Chapter 16

Sound Waves

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Speed of Sound Waves

• Use a compressible gas as an example with a setup as shown at right
• Before the piston is moved, the gas has uniform density
• When the piston is suddenly moved to the right, the gas just in front of it is compressed
  ➢ Darker region in the diagram
Speed of Sound Waves, cont

- When the piston comes to rest, the compression region of the gas continues to move
  - This corresponds to a longitudinal pulse traveling through the tube with speed \( v \)
  - The speed of the piston is not the same as the speed of the wave
Speed of Sound Waves, General

• The speed of sound waves in a medium depends on the compressibility and the density of the medium
• The compressibility can sometimes be expressed in terms of the elastic modulus of the material
• The speed of all mechanical waves follows a general form:

\[ v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \]
Speed of Sound in Liquid or Gas

• The bulk modulus of the material is $B$
• The density of the material is $\rho$
• The speed of sound in that medium is

$$v = \sqrt{\frac{B}{\rho}}$$
Speed of Sound in a Solid Rod

- The Young’s modulus of the material is \( Y \)
- The density of the material is \( \rho \)
- The speed of sound in the rod is

\[
v = \sqrt{\frac{Y}{\rho}}\]
Speed of Sound in Air

- The speed of sound also depends on the temperature of the medium.
- This is particularly important with gases.
- For air, the relationship between the speed and temperature is

\[ v = (331 \text{ m/s}) \sqrt{1 + \frac{T_C}{273^\circ C}} \]

- The 331 m/s is the speed at 0\(^\circ\) C.
- \( T_C \) is the air temperature in Celsius.
Speed of Sound in Gases, Example Values

<table>
<thead>
<tr>
<th>Gases</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen (0°C)</td>
<td>1286</td>
</tr>
<tr>
<td>Helium (0°C)</td>
<td>972</td>
</tr>
<tr>
<td>Air (20°C)</td>
<td>343</td>
</tr>
<tr>
<td>Air (0°C)</td>
<td>331</td>
</tr>
<tr>
<td>Oxygen (0°C)</td>
<td>317</td>
</tr>
</tbody>
</table>

Note: temperatures, speeds are in m/s
Speed of Sound in Liquids, Example Values

<table>
<thead>
<tr>
<th>Liquids at 25°C</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glycerol</td>
<td>1904</td>
</tr>
<tr>
<td>Seawater</td>
<td>1533</td>
</tr>
<tr>
<td>Water</td>
<td>1493</td>
</tr>
<tr>
<td>Mercury</td>
<td>1450</td>
</tr>
<tr>
<td>Kerosene</td>
<td>1324</td>
</tr>
<tr>
<td>Methyl alcohol</td>
<td>1143</td>
</tr>
<tr>
<td>Carbon tetrachloride</td>
<td>926</td>
</tr>
</tbody>
</table>

Speeds are in m/s
### Speed of Sound in Solids, Example Values

<table>
<thead>
<tr>
<th>Solids</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyrex glass</td>
<td>5 640</td>
</tr>
<tr>
<td>Iron</td>
<td>5 950</td>
</tr>
<tr>
<td>Aluminum</td>
<td>6 420</td>
</tr>
<tr>
<td>Brass</td>
<td>4 700</td>
</tr>
<tr>
<td>Copper</td>
<td>5 010</td>
</tr>
<tr>
<td>Gold</td>
<td>3 240</td>
</tr>
<tr>
<td>Lucite</td>
<td>2 680</td>
</tr>
<tr>
<td>Lead</td>
<td>1 960</td>
</tr>
<tr>
<td>Rubber</td>
<td>1 600</td>
</tr>
</tbody>
</table>

Speeds are in m/s; values are for bulk solids.
Speed of Sound in an Aluminum Rod, An Example

• Since we need the speed of sound in a metal rod,

\[
v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.0 \times 10^{10} \text{ Pa}}{2.70 \times 10^3 \text{ kg/m}^3}} = 5090 \text{ m/s}
\]
Last time....

Sound:
- A **longitudinal** wave in a medium.

How do we describe such a wave?
- Mathematically, just like **transverse** wave on string:
  \[ y(x,t) = A \cos(kx-\omega t) \]

Careful: the meaning of \( y(x,t) \) is quite different!!
Displacement vs. pressure (last time...)

- Can describe sound wave in terms of displacement or in terms of pressure.

$y(x,t) = A \cos(kx - \omega t)$

or

$p(x,t) = (A \cdot B \cdot k) \sin(kx - \omega t)$

Displacement and pressure out of phase by 90°.
Bulk Modulus (last time...)  
A measure of how easy it is to compress a fluid.

\[ B = -V \frac{dP}{dV} \]

Ideal gas:  
\[ B = \gamma P \]

\[ \gamma = \frac{C_P}{C_V} \]

\( \gamma \sim 1.7 \) monoatomic molecules (He, Ar,..)
\( \gamma \sim 1.4 \) diatomic molecules (O₂, N₂,..)
\( \gamma \sim 1.3 \) polyatomic molecules (CO₂,..)

Heat capacities at constant \( P \) or \( V \)
Speed of sound (last time...) 

\[ v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \]

For air: \( v = 344 \text{ m/sec at } 20^\circ \text{C} \)
Intensity (last time...) 

• The wave carries energy
• The intensity is the *time average* of the power carried by the wave crossing unit area.
• Intensity is measured in W/m²

\[ I = \frac{\langle P \rangle}{S} = \frac{\langle dE/dt \rangle}{S} \]
Decibel (last time...)

• A more convenient sound intensity scale
  more convenient than W/m².

• The sound intensity level $\beta$ is defined as
  $$\beta = (10 \text{ dB}) \log \frac{I}{I_0}$$

• Where $I_0 = 10^{-12}$ W/m²
  Approximate hearing threshold at 1 kHz

• It's a log scale
  A change of 10 dB corresponds to a factor of 10
Standing sound waves

Recall standing waves on a string

- A standing wave on a string occurs when we have interference between wave and its reflection.
- The reflection occurs when the medium changes, e.g., at the string support.
• We can have sound standing waves too.
• For example, in a pipe.
• Two types of **boundary conditions**:
  1. Open pipe
  2. Closed pipe
• In an closed pipe the boundary condition is that the **displacement is zero** at the end
  ➢ Because the fluid is constrained by the wall, it can't move!
• In an open pipe the boundary condition is that the **pressure fluctuation is zero** at the end
  ➢ Because the pressure is the same as outside the pipe (atmospheric)
Remember:

- Displacement and pressure are out of phase by 90°.
- When the displacement is 0, the pressure is ± $p_{\text{max}}$.
- When the pressure is 0, the displacement is ± $y_{\text{max}}$.
- So the nodes of the pressure and displacement waves are at different positions

➢ It is still the same wave, just two different ways to describe it mathematically!!
Exciting a Standing Sound Wave
Standing Waves in a Pipe That Is Open at One End
Getting a Tune-Up

Beats
4 Hz

Beats
5 Hz

440 Hz

440 Hz
More jargon: nodes and antinodes

- In a sound wave the pressure nodes are the displacement antinodes and vice versa.
Example

- A directional loudspeaker bounces a sinusoidal sound wave off the wall. At what distance from the wall can you stand and hear no sound at all?
- A key thing to realize is that the ear is sensitive to pressure fluctuations.
- Want to be at pressure node.
- The wall is a displacement node $\rightarrow$ pressure antinode.
Organ pipes

• Sound from standing waves in the pipe
• Remember:
  ➢ **Closed pipe:**
    • Displacement node (no displacement possible)
      ➔ Pressure Antinode
  ➢ **Open pipe:**
    • Pressure node (pressure is atmospheric)
      ➔ Displacement Antinode
Open Pipe (displacement picture)

(a) $f_1 = \frac{v}{2L}$

(b) $f_2 = 2 \frac{v}{2L} = 2f_1$

(c) $f_3 = 3 \frac{v}{2L} = 3f_1$

Closed Pipe (displacement picture)

(a) $f_1 = \frac{v}{4L}$

(b) $f_3 = 3 \frac{v}{4L} = 3f_1$

(c) $f_5 = 5 \frac{v}{4L} = 5f_1$
Organ pipe frequencies

• Open pipe:

\[ f = n \cdot \frac{v}{2L} \quad (n = 1, 2, 3, \ldots) \]

• Closed (stopped) pipe:

\[ f = n \cdot \frac{v}{4L} \quad (n = 1, 3, 5, \ldots) \]

\[ f_0 = \frac{v}{2L} \]
Sample Problem

• A pipe is filled with air and produces a fundamental frequency of 300 Hz.
  ➢ If the pipe is filled with He, what fundamental frequency does it produce?
  ➢ Does the answer depend on whether the pipe is open or stopped?

Open pipe: \[ f = n \cdot \frac{v}{2L} \]  
(\( n = 1, 2, 3.. \))

Closed (stopped) pipe: \[ f = n \cdot \frac{v}{4L} \]  
(\( n = 1, 3, 5.. \))

→ Fundamental frequency \( v/2L \) (open) or \( v/4L \) (stopped)

What happens when we substitute He for air?

The velocity of sound changes!
From last week, speed of sound:

\[ v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \]

\[ \gamma = \frac{C_P}{C_V} \]

\( \gamma \approx 1.7 \) monoatomic molecules (He, Ar,..)

\( \gamma \approx 1.4 \) diatomic molecules (O\(_2\), N\(_2\),..)

\( \gamma \approx 1.3 \) polyatomic molecules (CO\(_2\),..)

Heat capacities at constant \( P \) or \( V \)

\[ v_{\text{air}} = \sqrt{\frac{\gamma_{\text{air}}}{\gamma_{\text{He}}}} \sqrt{\frac{M_{\text{He}}}{M_{\text{air}}}} v_{\text{He}} \]
We had:

Open pipe: \[ f = n \cdot \frac{v}{2L} \quad (n = 1, 2, 3..) \]

Closed (stopped) pipe: \[ f = n \cdot \frac{v}{4L} \quad (n = 1, 3, 5..) \]

i.e., the fundamental frequency is proportional to velocity for both open and stopped pipes

\[
\begin{align*}
\frac{f_{He}}{f_{air}} &= \frac{v_{He}}{v_{air}} \quad f_{air} \\
\Rightarrow v_{air} &= \sqrt{\frac{\gamma_{air}}{\gamma_{He}}} \sqrt{\frac{M_{He}}{M_{air}}} v_{He} \\
\Rightarrow f_{He} &= \sqrt{\frac{\gamma_{He}}{\gamma_{air}}} \sqrt{\frac{M_{air}}{M_{He}}} f_{air}
\end{align*}
\]

So:

\[
\begin{align*}
f_{He} &= \sqrt{\frac{1.7}{1.4}} \sqrt{\frac{29 \text{ g/mol}}{4 \text{ g/mol}}} (300\text{Hz}) = 890\text{Hz}
\end{align*}
\]
Resonance

• Many mechanical systems have natural frequencies at which they oscillate.
  ➢ a mass on a spring: $\omega^2 = k/m$
  ➢ a pendulum: $\omega^2 = g/l$
  ➢ a string fixed at both ends: $f = n\nu/(2L)$

• If they are driven by an external force with a frequency equal to the natural frequency, they go into resonance:
  ➢ the amplitude of the oscillation grows
  ➢ in the absence of friction, the amplitude would $\to$ infinity
Interference

• Occur when two (or more) waves overlap.
• The resulting displacement is the sum of the displacements of the two (or more) waves.
  ➢ Principle of superposition.
  ➢ We already applied this principle to standing waves:
    • Sum of a wave moving to the right and the reflected wave moving to the left.
Interference (cont.)

• The displacements of the two waves can add to give
  ➢ a bigger displacement.
    • Constructive Interference.
  ➢ or they can even cancel out and give zero displacement.
    • Destructive interference.
    • Sometime, sound + sound = silence
    • Or, light + light = darkness
Interference Example

- Two loudspeakers are driven by the same amplifier and emit sinusoidal waves in phase. The speed of sound is $v = 350$ m/sec. What are the frequencies for (maximal) constructive and destructive interference.
• Wave from speaker A at P (x₁=AP)
  \[ y_1(t) = A_1 \cos(\omega t - kx_1) \]
• Wave from speaker B at P (x₂=BP)
  \[ y_2(t) = A_2 \cos(\omega t - kx_2) \]
• Total amplitude
  \[
  y(t) = y_1(t) + y_2(t) \\
  y(t) = A_1 \cos \omega t \cos kx_1 + A_1 \sin \omega t \sin kx_1 \\
  + A_2 \cos \omega t \cos kx_2 + A_2 \sin \omega t \sin kx_2
  \]
• When \( kx_1 = kx_2 + 2n\pi \) the amplitude of the resulting wave is largest.
  ➢ In this case \( \cos kx_1 = \cos kx_2 \) and \( \sin kx_1 = \sin kx_2 \).
• Conversely, when \( kx_1 = kx_2 + n\pi \) (with \( n \) odd), the amplitude of the resulting wave is the smallest.
  ➢ Then \( \cos kx_1 = -\cos kx_2 \) and \( \sin kx_1 = -\sin kx_2 \).
Constructive Interference

\[ kx_1 = kx_2 + 2n\pi \]
\[ \frac{2\pi}{\lambda} x_1 = \frac{2\pi}{\lambda} x_2 + 2n\pi \]
\[ x_1 - x_2 = n\lambda \]

Destructive Interference

\[ kx_1 = kx_2 + (2n + 1)\pi \]
\[ \frac{2\pi}{\lambda} x_1 = \frac{2\pi}{\lambda} x_2 + (2n + 1)\pi \]
\[ x_1 - x_2 = \frac{2n+1}{2}\lambda \]
• Constructive interference occurs when the difference in path length between the two waves is equal to an integer number of wavelengths.

• Destructive interference when the difference in path length is equal to a half-integer number of wavelengths.

• CAREFUL: this applies if
  - The two waves have the same wavelength.
  - The two waves are emitted in phase.

• What would happen if they were emitted (say) $180^\circ$ out of phase?
Back to our original problem:

The waves are generated in phase; $v=350$ m/sec

**Constructive interference:**
- $AP - BP = n\lambda$
- $\lambda = v/f$
- $f = n \frac{v}{AB - BP}$
- $f = n \frac{350}{0.35}$ Hz
- $f = 1, 2, 3, ...,$ kHz

**Destructive interference:**
- $AP - BP = n\lambda/2$ (n odd)
- $\lambda = v/f$
- $f = n \frac{v}{2(AB - BP)}$
- $f = n \frac{350}{0.70}$ Hz
- $f = 0.5, 1.5, 2.5$ kHz
Beats

• Consider interference between two sinusoidal waves with similar, but not identical, frequencies:

• The resulting wave looks like a single sinusoidal wave with a varying amplitude between some maximum and zero.

• The intensity variations are called beats, and the frequency with which these beats occur is called the beat frequency.
Beats, mathematical representation

- Consider two waves, equal amplitudes, different frequencies:
  - $y_1(x,t) = A \cos(2\pi f_1 t - k_1 x)$
  - $y_2(x,t) = A \cos(2\pi f_2 t - k_2 x)$

- Look at the total displacement at some point, say $x=0$.
  - $y(0,t) = y_1(0,t) + y_2(0,t) = A \cos(2\pi f_1 t) + A \cos(2\pi f_2 t)$

- Trig identity:
  - $\cos A + \cos B = 2 \cos[(A-B)/2] \cos[(A+B)/2]$

- This gives
  - $y(0,t) = 2A \cos_{1/2} (2\pi)(f_1-f_2)t \cos_{1/2} (2\pi)(f_1+f_2)t$
\[ y(0,t) = 2A \cos[\frac{1}{2} (2\pi)(f_1-f_2)t] \cos[\frac{1}{2} (2\pi)(f_1+f_2)t] \]

An amplitude term which oscillates with frequency \( \frac{1}{2}(f_1-f_2) \). If \( f_1 \neq f_2 \) then \( f_1-f_2 \) is small and the amplitude varies slowly.

A sinusoidal wave term with frequency \( f = \frac{1}{2}(f_1 + f_2) \).

Beat frequency is \( \frac{1}{2} |(f_1 - f_2)| \)
Beats

Amplitude: oscillates with beat frequency, $|f_1 - f_2|$

$y_{\text{total}}$: oscillates with average frequency, $\frac{f_1 + f_2}{2}$
Periodic Sound Waves, Example

- A longitudinal wave is propagating through a gas-filled tube
- The source of the wave is an oscillating piston
- The distance between two successive compressions (or rarefactions) is the wavelength
Periodic Sound Waves, cont

- As the regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave.

- The harmonic position function is

\[ s(x, t) = s_{\text{max}} \cos (kx - \omega t) \]

- \( s_{\text{max}} \) is the maximum position from the equilibrium position.

- This is also called the displacement amplitude of the wave.
Periodic Sound Waves, Pressure

• The variation in gas pressure, $\Delta P$, is also periodic

$\Delta P = \Delta P_{\text{max}} \sin (kx - \omega t)$

- $\Delta P_{\text{max}}$ is the pressure amplitude
- It is also given by $\Delta P_{\text{max}} = \rho v \omega s_{\text{max}}$

• $k$ is the wave number (in both equations)
- $\omega$ is the angular frequency (in both equations)
Periodic Sound Waves, final

- A sound wave may be considered either a displacement wave or a pressure wave.
- The pressure wave is $90^\circ$ out of phase with the displacement wave.
  - The pressure is a maximum when the displacement is zero, etc.
Energy of Periodic Sound Waves

- The piston transmits energy to the element of air in the tube.
- This energy is propagated away from the piston by the sound wave.
Energy, cont

• The speed of the element of air is the time derivative of its displacement

\[ v(x, t) = \frac{\partial}{\partial t} s(x, t) = -\omega s_{\text{max}} \sin(kx - \omega t) \]

• Once we know the speed, we can find its kinetic energy

\[ \Delta K = \frac{1}{2} \Delta m(v)^2 = \frac{1}{2} \rho A \Delta x (\omega s_{\text{max}})^2 \sin^2 kx \]
Energy, final

• The total kinetic energy in one wavelength is
  \[ K_\lambda = \frac{1}{4} \rho A (\omega s_{\text{max}})^2 \lambda \]

• The total potential energy for one wavelength is the same as the kinetic

• The total mechanical energy is
  \[ E_\lambda = K_\lambda + U_\lambda = \frac{1}{2} \rho A (\omega s_{\text{max}})^2 \lambda \]
Power of a Periodic Sound Wave

- The rate of energy transfer is the power of the wave

\[ \phi = \frac{\Delta E}{\Delta t} = \frac{E}{T} = \frac{1}{2} \rho A v (\omega s_{\text{max}})^2 \]

- This is the energy that passes by a given point during one period of oscillation
Intensity of a Periodic Sound Wave

• The **intensity** $I$ of a wave is defined as the power per unit area.
  
  ➢ This is the rate at which the energy being transported by the wave transfers through a unit area, $A$, perpendicular to the direction of the wave.

$$I = \frac{\mathcal{P}}{A}$$
Intensity, cont

- In the case of our example wave in air, \( I = \frac{1}{2} \rho v (\omega s_{\text{max}})^2 \)

- Therefore, the intensity of a periodic sound wave is proportional to the
  - square of the displacement amplitude
  - square of the angular frequency

- In terms of the pressure amplitude,

\[
I = \frac{\Delta P^2}{2 \rho v}
\]
Intensity of a Point Source

• A point source will emit sound waves equally in all directions
  ➢ This results in a spherical wave

• Identify an imaginary sphere of radius $r$ centered on the source

• The power will be distributed equally through the area of the sphere
Intensity of a Periodic Sound Wave

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• Identify an imaginary sphere of radius $r$ centered on the source
• The power will be distributed equally through the area of the sphere
Intensity of a Point Source, cont

• This is an inverse-square law

\[ I = \frac{\phi_{av}}{A} = \frac{\phi_{av}}{4\pi r^2} \]

• The intensity decreases in proportion to the square of the distance from the source
Sound Level

- The range of intensities detectible by the human ear is very large
- It is convenient to use a logarithmic scale to determine the intensity level, $\beta$

$$\beta = 10 \log \left( \frac{I}{I_0} \right)$$
Sound Level, cont

• $I_0$ is called the **reference intensity**
  - It is taken to be the threshold of hearing
  - $I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$
  - $I$ is the intensity of the sound whose level is to be determined

• $\beta$ is in decibels (dB)

• Threshold of pain: $I = 1.00 \text{ W/m}^2; \beta = 120 \text{ dB}$

• Threshold of hearing: $I_0 = 1.00 \times 10^{-12} \text{ W/m}^2; \beta = 0 \text{ dB}$
Sound Level, Example

• What is the sound level that corresponds to an intensity of 2.0 x 10^{-7} W/m^2?

\[ \beta = 10 \log \left( \frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.0 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log 2.0 \times 10^5 = 53 \text{ dB} \]

• Rule of thumb: A doubling in the loudness is approximately equivalent to an increase of 10 dB
## Sound Levels

<table>
<thead>
<tr>
<th>Source of Sound</th>
<th>$\beta$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearby jet airplane</td>
<td>150</td>
</tr>
<tr>
<td>Jackhammer; machine gun</td>
<td>130</td>
</tr>
<tr>
<td>Siren; rock concert</td>
<td>120</td>
</tr>
<tr>
<td>Subway; power mower</td>
<td>100</td>
</tr>
<tr>
<td>Busy traffic</td>
<td>80</td>
</tr>
<tr>
<td>Vacuum cleaner</td>
<td>70</td>
</tr>
<tr>
<td>Normal conversation</td>
<td>50</td>
</tr>
<tr>
<td>Mosquito buzzing</td>
<td>40</td>
</tr>
<tr>
<td>Whisper</td>
<td>30</td>
</tr>
<tr>
<td>Rustling leaves</td>
<td>10</td>
</tr>
<tr>
<td>Threshold of hearing</td>
<td>0</td>
</tr>
</tbody>
</table>
Loudness and Intensity

- Sound level in decibels relates to a physical measurement of the strength of a sound.
- We can also describe a psychological “measurement” of the strength of a sound.
- Our bodies “calibrate” a sound by comparing it to a reference sound.
- This would be the threshold of hearing.
- Actually, the threshold of hearing is this value for 1000 Hz.
Loudness and Frequency, cont

- There is a complex relationship between loudness and frequency
- The lower curve of the white area shows the threshold of hearing
- The upper curve shows the threshold of pain
Human Response to Sound
Doppler Effect

• When a car goes past you, the pitch of the engine sound that you hear changes.
• Why is that?
• This must have something to do with the velocity of the cars with respect to you (towards you vs. away from you).
  ➢ Unless it is because the driver is doing something "funny" like accelerating to try to run you over 😃
Consider listener moving towards sound source:

- Sound from source: velocity $v$, frequency $f_s$, wavelength $\lambda$, and $v = \lambda f_s$.
- The listener sees the wave crests approaching with velocity $v + v_L$.
- Therefore the wave crests arrive at the listener with frequency:

$$f_L = \frac{v + v_L}{v} f_S = \left(1 + \frac{v_L}{v}\right) f_S$$

→ The listener "perceives" a different frequency (Doppler shift)
Now imagine that the source is also moving:

- The wave speed relative to the air is still the same \( v \).
- The time between emissions of subsequent crests is the period \( T = 1/f_s \).
- Consider the crests in the direction of motion of the source (to the right)
  - A crest emitted at time \( t=0 \) will have travelled a distance \( vT \) at \( t=T \)
  - In the same time, the source has travelled a distance \( v_s T \).
  - At \( t=T \) the subsequent crest is emitted, and this crest is at the source.
  - So the distance between crests is \( vT - v_s T = (v-v_s)T \).
  - But the distance between crests is the wavelength
    \[ \lambda = (v-v_s)T \]
    \[ \Rightarrow \lambda = (v-v_s)/f_s \] (in front of the source)
• $\lambda = (v-v_s)/f_s$ (in front of the source)
• Clearly, behind the source $\lambda = (v+v_s)/f_s$
• For the listener, $f_L=(v+v_L)/\lambda$
  ➢ Since he sees crests arriving with velocity $v+v_L$

$$f_L = \frac{v+v_L}{v+v_s}f_S$$
Sample problem

- A train passes a station at a speed of 40 m/sec. The train horn sounds with $f=320$ Hz. The speed of sound is $v=340$ m/sec.

What is the change in frequency detected by a person on the platform as the train goes by.

Approaching train:

\[ f_L = \frac{v + v_L}{v + v_S} f_S \]
In our case $v_L = 0$ (the listener is at rest) and the source (train) is mowing towards rather than away from the listener.

$\rightarrow$ I must switch the sign of $v_S$

$$f_L = \frac{v + v_L}{v + v_S} f_S \quad \text{becomes} \quad f_{L1} = \frac{v}{v - v_{\text{train}}} f$$

When the train moves away:

Clearly I need to switch the sign of $v_{\text{train}}$:  \[ f_{L2} = \frac{v}{v + v_{\text{train}}} f \]

$$\Delta f = f_{L1} - f_{L2} = \ldots \text{ (algebra) } \ldots = -2 \frac{vv_{\text{train}}}{v^2 - v_{\text{train}}^2} f = 76 \text{ Hz}$$
Doppler Effect, Water Example

- A point source is moving to the right.
- The wave fronts are closer on the right.
- The wave fronts are farther apart on the left.
Doppler Effect, Submarine Example

- Sub A (source) travels at 8.00 m/s emitting at a frequency of 1400 Hz
- The speed of sound is 1533 m/s
- Sub B (observer) travels at 9.00 m/s
- What is the apparent frequency heard by the observer as the subs approach each other? Then as they recede from each other?
Doppler Effect, Submarine Example cont

- **Approaching each other:**

  \[
  f' = \left( \frac{v + v_o}{v - v_s} \right) f = \left( \frac{1533 \, \text{m/s} + (+9.00 \, \text{m/s})}{1533 \, \text{m/s} - (+8.00 \, \text{m/s})} \right)(1400 \, \text{Hz}) = 1416 \, \text{Hz}
  \]

- **Receding from each other:**

  \[
  f' = \left( \frac{v + v_o}{v - v_s} \right) f = \left( \frac{1533 \, \text{m/s} + (-9.00 \, \text{m/s})}{1533 \, \text{m/s} - (-8.00 \, \text{m/s})} \right)(1400 \, \text{Hz}) = 1385 \, \text{Hz}
  \]
Shock Wave

- The speed of the source can exceed the speed of the wave.
- The envelope of these wave fronts is a cone whose apex half-angle is given by $\sin \theta = v/v_s$.

  ➢ This is called the Mach angle.
Mach Number

• The ratio $v_s / v$ is referred to as the Mach number.

• The relationship between the Mach angle and the Mach number is

$$\sin \theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$
Shock Wave, final

- The conical wave front produced when $v_s > v$ is known as a shock wave
  - This is supersonic
- The shock wave carries a great deal of energy concentrated on the surface of the cone
- There are correspondingly great pressure variations
Sound Recording

- Encoding sound waveforms began as variations in depth of a continuous groove cut in tin foil wrapped around a cylinder.
- Sound was then recorded on cardboard cylinders coated with wax.
- Next were disks made of shellac and clay.
- In 1948, plastic phonograph disks were introduced.
Sound Recording, cont

• Disadvantages of phonograph records
  ➢ The recording quality diminishes with each playing as small pieces of the plastic are worn away or broken by the needle
  ➢ The natural irregularities in the plastic produce noise
  • The noise is particularly noticeable during quiet periods with high frequencies playing
Digital Recording

- In digital recording of sound, information is converted to binary code
- The waveforms of the sound are sampled
- During the sampling, the pressure of the wave is sampled and converted into a voltage
- The graph below shows the sampling process
Digital Recording, 2

• These voltage measurements are then converted to binary numbers (1’s and 0’s)
  ➢ Binary numbers are expressed in base 2

• Generally, the voltages are recorded in 16-bit “words”
  ➢ Each bit is a 1 or a 0
Digital Recording, 3

• The strings of ones and zeroes are recorded on the surface of the compact disc

• There is a laser playback system that detects *lands* and *pits*
  
  ➢ Lands are the untouched regions
    • They are highly reflective
  
  ➢ Pits are areas burned into the surface
    • They scatter light instead of reflecting it
Digital Recording, final

• The binary numbers from the CD are converted back into voltages
• The waveform is reconstructed
• Advantages
  ➢ High fidelity of the sound
  ➢ There is no mechanical wear on the disc
    • The information is extracted optically
Motion Picture Sound

• Early movies recorded sound on phonograph records
  ➢ They were synchronized with the action on the screen
• Then a *variable-area optical soundtrack* was introduced
  ➢ The sound was recorded on an optical track on the edge of the film
  ➢ The width of the track varied according to the sound wave
Motion Picture Sound, cont

• A photocell detecting light passing through the track converted the varying light intensity to a sound wave

• Problems

  ➢ Dirt or fingerprints on the track can cause fluctuations and loss of fidelity
Systems of Motion Picture Sound – Original

- **Cinema Digital Sound (CDS)**
  - First used in 1990
  - No backup
  - No longer used
  - Introduced the use of 5.1 channels of sound:
    - Left, Center, Right, Right Surround, Left Surround and Low Frequency Effects (LFE)
Systems of Motion Picture Sound – Current

- Dolby Digital
  - 5.1 channels stored between sprocket holes on the film
  - Has an analog backup
  - First used in 1992

- Digital Theater Sound (DTS)
  - 5.1 channels stored on a separate CD
  - Synchronized to the film by time codes
  - Has an analog backup
  - First used in 1993
Systems of Motion Picture Sound – Current, cont

• Sony Dynamic Digital Sound (SDDS)
  ➢ Eight full channels
  ➢ Optically stored outside the sprocket holes on both sides of the film
    • Both sides serve as a redundancy
  ➢ Analog optical backup
  ➢ The extra channels are a full channel LFE plus left center and right center behind the screen
Problems

16.24: Open Pipe: \[ \lambda_1 = 2L = \frac{v}{f_1} = \frac{v}{594 \text{ Hz}} \]

Closed at one end: \[ \lambda_1 = 4L = \frac{v}{f} \]

Taking ratios: \[ \frac{2L}{4L} = \frac{\sqrt{594 \text{ Hz}}}{\sqrt{f}} \]

\[ f = \frac{594 \text{ Hz}}{2} = 297 \text{ Hz} \]
Problems

16.34: We are to assume

\[ v = 344 \text{ m/s} , \text{ so } \lambda = \frac{v}{f} = \frac{(344 \text{ m/s})}{(172 \text{ Hz})} = 2.00 \text{ m}. \]

\[ r_A = 8.00 \text{ m} \text{ and } r_B \]

are the distances of the person from each speaker, the condition for destructive interference is

\[ r_B - r_A = (n + \frac{1}{2})\lambda, \]

where \( n \) is any integer. Requiring

\[ r_B = r_A + \left( n + \frac{1}{2} \right)\lambda > 0 \]

gives

\[ n + \frac{1}{2} > -\frac{r_A}{\lambda} = -\frac{(8.00 \text{ m})}{(2.00 \text{ m})} = -4, \]

so the smallest value of \( n \) occurs when \( n = -4 \), and the closest distance to \( B \) is

\[ r_B = 8.00 \text{ m} + \left( -4 + \frac{1}{2} \right)(2.00 \text{ m}) = 1.00 \text{ m}. \]
Problems

16.16: From Eq. (16.13), \( I = \frac{v p_{\text{max}}^2}{2B} \), and from Eq. (19.21),

\[ v^2 = \frac{B}{\rho} \]

Using Eq. (16.7) to eliminate \( v \),

\[ v = \sqrt{\frac{B}{\rho}} \]

\[ I = \left( \frac{B}{\rho} \right) p_{\text{max}}^2 / 2B = p_{\text{max}}^2 / 2\sqrt{\rho B} \]

Using Eq. (16.7) to eliminate \( B \),

\[ I = \frac{v p_{\text{max}}^2}{2(v^2 \rho)} = p_{\text{max}}^2 / 2\rho v \]

Taking ratios:

16.19: a) As in Example 16.6,

\[ I = \frac{(6.0 \times 10^{-5} \text{ Pa})^2}{2(1.20 \text{ kg/m}^3)(344 \text{ m/s})} = 4.4 \times 10^{-12} \text{ W/m}^2. \beta = 6.40 \text{ dB}. \]
Problems

16.38: Solving Eq. (16.17) for $v$, with $v_L = 0$

$$v = \frac{f_L}{f_s - f_L} v_s = \left( \frac{1240 \text{ Hz}}{1200 \text{ Hz} - 1240 \text{ Hz}} \right) (-25.0 \text{ m/s}) = 775 \text{ m/s},$$

$v_S < 0$, since the source is moving toward the listener.

a)

16.41: In terms of wavelength, Eq. (16.29) is

$$\lambda_L = \frac{v + v_s}{v + v_L} \lambda_s.$$

$v_L = 0$, $v_s = -25.0 \text{ m}$ and $\lambda_L = \left( \frac{319}{344} \right) (344 \text{ m/s})/(400 \text{ Hz}) = 0.798 \text{ m}.$

This is, of course, the same result as obtained directly from Eq. (16.27).

$v_s = 25.0 \text{ m/s}$ and

$$\lambda_L = (369 \text{ m/s})/(400 \text{ Hz}) = 0.922 \text{ m}.$$

The frequencies corresponding to these wavelengths are c) 431 Hz and d) 373 Hz.
Problems

16.44: For a stationary source,

\[ v_s = 0, \text{ so } f_L = \frac{v + v_L}{v - v_s} f_S = (1 + v_L/v_s) f_S, \]

which gives

\[ v_L = v \left( \frac{f_L}{f_S} - 1 \right) = (344 \text{ m/s}) \left( \frac{490 \text{ Hz}}{520 \text{ Hz}} - 1 \right) = -19.8 \text{ m/s}. \]

This is negative because the listener is moving away from the source.

16.50: a) \( p = IA = I_0 10^{(\beta/10 \text{ dB})} A. \)

\[ b) \left( 1.00 \times 10^{-12} \text{ W/m}^2 \right) \left( 10^{5.50} \right) (1.20 \text{ m}^2) = 3.79 \times 10^{-7} \text{ W}. \]

16.45: a) \( v_L = 18.0 \text{ m/s}, v_S = -30.0 \text{ m/s}, \) and

Eq. (16.29) gives \( f_L = \left( \frac{362}{314} \right) (262 \text{ Hz}) = 302 \text{ Hz}. \)

b) \( v_L = -18.0 \text{ m/s}, v_S = 30.0 \text{ m/s} \) and \( f_L = 228 \text{ Hz}. \)