EQUATIONS + INEQUALITIES WITH ABSOLUTE VALUES

I. Equations Of The Form \(|x| = k\)

If \(k > 0\), then \(|x| = k\) is equivalent to \(x = k\) or \(x = -k\)

Example 1: \(|x - 3| = 7\)
\(A)\) \(x - 3 = 7\) \(\text{or}\) \(B)\) \(x - 3 = -7\)
\(\begin{align*}
    x &= 7 + 3 & x &= -7 + 3 \\
    x &= 10 & x &= -4 \\
\end{align*}\)
\(\{-4, 10\}\)

II. Equations With Two Absolute Values

If \(a\) and \(b\) represent algebraic expressions, the equation \(|a| = |b|\)
is equivalent to the pair of equations \(a = b\) \(\text{or}\) \(a = -b\).

Example 2: \(|2x - 3| = |4x + 9|\)
\(A)\) \(2x - 3 = 4x + 9\) \(\text{or}\) \(B)\) \(2x - 3 = -(4x + 9)\)
\(\begin{align*}
    2x - 4x &= 9 + 3 & 2x - 3 &= -4x - 9 \\
    -2x &= 12 & 2x + 4x &= -9 + 3 \\
    x &= -6 & 6x &= -6 \\
    x &= -1 \\
\end{align*}\)
\(\{-6, -1\}\)

III. Inequalities Of The Form \(|x| < k\) \(\text{(Dog On A Leash!)}\)

If \(k \gg 0\), then
\(|x| < k\) is equivalent to \(-k < x < k\).
\(|x| \leq k\) is equivalent to \(-k \leq x \leq k\). \((k \geq 0)\)

Example 3: \(|2x - 5| < 25\)
\(-25 < 2x - 5 < 25\)
\(-20 < 2x < 30\)
\(-10 < x < 15\)
\((-10, 15)\)

Example 4: \(|-1 - 2x| \leq 5\)
\(-5 \leq -1 - 2x \leq 5\)
\(-4 \leq -2x \leq 6\)
\(\frac{-4x}{2} \geq \frac{-2x}{-2} \geq \frac{6}{-2}\)
\(2 \geq x \geq -3\)
\(-3 \leq x \leq 2\)
\([-3, 2]\)
IV. Inequalities Of The Form \(|x| > k\) (Skunk On The Run!)

If \(k \geq 0\), then
\[ |x| > k \text{ is equivalent to } x < -k \text{ or } x > k \]
\[ |x| \geq k \text{ is equivalent to } x \leq -k \text{ or } x \geq k \]

Example 5: \(|2x + 4| \geq 16\)

A) \(2x + 4 \leq -16\) or B) \(2x + 4 \geq 16\)
\[
\begin{align*}
2x & \leq -20 \\
x & \leq -10 \\
2x & \geq 12 \\
x & \geq 6
\end{align*}
\]
\((-\infty, -10] \cup [6, \infty)\)

Example 6: \(|4 - 2x| > 2\)

A) \(4 - 2x < -2\) or B) \(4 - 2x > 2\)
\[
\begin{align*}
-2x & < -6 \\
x & > 3 \\
-2x & > -2 \\
-x & > 1
\end{align*}
\]
\((-\infty, 1) \cup (3, \infty)\)

Example 7: \(|5x + 2| > -20\)

No matter what values we use for \(x\), when we take the absolute value of the expression \(5x + 2\), we will always get a result which is \(\geq 0\), so the solution is all real numbers. \((-\infty, \infty)\).

Example 8: \(|5x + 2| < -20\)

No matter what values we use for \(x\), when we take the absolute value of the expression \(5x + 2\), we will always get a result which is \(\geq 0\), so this inequality has no solution!