table>

Source: Budget of the U.S. Government

79. a. In 2004, the government collected $1880 billion and spent $2293 billion. Find $1880 + (−2293)$ and determine the deficit, in billions of dollars, for 2004. $−413$ billion

b. In 2005, the government collected $2154 billion and spent $2472 billion. Find the deficit, in billions of dollars, for 2005. $−318$ billion

c. Use your answers from parts (a) and (b) to determine the combined deficit, in billions of dollars, for 2004 and 2005. $−731$ billion

80. a. In 2002, the government collected $1853 billion and spent $2011 billion. Find $1853 + (−2011)$ and determine the deficit, in billions of dollars, for 2002. $−158$ billion

b. In 2003, the government collected $1782 billion and spent $2160 billion. Find the deficit, in billions of dollars, for 2003. $−378$ billion

c. Use your answers from parts (a) and (b) to determine the combined deficit, in billions of dollars, for 2002 and 2003. $−556$ billion

Writing in Mathematics

81. Explain how to add two numbers with a number line. Provide an example with your explanation. 81–87. Answers will vary.

82. What are additive inverses?

83. Describe how the inverse property of addition

$$a + (−a) = 0$$

can be shown on a number line.

84. Without using a number line, describe how to add two numbers with the same sign. Give an example.

85. Without using a number line, describe how to add two numbers with different signs. Give an example.

86. Write a problem that can be solved by finding the sum of at least three numbers, some positive and some negative. Then explain how to solve the problem.

87. Without a calculator, you can add numbers using a number line, using absolute value, or using gains and losses. Which method do you find most helpful? Why is this so?

Critical Thinking Exercises

Make Sense? In Exercises 88–91, determine whether each statement makes sense or does not make sense and explain your reasoning. Explanations will vary. These exercises can also be used to start group discussions.

88. It takes me too much time to add real numbers with a number line. makes sense

89. I found the sum of $−13$ and $4$ by thinking of the temperatures above and below zero: If it's $13$ below zero and the temperature rises $4$ degrees, the new temperature will be $9$ below zero, so $−13 + 4 = −9$. makes sense

90. I added two negative numbers and obtained a positive sum. does not make sense

91. Without adding numbers, I can see that the sum of $−227$ and $319$ is greater than the sum of $227$ and $−319$. makes sense

In Exercises 92–95, determine whether each statement is true or false. IF the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

92. $\frac{3}{4} + (−\frac{3}{5}) = −\frac{3}{20}$ false

93. The sum of zero and a negative number is always a negative number. true

94. If one number is positive and the other negative, then the absolute value of their sum equals the sum of their absolute values. false

95. The sum of a positive number and a negative number is always a positive number. false

In Exercises 96–99, use the number line to determine whether each expression is positive or negative.

96. $a + b$ negative

97. $a + c$ negative

98. $b + c$ positive

99. $|a + c|$ positive

Technology Exercises

100. Use a calculator to verify any five of the sums that you found in Exercises 92–96. 100–101. Answers will vary.

101. Use a calculator to verify any three of the answers that you obtained in Application Exercises 71–78.
1.6 EXERCISE SET

Practice Exercises

1. Consider the subtraction $5 - 12$.
   a. Find the opposite, or additive inverse, of $12$. $\text{ } -12$
   b. Rewrite the subtraction as the addition of the opposite of $12$. $5 + (-12)$

2. Consider the subtraction $4 - 10$.
   a. Find the opposite, or additive inverse, of $10$. $-10$
   b. Rewrite the subtraction as the addition of the opposite of $10$. $4 + (-10)$

3. Consider the subtraction $5 - (-7)$.
   a. Find the opposite, or additive inverse, of $-7$. $7$
   b. Rewrite the subtraction as the addition of the opposite of $-7$. $5 + 7$

4. Consider the subtraction $2 - (-8)$.
   a. Find the opposite, or additive inverse, of $-8$. $8$
   b. Rewrite the subtraction as the addition of the opposite of $-8$. $2 + 8$

In Exercises 5–50, perform the indicated subtraction.

5. $14 - 8 = 6$
6. $15 - 2 = 13$
7. $8 - 14 = -6$
8. $2 - 15 = -13$
9. $9 - (-20) = 23$
10. $5 - (-17) = 22$
11. $7 - (-18) = 11$
12. $-5 - (-19) = 14$
13. $-13 - (-2) = -11$
14. $-21 - (-3) = -18$
15. $-21 - 17 = -38$
16. $-29 - 21 = -50$
17. $-45 - (-45) = 0$
18. $-65 - (-65) = 0$
19. $23 - 23 = 0$
20. $26 - 26 = 0$
21. $-13 - (-13) = 26$
22. $15 - (-15) = 30$
23. $0 - 13 = -13$
24. $0 - 15 = -15$
25. $0 - (-13) = 13$
26. $0 - (-15) = 15$
27. $\frac{3}{7} - \frac{5}{7} = -\frac{2}{7}$
28. $\frac{4}{9} - \frac{7}{9} = -\frac{1}{3}$
29. $\frac{1}{5} - \left(-\frac{3}{5}\right) = \frac{4}{5}$
30. $\frac{1}{7} - \left(-\frac{3}{7}\right) = \frac{4}{7}$
31. $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$
32. $\frac{4}{9} - \frac{1}{9} = \frac{5}{9}$
33. $\frac{4}{5} - \left(-\frac{1}{5}\right) = \frac{3}{5}$
34. $\frac{4}{9} - \left(-\frac{1}{9}\right) = \frac{5}{9}$
35. $\frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$
36. $\frac{2}{5} - \left(-\frac{1}{10}\right) = \frac{1}{2}$
37. $\frac{2}{5} - \frac{1}{3} = \frac{3}{10}$
38. $\frac{2}{5} - \frac{1}{10} = \frac{3}{10}$
39. $9.8 - 2.2 = 7.6$
40. $5.7 - 3.3 = 2.4$
41. $-3.1 - (-1.1) = -2$
42. $-4.6 - (-1.1) = -3.5$
43. $1.3 - (-1.3) = 2.6$
44. $1.4 - (-1.4) = 2.8$
45. $-2.06 - (-2.06) = 0$
46. $-3.47 - (-3.47) = 0$
47. $5\pi - 2\pi = 3\pi$
48. $9\pi - 7\pi = 2\pi$
49. $3\pi - (-10\pi) = 13\pi$
50. $4\pi - (-12\pi) = 16\pi$

51. $13 - 2 - (-8) = 19$
52. $14 - 3 - (-7) = 18$
53. $9 - 8 + 3 - (-7) = -3$
54. $8 - 2 + 5 - 13 = -2$
55. $-6 - 2 + 3 - 10 = -15$
56. $-9 - 5 + 4 - 17 = -27$
57. $-10 - (-5) + 7 - 2 = 0$
58. $-6 - (-3) + 8 - 11 = -2$
59. $-23 - 11 - (-7) + (-25) = -52$
60. $-19 - 8 - (-6) + (-21) = -42$
61. $-823 - 146 - 50 - (-382) = -187$
62. $-726 - 422 - 921 - (-816) = -1253$
63. $\frac{1}{3} - \frac{2}{3} - \left(-\frac{5}{6}\right) = \frac{17}{6}$
64. $2 - \frac{3}{4} - \left(-\frac{7}{8}\right) = \frac{1}{8}$
65. $-0.16 - 5.2 - (-0.87) = -4.49$
66. $-1.9 - 3 - (-0.26) = -4.64$
67. $\frac{3}{4} - \frac{1}{4} - \left(-\frac{5}{8}\right) = \frac{3}{8}$
68. $\frac{1}{2} - \frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{5}{6}$

In Exercises 69–72, identify the terms in each algebraic expression.

69. $-3x - 8y - 3x, -8y$
70. $-9a - 4b -9a, -4b$
71. $12x - 5xy - 4 - 5xy, -4$
72. $8a - 7ab - 13 - 8a, -7ab, -13$

In Exercises 73–84, simplify each algebraic expression.

73. $3x - 9x -6x$
74. $2x - 10x -8x$
75. $4 + 7y - 17y -4 - 10y$
76. $5 + 9y - 29y -5 - 20y$
77. $2a + 5 = 9a - 5 - 7a$
78. $3a + 7 = 11a - 7 - 8a$
79. $4 - 6b - 8 - 3b -4 - 9b$
80. $5 - 7b - 13 + 4b -8 - 11b$
81. $13 - (-7)x + 4x - (11) -24 + 11x$
82. $15 - (-3)x + 8x - (-10) -25 + 11x$
83. $-5x - 10y - 3x + 13y -3y - 8x$
84. $-6x - 9y = 4x + 15y -6y - 10x$

Practice PLUS

In Exercises 85–90, find the value of each expression.

85. $-9 - (-6)| - (-12) - 9$
86. $-8 - (-2) = (-6) 0$
87. $\frac{5}{8} - \left(\frac{1}{2} \frac{3}{4}\right) = \frac{7}{8}$
88. $\frac{9}{10} - \left(-\frac{1}{4} \frac{7}{10}\right) = \frac{27}{20}$ or $\frac{1}{7} \frac{20}{20}$
89. $-9 - (-3) + 7 | - 17 - (-2) = 2$
90. $24 - (-16) -1 -51 - (-31 + 2) = 18$

In Exercises 91–94, write each English phrase as an algebraic expression. Then simplify the expression. Let \( x \) represent the number.
91. The difference between 6 times a number and $5$ times a number $6x - (-5x); 11x$
92. The difference between 9 times a number and \(-4\) times a number \(9x - (-4x) = 13x\)

93. The quotient of \(-2\) and a number subtracted from the quotient of \(-5\) and a number \(\frac{-5}{x} - \left(\frac{-2}{x}\right) = \frac{3}{x}\)

94. The quotient of \(-7\) and a number subtracted from the quotient of \(-12\) and a number \(\frac{-12}{x} - \left(\frac{-7}{x}\right) = \frac{5}{x}\)

Application Exercises

95. The peak of Mount Kilimanjaro, the highest point in Africa, is 19,321 feet above sea level. Qattara Depression, Egypt, one of the lowest points in Africa, is 436 feet below sea level. What is the difference in elevation between the peak of Mount Kilimanjaro and the Qattara Depression? \(19,757\) feet

96. The peak of Mount Whitney is 14,494 feet above sea level. Mount Whitney can be seen directly above Death Valley, which is 282 feet below sea level. What is the difference in elevation between these geographic locations? \(14,112\) feet

The bar graph shows the average daily low temperature for each month in Fairbanks, Alaska. Use the graph to solve Exercises 97–100.

97. What is the difference between the average daily low temperatures for March and February? \(21^\circ F\)

98. What is the difference between the average daily low temperatures for December and November? \(18^\circ F\)

99. How many degrees warmer is February’s average low temperature than January’s average low temperature? \(3^\circ F\)

100. How many degrees warmer is November’s average low temperature than December’s average low temperature? \(7^\circ F\)

The line graphs at the top of the next column show that from 1994 through 2004, the United States imported goods and services from other countries worth more than the value of its exports to those countries. As a result, we had a trade deficit for each year during this period. Exercises 101–102 are based on the data displayed by the graphs. Express all answers in billions of dollars.

101. a. How much did we export to other countries in 2002? \(\$1.0\) billion
   b. How much did we import from other countries in 2002? \(\$1.4\) billion
   c. Find the trade deficit, the difference between exports and imports, for 2002. \(-\$0.4\) billion

102. a. How much did we export to other countries in 1998? \(\$0.9\) billion
   b. How much did we import from other countries in 1998? \(\$1.2\) billion
   c. Find the trade deficit, the difference between exports and imports, for 1998. \(-\$0.3\) billion

The data for U.S. international trade, shown by the line graphs in Exercises 101–102, can be approximated by the following mathematical models:

Exports, in billions of dollars: \(E = 0.04n + 0.7\)

Imports, in billions of dollars: \(I = 0.09n + 0.8\)

Use these formulas to solve Exercises 103–104. Express all answers in billions of dollars.

103. a. How much did we export to other countries in 2002? \(\$1.02\) billion
   b. How much did we import from other countries in 2002? \(\$1.52\) billion
   c. According to the models, what was the trade deficit for 2002? \(\$0.5\) billion
   d. What is the difference between the actual trade deficit for 2002 that you found in Exercise 101(c) and the trade deficit given by the models? \(\$0.1\) billion

104. a. How much did we export to other countries in 1998? \(\$0.86\) billion
   b. How much did we import from other countries in 1998? \(\$1.16\) billion
   c. According to the models, what was the trade deficit for 1998? \(-\$0.3\) billion
   d. What is the difference between the actual trade deficit for 1998 that you found in Exercise 102(c) and the trade deficit given by the models? \$0. There is no difference. The models give the actual deficit.
Writing in Mathematics

105. Explain how to subtract real numbers. Answers will vary.

106. How is 4 + (−2) read? Four minus negative two

107. Explain how to simplify a series of additions and subtractions. Provide an example with your explanation. Answers will vary.

108. Explain how to find the terms of the algebraic expression 5x − 2y − 7.

109. Write a problem that can be solved by finding the difference between two numbers. At least one of the numbers should be negative. Then explain how to solve the problem.

Critical Thinking Exercises

Make Sense? In Exercises 110–113, determine whether each statement "makes sense" or "does not make sense" and explain your reasoning. Explanations will vary. These exercises can also be used to start group discussions.

110. I already knew how to add positive and negative numbers, so there was not that much new to learn when it came to subtracting them. makes sense

111. I can find the closing price of stock PQR on Wednesday by subtracting the change in price, −1.23, from the closing price on Thursday, 47.19. makes sense

<table>
<thead>
<tr>
<th>Stock</th>
<th>Thursday Close</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>31.54</td>
<td>0.47</td>
</tr>
<tr>
<td>XYZ</td>
<td>16.23</td>
<td>−0.87</td>
</tr>
<tr>
<td>PQR</td>
<td>47.19</td>
<td>−1.23</td>
</tr>
<tr>
<td>DEF</td>
<td>21.54</td>
<td>0.21</td>
</tr>
</tbody>
</table>

112. I found the variation in elevation between two heights by taking the difference between the high point and the low point. makes sense

113. I found the variation in U.S. temperature by subtracting the record low temperature, a negative number, from the record high temperature, a positive number. makes sense

In Exercises 114–117, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

114. If a and b are negative numbers, then a − b is sometimes a negative number. true

115. 7 − (−2) = 5 false

116. The difference between 0 and a negative number is always a positive number. true

117. |a − b| = |b − a| true

In Exercises 118–121, use the number line to determine whether each difference is positive or negative.

118. c − a positive 119. a − b negative

120. b − c negative 121. 0 − b positive

122. Order the expressions |x − y|, |x| − |y|, and |x + y| from least to greatest for x = −6 and y = −8.

Technology Exercises

123. Use a calculator to verify any five of the differences that you found in Exercises 5–46. Answers will vary.

124. Use a calculator to verify any three of the answers you found in Exercises 51–68. Answers will vary.

Review Exercises

125. Determine whether 2 is a solution of 13x + 3 = 3(5x − 1). (Section 1.1, Example 4) not a solution

126. Simplify: 5(3x + 2y) + 6(5y). (Section 1.4, Example 11) 15x + 40y

127. Give an example of an integer that is not a natural number. (Section 1.3, Example 5) Answers will vary.

Preview Exercises

Exercises 128–130 will help you prepare for the material covered in the next section. In Exercises 128–129, a multiplication is expressed as a repeated addition. Find this sum, indicated by a question mark.

128. 4(−3) = (−3) + (−3) + (−3) + (−3) = ? −12

129. 3(−3) = (−3) + (−3) + (−3) = ? −9

130. The list shows a pattern for various products.

| 2(−3) | = −6 |
| 1(−3) | = −3 |
| 0(−3) | = 0  |
| −1(−3) | = 3 |
| −2(−3) | = 6 |
| −3(−3) | = 9 |

Reading down the list, products keep increasing by 3.

Use this pattern to find −4(−3). 12
CHECK POINT 9  The data for doctorate degrees shown in Figure 1.13 on page 77 can be described by a linear model: $y = -0.6x + 64.4$, where $y$ is the percentage of doctorate degrees awarded to men $n$ years after 1989. According to this mathematical model, what percentage of doctorate degrees are projected to be received by men in 2014? Does this underestimate or overestimate the projection shown in Figure 1.13? By how much? 49.4%, overestimates by 0.4%

1.7 EXERCISE SET

Practice Exercises
In Exercises 1–34, perform the indicated multiplication.
1. 5(-9) -45  
2. 10(-7) -70  
3. (-8)(-3) 24  
4. (-9)(-5) 45  
5. (-3)(7) -21  
6. (-4)(8) -32  
7. (-19)(-1) 19  
8. (-11)(-1) 11  
9. 0(-19) 0  
10. 0(-11) 0  
11. \(\frac{1}{2}(-24)\) -12  
12. \(\frac{1}{3}(-21)\) -7  
13. \(-\frac{3}{4}(-12)\) 9  
14. \(-\frac{4}{5}(-30)\) 24  
15. \(-\frac{3}{5}\left(-\frac{4}{7}\right)\) \(\frac{12}{35}\)  
16. \(-\frac{5}{7}\left(-\frac{3}{8}\right)\) \(\frac{15}{56}\)  
17. \(-\frac{7}{9}\left(-\frac{2}{3}\right)\) \(\frac{14}{27}\)  
18. \(-\frac{5}{11}\left(-\frac{2}{7}\right)\) \(\frac{10}{77}\)  
19. 3(-1.2) -3.6  
20. 4(-1.2) -4.8  
21. -0.2(-0.6) 0.12  
22. -0.3(-0.7) 0.21  
23. (-5)(-2)(3) 30  
24. (-6)(-3)(10) 180  
25. (-4)(-3)(-1)(6) -72  
26. (-2)(-7)(-1)(3) -42  
27. -(2)(-3)(-4)(-1) 24  
28. -(5)(-2)(-5)(-1) 30  
29. -(3)(-3)(-3) -27  
30. -(4)(-4)(-4) -64  
31. 5(-3)(-1)(2)(3) 90  
32. 2(-5)(-2)(3)(1) 60  
33. -(8)(-4)(0)(-17)(-6) 0  
34. -(9)(-12)(-18)(0)(-3) 0  

In Exercises 35–42, find the multiplicative inverse of each number.
35. 4 \(\frac{1}{4}\)  
36. 3 \(\frac{1}{3}\)  
37. \(\frac{1}{5}\) \(\frac{5}{1}\)  
38. \(\frac{1}{7}\) 7  
39. -10 \(-\frac{1}{10}\)  
40. -12 \(-\frac{1}{12}\)  
41. \(-\frac{2}{5}\) \(-\frac{5}{2}\)  
42. \(-\frac{4}{9}\) \(-\frac{9}{4}\)  

In Exercises 43–46,

a. Rewrite the division as multiplication involving a multiplicative inverse.

b. Use the multiplication from part (a) to find the given quotient.
43. -32 \(\div\) \(\frac{1}{4}\) a. \(-32 \cdot \frac{1}{4}\) b. -8  
44. -18 \(\div\) \(\frac{1}{6}\) a. \(-18 \cdot \frac{1}{6}\) b. -3  
45. \(-\frac{60}{-5}\) a. \(-60 \cdot \frac{-1}{5}\) b. 12  
46. \(-\frac{30}{-5}\) a. \(-30 \cdot \frac{1}{5}\) b. 6  

In Exercises 47–76, perform the indicated division or state that the expression is undefined.
47. \(\frac{12}{-4}\) -3  
48. \(\frac{40}{-5}\) -8  
49. \(\frac{-21}{3}\) -7  
50. \(\frac{-60}{6}\) -10  
51. \(\frac{-90}{-3}\) 30  
52. \(\frac{-66}{-6}\) 11  
53. \(\frac{0}{-7}\) 0  
54. \(\frac{0}{-8}\) 0  
55. \(\frac{7}{0}\) undefined  
56. \(-\frac{8}{0}\) undefined  
57. \(-15 \div -5\) 3  
58. \(-80 \div 8\) -10  
59. \(\frac{120}{-10}\) -12  
60. \(\frac{130}{-10}\) -13  
61. \(\frac{-180}{-30}\) 6  
62. \(\frac{-150}{-25}\) 6  
63. \(0 \div -4\) 0  
64. \(0 \div -10\) 0  
65. \(-4 \div 0\) undefined  
66. \(-10 \div 0\) undefined  
67. \(-\frac{12.9}{3}\) -4.3  
68. \(-\frac{21.6}{3}\) -7.2  
69. \(-\frac{1}{2} \div \left(-\frac{3}{5}\right)\) \(\frac{5}{6}\)  
70. \(-\frac{1}{2} \div \left(-\frac{7}{9}\right)\) \(\frac{9}{14}\)  
71. \(-\frac{14}{9} \div \frac{7}{8}\) \(\frac{16}{9}\)  
72. \(-\frac{5}{16} \div \frac{25}{8}\) \(-\frac{1}{10}\)  
73. \(\frac{1}{3} \div \left(-\frac{1}{3}\right)\) -1  
74. \(\frac{1}{5} \div \left(-\frac{1}{5}\right)\) -1  
75. \(6 \div \left(-\frac{2}{5}\right)\) -15  
76. \(8 \div \left(-\frac{2}{9}\right)\) -36

In Exercises 77–96, simplify each algebraic expression.
77. \(-5(2x)\) -10x  
78. \(-9(3x)\) -27x  
79. \(-4\left(-\frac{3}{4}y\right)\) 3y  
80. \(-5\left(-\frac{3}{5}y\right)\) 3y  
81. \(8x + x\) 9x  
82. \(12x + x\) 13x  
83. \(-5x + x\) -4x  
84. \(-6x + x\) -5x  
85. \(6b - 7b\) -b  
86. \(12b - 13b\) -b
87. \(-y + 4y = 3y\) \hspace{1cm} 88. \(-y + 9y = 8y\)
89. \(-4(2x - 3) = -8x + 12\) \hspace{1cm} 90. \(-3(4x - 5) = -12x + 15\)
91. \(-3(-2x + 4) = 6x - 12\) \hspace{1cm} 92. \(-4(-3x + 2) = 12x - 8\)
93. \([-2y - 5] = 2y + 5\) \hspace{1cm} 94. \((-3y - 1) = -3y + 1\)
95. \((4y - 3) = (7y + 2)\) \hspace{1cm} 96. \((3y - 1) = (14y - 2)\)

In Exercises 97–108, determine whether the given number is a solution of the equation.

97. \(4x = 2x - 10\); \(-5\) \hspace{1cm} solution
98. \(5x = 3x - 6\); \(-3\) \hspace{1cm} solution
99. \(-7y + 18 = -10y + 6\); \(-4\) \hspace{1cm} solution
100. \(-4y + 21 = -7y + 15\); \(-2\) \hspace{1cm} solution

\(\textbf{101.} 5(w + 3) = 2w - 21\); \(-10\) \hspace{1cm} not a solution

102. \(6(w + 2) = 4w - 10\); \(-9\) \hspace{1cm} not a solution
103. \(4(6 - z) + 7z = 0\); \(-8\) \hspace{1cm} solution
104. \(5(7 - z) + 12z = 0\); \(-5\) \hspace{1cm} solution

105. \(12 = 2x - 4x + 7\); \(-2\frac{1}{2}\) \hspace{1cm} not a solution
106. \(16 = 4x - 2x + 21\); \(-3\frac{1}{2}\) \hspace{1cm} not a solution

107. \(5m - 1 = \frac{3m - 2}{6}; -4\) \hspace{1cm} solution

\(6m - 5 = \frac{3m - 2}{11}; -1\) \hspace{1cm} solution

\textbf{Practice PLUS}

In Exercises 109–116, write a numerical expression for each phrase. Then simplify the numerical expression by performing the given operations.

109. 8 added to the product of 4 and \(-10\) \hspace{1cm} \(4(-10) + 8 = -32\)
110. 14 added to the product of 3 and \(-15\) \hspace{1cm} \(3(-15) + 14 = -31\)

111. The product of \(-9\) and \(-3\), decreased by \(-2\) \hspace{1cm} \((-9)(-3) - (-2) = 29\)
112. The product of \(-6\) and \(-4\), decreased by \(-5\) \hspace{1cm} \((-6)(-4) - (-5) = 29\)

113. The quotient of \(-18\) and the sum of \(-15\) and 12 \hspace{1cm} \(-18 + 12 = 6\)
114. The quotient of 25 and the sum of 31 and 16 \hspace{1cm} \(25 + 16 = 5\)
115. The difference between \(-6\) and the quotient of 12 and \(-4\) \hspace{1cm} \(-6 - \left(\frac{12}{-4}\right) = -3\)
116. The difference between \(-11\) and the quotient of 20 and \(-5\) \hspace{1cm} \(-11 - \left(\frac{20}{-5}\right) = -7\)

\textbf{Application Exercises}

In Exercises 117–118, use the formula \(C = \frac{5}{9}(F - 32)\) to express each Fahrenheit temperature, \(F\), as its equivalent Celsius temperature, \(C\).

117. \(-22°F = -30°C\)
118. \(-31°F = -35°C\)

The graph shows the percentage of high school seniors who used alcohol or marijuana during the 30 days prior to being surveyed for the University of Michigan's Monitoring the Future study.

\textbf{Use this information to solve Exercises 119–120.}

119. a. Use the appropriate line graph to determine the percentage of seniors who used alcohol in 2000. 50\%  
b. Use the appropriate formula to determine the percentage of seniors who used alcohol in 2000. What do you observe? 50\%; The model provides an exact description of the data.

c. Use the appropriate line graph to estimate the percentage of seniors who used marijuana in 2000. Answers will vary; approximately 22\%.

d. Use the appropriate formula to determine the percentage of seniors who used marijuana in 2000. How does this compare with your estimate in part (c)? 18\%; It's less than the estimate.

e. Write a formula that describes the ratio of the percentage of seniors who used marijuana to the percentage who used alcohol. Name this new mathematical model \(R\), for ratio. \(R = \frac{-0.5n + 28}{-n + 70}\)

120. a. Use the appropriate line graph to estimate the percentage of seniors who used alcohol in 2004. Answers will vary; approximately 47\%.

b. Use the appropriate formula to determine the percentage of seniors who used alcohol in 2004. How does this compare with your estimate in part (a)? 46\%; It's very close to the estimate.

c. Use the appropriate line graph to determine the percentage of seniors who used marijuana in 2004. 30\% 

d. Use the appropriate formula to determine the percentage of seniors who used marijuana in 2004.
Does this underestimate or overestimate the percent that you found in part (c)? By how much?

16%, underestimates by 4%.

e. Write a formula that describes the ratio of the percentage of seniors who used alcohol to the percentage who used marijuana. Name this new mathematical model \( R \), for ratio.

\[
R = \frac{-0.5n + 28}{-0.1n + 70}
\]

121. The graph shows the sources of income for the federal government in 1960 and 2005.

**Sources of Income for the United States Government**

<table>
<thead>
<tr>
<th>Year</th>
<th>Individual income tax</th>
<th>Corporate income tax</th>
<th>Social security taxes</th>
<th>Excise taxes</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>44%</td>
<td>23%</td>
<td>16%</td>
<td>13%</td>
<td>4%</td>
</tr>
<tr>
<td>2005</td>
<td>44%</td>
<td>11%</td>
<td>38%</td>
<td>4%</td>
<td>3%</td>
</tr>
</tbody>
</table>


The graph indicates that the percentage of money collected from corporations has decreased and the percentage coming from income taxes has not changed.

a. Use the data in the graph to write a simplified fraction that represents the 2005 ratio of percentage of income taxes from corporations to percentage from individual income taxes. According to this rational number, for every $10,000 collected in individual income taxes, how much was collected from corporate income taxes? \( \frac{1}{3}; $2500\)

b. Write the rational number in part (a) in decimal form. 0.25

c. Here is a mathematical model that describes the diminishing share, \( D \), of the tax burden by corporations \( n \) years after 1960:

\[
D = \frac{-0.27n + 23}{44}
\]

Use the model to write a quotient of two numbers that represents the 2005 ratio of percentage of corporate income taxes to percentage of individual income taxes. \( \frac{10.85}{44} \)

d. Express the quotient of the two numbers in part (c) in decimal form, correct to two decimal places. How well does this rounded decimal describe the value obtained from the data in part (b)? 0.25; Perfectly well; The rounded decimal gives the value obtained from the data.

**Critical Thinking Exercises**

**Make Sense?** In Exercises 130–133, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning. *Explanations will vary.* These exercises can also be used to start group discussions.

130. I've noticed that the sign rules for dividing real numbers are slightly different than the sign rules for multiplying real numbers. **does not make sense**

131. Just as two negative factors give a positive product, I've seen the same thing occur with double negatives in English:

**That number is unknown** means **That number is known**

**makes sense**

132. This pattern suggests that multiplying two negative numbers results in a positive answer:

\[
-2(-3) = -6; 1(-3) = -3; 0(-3) = 0; -1(-3) = 3; -2(-3) = 6.
\]

**Decreasing by 1 from left to right**

**Increasing by 3 from left to right**

**makes sense**

133. When I used

\[
R = \frac{M}{W} = \frac{-0.28n + 47}{0.28n + 53}
\]

**Number of years after 1989**

to project the ratio of bachelor's degrees received by men to degrees received by women in 2020, I had to find the quotient of two negative numbers. **does not make sense**
In Exercises 134–137, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

134. Both the addition and the multiplication of two negative numbers result in a positive number.  
false

135. Multiplying a negative number by a nonnegative number will always give a negative number.  
false

136. $0 + (-\sqrt{2})$ is undefined.  
false

137. If $a$ is negative, $b$ is positive, and $c$ is negative, then $\frac{a}{bc}$ is positive.  
true

In Exercises 138–141, write an algebraic expression for the given English phrase.

138. The value, in cents, of $x$ nickels  
$5x$

139. The distance covered by a car traveling at 50 miles per hour for $x$ hours  
$50x$

140. The monthly salary, in dollars, for a person earning $x$ dollars per year  
$\frac{x}{12}$

141. The fraction of people in a room who are women if there are 40 women and $x$ men in the room  
$\frac{40}{x+40}$

Technology Exercises

142. Use a calculator to verify any five of the products that you found in Exercises 1–34.  
Answers will vary.

143. Use a calculator to verify any five of the quotients that you found in Exercises 47–76.  
Answers will vary.

144. Simplify using a calculator:

$$0.3(4.7x - 5.9) - 0.07(3.8x - 61).$$

$$1.144x + 2.5$$

145. Use your calculator to attempt to find the quotient of $-3$ and 0. Describe what happens. Does the same thing occur when finding the quotient of 0 and $-3$? Explain the difference. Finally, what happens when you enter the quotient of 0 and itself?  
Answers will vary.

Review Exercises

In Exercises 146–148, perform the indicated operation.

146. $-6 + (-3)$ (Section 1.5, Example 3)  
$-9$

147. $-6 - (-3)$ (Section 1.6, Example 1)  
$-3$

148. $-6 + (-3)$ (Section 1.7, Example 4)  
$-2$

Preview Exercises

Exercises 149–151 will help you prepare for the material covered in the next section. In each exercise, an expression with an exponent is written as a repeated multiplication. Find this product, indicated by a question mark.

149. $(-6)^2 = (-6)(-6) = ?$  
$36$

150. $(-5)^3 = (-5)(-5)(-5) = ?$  
$-125$

151. $(-2)^4 = (-2)(-2)(-2)(-2) = ?$  
$16$

---

### 1.8 Exponents and Order of Operations

**Objectives**

1. Evaluate exponential expressions.
2. Simplify algebraic expressions with exponents.
3. Use the order of operations agreement.
4. Evaluate mathematical models.

Traffic is really backed up on the highway. Finally, you see the source of the traffic jam—a minor fender-bender. Still stuck in traffic, you notice that the driver appears to be quite young. This might seem like a strange observation. After all, what does a driver's age have to do with his or her chance of getting into an accident? In this section, we continue to see how mathematical models describe your world, including a formula that provides a relationship between age and number of car accidents.

**Natural Number Exponents**

Although people do a great deal of talking, the total output since the beginning of gabble to the present day, including all baby talk, love songs, and congressional debates, only amounts to about 10 million billion words. This can be expressed as 16 factors of 10, or $10^{16}$ words.
1.8 EXERCISE SET

Practice Exercises

In Exercises 1–14, evaluate each exponential expression.
1. 2^3 = 8
2. 3^2 = 9
3. 4^3 = 64
4. 6^3 = 216
5. (-4)^2 = 16
6. (-10)^2 = 100
7. (-4)^3 = -64
8. (-10)^3 = -1000
9. (-5)^4 = 625
10. (-1)^5 = 1
11. -5^4 = -625
12. -6^5 = -1
13. -10^2 = -100
14. -8^2 = -64

In Exercises 15–28, simplify each algebraic expression, or explain why the expression cannot be simplified.
15. 7x^2 + 12x^2 = 19x^2
16. 6x^2 + 18x^2 = 24x^2
17. 10x^3 + 5x^3 = 15x^3
18. 14x^3 + 8x^2 = 22x^2
19. 8x^4 + x^4 = 9x^4
20. 14x^4 + x^4 = 15x^4
21. 26x^2 - 27x^2 = -x^2
22. 29x^2 - 30x^2 = -x^2
23. 27x^3 - 26x^3 = x^3
24. 30x^3 - 29x^3 = x^3
25. 5x^2 + 5x^3 cannot be simplified
26. 8x^2 + 8x^3 cannot be simplified
27. 16x^2 - 16x^2 = 0
28. 34x^2 - x^2 = 33x^2

In Exercises 29–72, use the order of operations to simplify each expression.
29. 7 + 6 · 3 = 25
30. 3 + 4 · 5 = 23
31. 45 + 5 · 3 = 37
32. 40 + 4 · 2 = 20
33. 6 · 8 + 4 = 52
34. 8 · 6 + 2 = 50
35. 14 - 2 - 6 + 3 = 5
36. 36 - 12 ÷ 4 + 2 = 38
37. 8^2 - 16 + 2^2 - 4 = 35
38. 10^2 - 100 ÷ 5^2 - 2 = 91
39. 3(-2)^2 - 4(-3)^2 = -24
40. 5(-3)^2 - 2(-4)^2 = 13
41. (4 · 3)^2 - 3 = 57
42. (3 · 5)^2 - 3 · 5^2 = 150
43. (2 - 6)^2 = (3 - 7)^2 = 16
44. (4 - 6)^2 - (5 - 9)^2 = -12
45. 6(3 - 5)^2 - 2(1 - 3)^2 = 32
46. -3(-6 + 8)^3 - 5(-3 + 5)^3 = 64
47. [2(6 - 2)^2 = 64
48. [3(4 - 6)^3 = -216
49. [2(5 + 2(9 - 4)] = 36
50. [3(4 + 3(10 - 8)) = 30
51. [7 + 3(2^2 - 1)] = 21
52. [11 - 4(2 - 3^3)] = 37
53. 10 + 8
54. 6^2 - 4^2
55. 37 + 15 + (-3)
56. 3^2 + 2^4
57. (-11)(-4) + 2(-7)
58. 5(7 - 2) - 3(4 - 7)
59. 4(10 - (8 - 20)]
60. 6[7 · 4 - 3]
61. 8(-10) + |4(-5)| = -60
62. 4(-15) + |3(-10)| = -30
63. -2^2 + 4(16 + (3 - 5)] = -36
64. -3^2 + 2[20 + (7 - 11)] = -19
65. 24 ÷ 3^2 = 8 ÷ 5 = 14
66. 30 ÷ 5^2 = 7 ÷ 12 = 100
67. 1 - 1
68. 3 - 7 = 10
69. 3 - 5 = 10
70. 1
71. 3 - 7 = 10
72. 3 - 10 = 10
73. 7 + 3 = 7
74. x^2 - 10x; x = 1 = 9
75. 3x^2 - 8x; x = 2 = 28
76. 4x^2 - 2x; x = -3 = 42
77. -x^2 - 13x; x = 1 = 7
78. -x^2 - 14x; x = -1 = 13
79. -1 = 7
80. -1 = 3

In Exercises 73–80, evaluate each algebraic expression for the given value of the variable.
73. x^2 + 5x; x = 3 = 24
74. x^2 - 2x; x = 6 = 24
75. 3x^2 - 8x; x = -2 = 28
76. 4x^2 - 2x; x = -3 = 42
77. -x^2 - 10x; x = -1 = 9
78. -x^2 - 14x; x = 1 = 13
79. -1 = 7
80. -1 = 3

In Exercises 81–88, simplify each algebraic expression by removing parentheses and brackets.
81. 3[5(x - 2) + 1] = 15x - 27
82. 4[6(x - 3) + 1] = 24x - 68
83. 3(6 + (y + 1)) = 15 - 3y
84. 5[2 - (y + 3)] = -5y - 5
85. 7 - 4[3 - (4y - 5)] = 16y - 25
86. 6 - 5[8 - (2y - 4)] = 10y - 54
87. 2[3x^2 - 5] - [4(2x^2 - 1)] = -2x^2 - 9
88. 4(6x^2 - 3) - [2(5x^2 - 1)] = 14x^2 - 11

Practice PLUS

In Exercises 89–92, express each sentence as a single numerical expression. Then use the order of operations to simplify the expression.
89. Cube -2. Subtract this exponential expression from -10. -10 - (-2)^3 = -2
90. Cube -5. Subtract this exponential expression from -100. -100 - (-5)^3 = 25
91. Subtract 10 from 7. Multiply this difference by 2. Square this product. \(2(7 - 10)^2 = 36\)

92. Subtract 11 from 9. Multiply this difference by 2. Raise this product to the fourth power. \(2(9 - 11)^4 = 256\)

In Exercises 93–96, let \(x\) represent the number. Express each sentence as a single algebraic expression. Then simplify the expression.

93. Multiply a number by 5. Add 8 to this product. Subtract this sum from the number. \(x - (5x + 8) = -4x - 8\)

94. Multiply a number by 3. Add 9 to this product. Subtract this sum from the number. \(x - (3x + 9) = -2x - 9\)

95. Cube a number. Subtract 4 from this exponential expression. Multiply this difference by 5. \(5(x^3 - 4) = 5x^3 - 20\)

96. Cube a number. Subtract 6 from this exponential expression. Multiply this difference by 4. \(4(x^3 - 6) = 4x^3 - 24\)

Application Exercises

The United States has more people in prison, as well as more people in prison per capita, than any other western industrialized nation. The bar graph shows the number of inmates in U.S. state and federal prisons for five selected years from 1980 through 2004.

Source: U.S. Justice Department

The mathematical model

\[I = -0.09n^2 + 53n + 315\]

describes the number of inmates, \(I\), in thousands, \(n\) years after 1980. Use this information to solve Exercises 97–98.

97. Use the formula to find the number of inmates, in thousands, in 1990. Does this underestimate or overestimate the actual number shown by the graph? By how much?

836 thousand; overestimates by 62 thousand

98. Use the formula to find the number of inmates, in thousands, in 2000. Does this underestimate or overestimate the actual number shown by the graph? By how much?

1339 thousand; underestimates by 33 thousand

The line graphs show the percentage of people who used the Internet for four selected years, by level of education. Exercises 99–101 involve mathematical models for the data.

Source: Pew Internet and American Life Project

99.a. Use the appropriate line graph to estimate the percentage of people at the college plus level of education who used the Internet in 2005. 92%, although answers may vary by ±1%.

b. The mathematical model

\[C = 0.006n^2 + 3.3n + 75\]
describes the percentage of people in the college plus group, \(C\), who used the Internet \(n\) years after 2000. Use the formula to find the percentage of people at this education level who used the Internet in 2005. How does this value compare to your estimate in part (a)? 91.65%; very close to the estimate

100. a. Use the appropriate line graph to estimate the percentage of people with some college who used the Internet in 2004. 75%

b. The mathematical model

\[S = -0.01n^2 + 3.5n + 63.5\]
describes the percentage of people with some college, \(S\), who used the Internet \(n\) years after 2000. Use the formula to find the percentage of people at this education level who used the Internet in 2004. How does this value compare to your estimate in part (a)? 77.34%; slightly exceeds the estimate

101. a. Use the appropriate line graph to estimate the percentage of people at the high school graduate level of education who used the Internet for each of the four years shown. 2000: 34%; 2002: 43%; 2004: 52%; 2005: 62%; Answers may vary by ±1%.

b. Based on your estimates in part (a), select the correct mathematical model, model 1 or model 2, that describes the percentage of people in the high school graduate group, \(H\), who used the Internet \(n\) years after 2000.

Model 1: \[H = 0.41n^2 + 2.1n + 17\]
Model 2: \[H = 0.26n^2 + 3.9n + 34.5\]
In Palo Alto, California, a government agency ordered computer-related companies to contribute to a pool of money to clean up underground water supplies. (The companies had stored toxic chemicals in leaking underground containers.) The mathematical model

\[ C = \frac{200x}{100 - x} \]

describes the cost, \( C \), in tens of thousands of dollars, for removing \( x \) percent of the contaminants. Use this formula to solve Exercises 102–103.

102. a. Find the cost, in tens of thousands of dollars, for removing 50% of the contaminants.
\$2,000,000 (or $200 tens of thousands)

b. Find the cost, in tens of thousands of dollars, for removing 80% of the contaminants.
\$8,000,000 (or $800 tens of thousands)

c. Describe what is happening to the cost of the cleanup as the percentage of contaminant removed increases. Cost increases.

103. a. Find the cost, in tens of thousands of dollars, for removing 60% of the contaminants.
\$3,000,000 (or $300 tens of thousands)

b. Find the cost, in tens of thousands of dollars, for removing 90% of the contaminants.
\$18,000,000 (or $1800 tens of thousands)

c. Describe what is happening to the cost of the cleanup as the percentage of contaminants removed increases. Cost increases.

Writing in Mathematics

104. Describe what it means to raise a number to a power. In your description, include a discussion of the difference between \(-5^2\) and \((-5)^2\). 104–106. Answers will vary.

105. Explain how to simplify \(4x^2 + 6x^2\). Why is the sum not equal to \(10x^4\)?

106. Why is the order of operations agreement needed?

Critical Thinking Exercises

Make Sense? In Exercises 107–110, determine whether each statement “makes sense” or “does not make sense” and explain your reasoning. Explanations will vary. These exercises can also be used to start group discussions.

107. Without parentheses, an exponent has only the number next to it as its base. makes sense

108. I read that a certain star is \(10^6\) light-years from Earth, which means 100,000 light-years. does not make sense

109. When I evaluated \((-1)^n\), I obtained positive numbers when \(n\) was even and negative numbers when \(n\) was odd. makes sense

110. The rules for the order of operations avoid the confusion of obtaining different results when I simplify the same expression. makes sense

In Exercises 111–114, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement. Changes to false statements will vary.

111. If \(x = -3\), then the value of \(-3x - 9\) is \(-18\). false

112. The algebraic expression \(6x + 6\) cannot have the same value when two different replacements are made for \(x\) such as \(x = -3\) and \(x = 2\). false

113. The value of \(\frac{3 - 7}{-2} - 2^3\) is the fraction that results when \(\frac{1}{3}\) is subtracted from \(-\frac{1}{3}\). true

114. \(-2(6 - 4)^3 = -2(6 - 16)^3\)
\[= -2(-10)^3 = (-20)^3 = -8000 \quad \text{false} \]

115. Simplify: \(\frac{1}{4} - 6(2 + 8) + \left( \frac{1}{3} \right) \left( \frac{1}{9} \right) \cdot \frac{79}{4} \)

In Exercises 116–117, insert parentheses in each expression so that the resulting value is 45.

116. \(2 \cdot 3 + 3 \cdot 5 = (2 \cdot 3 + 3) \cdot 5 = 45\)

117. \(2 \cdot 5 - \frac{1}{2} \cdot 10 \cdot 9 = (2 \cdot 5 - \frac{1}{2} \cdot 10) \cdot 9 = 45\)

Review Exercises

118. Simplify: \(-8 - 2 - ( - 5 ) + 11\). (Section 1.6, Example 3) \(\frac{6}{6}\)

119. Multiply: \(-4(-1)(-3)(2)\). (Section 1.7, Example 2) \(-24\)

120. Give an example of a real number that is not an irrational number. (Section 1.3, Example 5). Answers will vary: \(-3\)

is an example.

Preview Exercises

Exercises 121–123 will help you prepare for the material covered in the first section of the next chapter. In each exercise, determine whether the given number is a solution of the equation.

121. \(-\frac{1}{2} = x - \frac{2}{3}\) \(\frac{1}{6}\) solution

122. \(5y + 3 - 4y - 8 = 15\); \(20\) solution

123. \(4x + 2 = 3(x - 6) + 8; -11\) not a solution

GROUP PROJECT

One measure of physical fitness is your resting heart rate. Generally speaking, the more fit you are, the lower your resting heart rate. The best time to take this measurement is when you first awaken in the morning, before you get out of bed. Lie on your back with no body parts crossed and take your pulse in your neck or wrist. Use your index and second fingers and count your pulse beat for one full minute to get your resting heart rate. A resting heart rate under 48 to 57 indicates high fitness, 58 to 62, above average fitness, 63 to 70, average fitness, 71 to 82, below average fitness, and 83 or more, low fitness.

(The project continues on the next page.)
CHAPTER 1 TEST

In Exercises 1–10, perform the indicated operation or operations.

1. \[1.4 - (-2.6) = 4\]
2. \[-9 + 3 + (-11) + 6 = -11\]
3. \[3(-17) = -51\]
4. \[
\left(-\frac{3}{4}\right) + \left(-\frac{15}{7}\right) = \frac{1}{5}
\]
5. \[
\left(3 - \frac{1}{3}\right) \left(-\frac{3}{4}\right) = \frac{-35}{6} \text{ or } -\frac{5}{6}
\]
6. \[-50 + 10 = -5\]
7. \[-6 - (5 - 12) = 1\]
8. \[-3(-4) + (7 - 10) = -4\]
9. \[(6 - 8)^2(5 - 7)^3 = -32\]
10. \[
\frac{3(-2) - 2(2)}{-2(8 - 3)} = 1
\]

In Exercises 11–13, simplify each algebraic expression.
11. \[11x - (7x - 4) = 4x + 4\]
12. \[5(3x - 4y) - (2x - y) = 13x - 19y\]
13. \[6 - 2[3(x + 1) - 5] = 10 - 6x\]

14. List all the rational numbers in this set.
\[\left\{-\frac{4}{5}, 0, 0.25, \sqrt{3}, \sqrt{4}, \frac{22}{7}, \pi\right\}\]

15. Insert either < or > in the shaded area to make a true statement: \(-1 \quad \square \quad -100.\)  \(>\)

16. Find the absolute value: \[| -12.8 | = 12.8\]

In Exercises 17–18, evaluate each algebraic expression for the given value of the variable.
17. \[5(x - 7); x = 4 = -15\]
18. \[x^2 - 5x; x = -10 = 150\]

19. Use the commutative property of addition to write an equivalent algebraic expression: \[2(x + 3).
\[2(3 + x)\]

20. Use the associative property of multiplication to rewrite \(-6(4x).\) Then simplify the expression.
\[(-6 \cdot 4)x = -24x\]

21. Use the distributive property to rewrite without parentheses: \[7(5x - 1 + 2y).
\[35x - 7 + 14y\]

22. What is the difference in elevation between a plane flying 16,200 feet above sea level and a submarine traveling 830 feet below sea level? \[17,030\text{ feet}\]

In Exercises 23–24, determine whether the given number is a solution of the equation.
23. \[
\frac{1}{3}(x + 2) = \frac{1}{10}x + \frac{3}{5} ; 3
\]
not a solution

24. \[3(x + 2) - 15 = 4x - 9 \quad \text{solution}\]

In Exercises 25–26, translate from English to an algebraic expression or equation, whichever is appropriate. Let the variable \(x\) represent the number.
25. \[\frac{1}{4}\text{ of a number, decreased by 5, is 32.} \quad \frac{1}{4}x - 5 = 32\]

26. Seven subtracted from the product of 5 and 4 more than a number \[5(x + 4) - 7\]

27. The Sopranos saga showed just how complex and involving TV storytelling could be. The bar graph shows the number of viewers in the series opening episodes.

![Bar Graph](image)

Number of Viewers of Opening Episodes of The Sopranos

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Viewers (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>3.5</td>
</tr>
<tr>
<td>2000</td>
<td>7.6</td>
</tr>
<tr>
<td>2001</td>
<td>11.3</td>
</tr>
<tr>
<td>2002</td>
<td>13.4</td>
</tr>
<tr>
<td>2003</td>
<td>12.1</td>
</tr>
<tr>
<td>2004</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Source: Nielsen Media Research

The number of viewers of the opening episodes of The Sopranos, \(V,\) in millions, can be described by the mathematical model
\[V = -n^2 + 8n - 4,\]

where \(n\) is the season number. Use the formula to find the number of viewers of the opening episode of season 3. Does the mathematical model underestimate or overestimate the actual number of viewers shown by the bar graph? By how many million? \[11\text{ million};\] underestimate by 0.3 million.
28. Electrocardiograms are used in exercise stress tests to determine a person's fitness for strenuous exercise. The target heart rate, in beats per minute, for such tests depends on a person's age. The line graph shows target heart rates for stress tests for people of various ages.

Target Heart Rates for Stress Tests

Use the graph to estimate the target heart rate for a 40-year-old taking a stress test. 144 beats per minute, although answers may vary by ±1.

29. The formula \( H = \frac{2}{3}(220 - a) \) gives the target heart rate, \( H \), in beats per minute, on a stress test for a person of age \( a \). Use the formula to find the target heart rate for a 40-year-old. How does this compare with your estimate from Example 28? 144 beats per minute; very well; it is the same.