Math 52  Answers to QUIZ 3

(1) Do you think each of the following numbers can be a probability number? (Yes, No).

A probability is a real number within \([0, 1]\)

(a) \(\frac{7}{5}\) (No, \(\frac{7}{5} > 1\))
(b) 0.01 (Yes)
(c) −2 (No, \(-2 < 0\))
(d) \((\sqrt{2} - 1)\) (Yes, \(\sqrt{2} - 1 \approx 0.4142\))
(e) \(|-\frac{3}{4}|\) (Yes, \(|-\frac{3}{4}| = \frac{3}{4} = 0.75\))
(f) 1 (Yes)

(2) Let \(A\) be an event, how do you understand the likelihood of the occurrence of this event?

(a) \(P(A) = 0.1\) ⇒ Event A is very UNlikely to occur.
(b) \(P(A) = 1\) ⇒ Event A is a Certain event.
(c) \(P(A) = 0\) ⇒ Event A is an Impossible event.
(d) \(P(A) = 0.95\) ⇒ Event A is very Likely to occur.
(95% is very close to 1, ⇒ Event A is almost sure to occur.)

(3) Rolling a balanced die twice, each time a number shows up.

(a) Let event A be "the sum of the two numbers is 10", find \(P(A)\)
Event A: \(A = \{(4,6), (5,5), (6,4)\}\) \(\Rightarrow x = 3\) \(\Rightarrow P(A) = \frac{3}{36} = \frac{1}{12}\)
(b) Let event B be "the sum of the two numbers is at least 10", find \(P(B)\)
Event B: \(B = \{(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)\}\) \(\Rightarrow x = 6\) \(\Rightarrow P(B) = \frac{6}{36} = \frac{1}{6}\)
(Note: "At least" is "greater than or equal")
(c) Let event C be "the two numbers are both odd numbers", find \(P(C)\)
Event C: \(C = \{(1,1), (1,3), (1,5), \ldots, (5,5)\}\) \(\Rightarrow x = 9\) \(\Rightarrow P(C) = \frac{9}{36} = \frac{1}{4}\)

(4) A standard deck of playing cards contain 52 cards with 4 suits (hearts, diamonds, clubs, and spades).
Let \(A\) be the event that the card is spade,
Let \(B\) be the event that the card is a face card (J, Q, K).

(a) (there are 13 spade cards in a deck.) \(\Rightarrow n(A) = 13\) \(\Rightarrow P(A) = \frac{13}{52} = \frac{1}{4}\)
(b) (there are 12 face cards in a deck.) \(\Rightarrow n(B) = 12\) \(\Rightarrow P(B) = \frac{12}{52} = \frac{3}{13}\)
(c) \(P(A \text{ or } B)\) Hint: \((A \text{ or } B)\) means that the drawn card is either a spade card or a face card.
Event (A or B) contains 13 Spade cards and 9 J, Q, K cards (of Club, Heart, and Diamond),
\[\Rightarrow P(A \text{ or } B) = \frac{22}{52} = \frac{11}{26}\]
(d) \(P(A \text{ and } B)\) Hint: \((A \text{ and } B)\) means that the drawn card is both a spade card and a face card at the same time.
Event (A and B) contains 3 Spade J, Q, K cards,
\[\Rightarrow P(A \text{ and } B) = \frac{3}{52}\]

(5) Determine whether the following tables define a probability distribution. If NO, why? If YES, find the mean \(\mu\), variance \(\sigma^2\) and standard deviation \(\sigma\) of the random variable \(X\) with that probability distribution.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(p(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(a) \(\Rightarrow\) No, it does not describe a probability distribution.
(the sum of the 2nd column is 1.5 > 1)
Yes, this table describes a probability distribution.

\[
\mu = \sum xp(x) = (-5)(0.1) + (-2)(0.3) + (-1)(0.2) + (3)(0.2) + (5)(0.1) + (10)(0.1) = 0.8
\]

\[
\sigma^2 = \sum x^2p(x) - \mu^2 = ((-5)^2)(0.1) + (-2)^2(0.3) + (-1)^2(0.2) + (3)^2(0.2) + (5)^2(0.1) + (10)^2(0.1)) - (0.8)^2 = 18.2 - 0.64 = 17.56
\]

\[
\sigma = \sqrt{\sigma^2} = \sqrt{17.56} \approx 4.2
\]

No, it does not describe a probability distribution. (the 2nd column contains a negative number, which cannot be a probability.)

(6) Do you think the following experiments are Binomial Experiments? If NO, why? If YES, find:

- \(n\) (the total number of trials), \(S\) (what is a Success), \(F\) (what is a Failure),
- \(p\) (success probability), and \(q\) (failure probability).

(a) Rolling a fair die 20 times and keeping track of the numbers shown up.

⇒ No, because each trial results in 6 possible outcomes.

(b) Rolling a fair die 55 times, keeping track of the number of the "sixes" rolled.

⇒ Yes, because each trial will result in 2 possible outcomes, each of which is with a fix probability.

⇒ \(n = 55,\)
  - a "Success" = the number "Six" is shown up,
  - a "Failure" = the number shown up is not "Six",
  - (the success probability) \(p = \frac{1}{6},\)
  - (the failure probability) \(q = 1 - p = \frac{5}{6}.\)

(c) Pick a ball randomly from box that contains 20 balls (7 red and 13 green) and record its color. Put the ball back into the box and mix the balls, pick one randomly again and record the color, then put it back. Repeat this process for 15 times and keep track of the numbers of red balls you got.

⇒ Yes, because each trial will result in 2 possible outcomes, each of which is with a fix probability.

⇒ \(n = 15,\)
  - a "Success" = a red ball is picked,
  - a "Failure" = a green ball is picked,
  - (the success probability) \(p = \frac{7}{20},\)
  - (the failure probability) \(q = 1 - p = \frac{13}{20}.\)

(d) Do the same as is in (c) but don’t put the ball back to the box. Pick 15 balls one by one and find the number of red balls you got.

No, because each time when you pick a ball, the probability of "getting a red ball" is not the same. (the trials are NOT independent of each other.)

(for example,)
At the 1st time, \(P(\text{a red ball is picked}) = \frac{7}{20},\)

At the 2nd time,

(if the 1st ball picked is red, then there are 6 red balls and 13 green balls in the box.)
⇒ \(P(\text{a red ball is picked}) = \frac{6}{19},\)

(if the 1st ball picked is green, then there are 7 red balls and 12 green balls in the box.)
⇒ \(P(\text{a red ball is picked}) = \frac{7}{19},\)

(7) Assume that the probability of a newly born baby being a boy is 0.513.

(a) Find the mean value of boys born in a randomly selected group of 1000 babies.
Set-ups: This can be regarded as a Binomial experiment:

1. For example, in a Binomial experiment with $n = 1000$, $p = 0.1$, $q = 0.9$
2. The mean value of $x$ is $\mu = np = (1000)(0.1) = 100$   
3. the standard deviation of $x$ is $\sigma = \sqrt{npq} = \sqrt{(1000)(0.1)(0.9)} = 9.487$

(8) The Jordan Sports Equipment company finds that 10% of the general population is left-handed. A random sample of 12 people is selected.

(a) What is the probability that exactly three people are left-handed in this sample?

Let $x$ be the number of left-handed people in the group, then $x$ is a random variable, $x \sim Bin(12,\, 0.1)$

Then, $P(x = 3) = 12 C_3 p^3 q^{12-3} = (220)(0.1)^3(0.9)^9 \approx 0.0852$

(b) What is the probability that more than two are left-handed in this sample?

$P(x > 2) = 1 - P(x \leq 2) = 1 - (P(0) + P(1) + P(2)) \approx 1 - (0.282 + 0.377 + 0.230) = 0.111$

(c) If 4 people in this sample are left-handed, is it a usual or unusual value? Explain.

If you calculate the probability: $P(x \geq 4) = 1 - P(x < 4)$

$= 1 - (P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)) = 1 - (0.282 + 0.377 + 0.230 + 0.085) = 0.026 < 0.05$

That means, 4 left-handed people found in 12 randomly selected people is an **Unusually high** value. or, a **rare** event. (It is very unlikely to occur.)

(9) In a Binomial probability distribution with $n = 275$ and $p = 0.2$, find

Let a random variable $x \sim Bin(275,\, 0.2)$

(a) the minimum usual value is $x_{usual\, min} = \mu - 2\sigma = 55 - 2(6.63) = 41.74$

(b) the maximum usual value. $x_{usual\, max} = \mu + 2\sigma = 55 + 2(6.63) = 68.26$
An airline estimates that 90% of people booked on their flights actually show up. If the airline books 71 people on a flight for which the maximum number is 69, what is the probability that the number of people who show up will exceed the capacity of the plane?

(Set-ups:)

Each passenger who booked the flight is regarded as a trial, if he/she shows up, it is regarded as a "Success", with $p = 0.9$, if not showing up, it is a "Failure", with $q = 0.1$.

Let $x$ be the number of passengers who will show up among the 71 ones who booked the flight, then, $x$ is a Binomial random variable, $x \sim Bin(71, 0.9)$.

To find the probability that the number of people who show up will exceed the capacity of the plane:

$$\implies P(x > 69) = P(x = 70) + P(x = 71)$$

$$= 71 \binom{70}{70}(0.9)^{70}(0.1)^{71} + 71 \binom{71}{71}(0.9)^{71}(0.1)^{0} = 0.00445 + 0.000564 \approx 0.005 = 0.5\%$$

Note: An event with 0.5% probability is very unlikely to occur, almost impossible. So, the airline companies always sell a little more tickets than the actual capacity of the plane.

In a shipment of thousands of machine components actually having a 4% rate of defects, a random sample of 24 components are selected. What is the probability that there are fewer than 3 defectives in the sample?

If one component is randomly selected and it is a defective one, it is regarded as a "Success" with the probability $p = 0.04$.

Let $x$ be the number of defective components in the 24 randomly selected ones, then $x$ is a Binomial random variable, $x \sim Bin(24, 0.04)$

Then, $P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2) = 0.3754 + 0.3754 + 0.1799 = 0.9307$