Writing the Equation of A Line And More!

We can write the equation of a line when we know at least one point on the line and the slope of the line. We use Point-Slope Form \( y - y_1 = m(x - x_1) \) to write the equation if we have the necessary information.

Example 1) Write the equation of the line with a slope of 2 and passing through the point \((3, 5)\). Write the equation in slope-intercept form \( y = mx + b \).

Write the formula : \[ y - y_1 = m(x - x_1) \] Substitute into the formula : \[ y - 5 = 2(x - 3) \] Distribute : \[ y - 5 = 2x - 6 \] Isolate for \( y \): \[ +5 \quad +5 \] \[ y = 2x - 1 \star \]

Check : \((3, 5)\)
\[ y = 2x - 1 \]
\[ 5 = 2 \cdot 3 - 1 \]
\[ 5 = 5 \checkmark \]

Example 2) Write the equation of the line with a slope of \( \frac{1}{3} \) and passing through the point \((-7, -1)\). Write the equation in slope-intercept form \( y = mx + b \).

Write the formula : \[ y - y_1 = m(x - x_1) \] Substitute into the formula : \[ y - (-1) = \frac{1}{3}(x - (-7)) \] Add the opposite : \[ y + 1 = \frac{1}{3}(x + 7) \] Multiply both sides by 3 : \[ 3[y + 1] = 3 \cdot \frac{1}{3}(x + 7) \] Distribute : \[ 3y + 3 = x + 7 \] Eliminate constant on left : \[ -3 \quad -3 \] \[ 3y = x + 4 \] Divide each term by 3 : \[ \frac{3y}{3} = \frac{x}{3} + \frac{4}{3} \]
\[ y = \frac{1}{3}x + \frac{4}{3} \star \]

Check : \((-7, -1)\)
\[ y = \frac{1}{3}x + \frac{4}{3} \]
\[ -1 \neq \frac{2}{3} \cdot (-7) + \frac{4}{3} \]
\[ -1 = -1 \checkmark \]

Example 3) Write the equation of the line passing through \((-2, -3)\) and \((-4, -13)\). Write the equation in slope-intercept form \( y = mx + b \).

Part I) We need to first find the slope to be able to find the equation of the line.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-13 - (-3)}{-4 - (-2)} = \frac{-10}{-2} = 5 \]
Part II) We then can use point-slope form because we know the slope and at least one point on the line.

Write the formula: \( y - y_1 = m(x - x_1) \)

Substitute into the formula: \( y - (-3) = 5(x - (-2)) \)

Add the opposite: 
\[-3 = 5 \cdot (-2) + 7\]

Distribute: 
\[y + 3 = 5x + 10\]

Isolate for \( y \):
\[-3 = 5x + 7\]

\[\frac{-3}{5} = \frac{5x}{5} + \frac{7}{5}\]

Check: \((-2, -3)\)

\[y = 5x + 7\]

Example 4) Write the equation of the line passing through \((-6, -2)\) and \((1, 4)\). Write the equation in slope-intercept form \(y = mx + b\).

Part I) We need to first find the slope to be able to find the equation of the line.

\[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{1 - (-6)} = \frac{4 + 2}{1 + 6} = \frac{6}{7}\]

Part II) We then can use point-slope form because we know the slope and at least one point on the line. Either point may by used because they are both points on the line.

Write the formula: \( y - y_1 = m(x - x_1) \)

Substitute into the formula: \( y - 4 = \frac{6}{7}(x - 1) \)

Multiply both sides by 7:
\[7[y - 4] = 7[\frac{6}{7}(x - 1)]\]

Distribute:
\[7y - 28 = 6x - 6\]

Eliminate constant on left:
\[\frac{7y}{7} = \frac{6x}{7} + \frac{22}{7}\]

Divide each term by 7:
\[y = \frac{6x}{7} + \frac{22}{7}\]

Check: \((1, 4)\)

\[y = \frac{6}{7}x + \frac{22}{7}\]

Example 5) Show that the two lines \(y = \frac{1}{3}x - 4\) and \(y = \frac{1}{3}x + 2\) are parallel by graphing the two lines using their slopes and \(y\)-intercepts on the provided rectangular grid.

Recall that to graph \(y = \frac{1}{3}x - 4\), first plot the \(y\)-intercept \((0, -4)\) and then from that point move over horizontally \(+3\) and vertically \(+1\) and plot a second point. Graph the line through the two points and label it \(y = \frac{1}{3}x - 4\). Recall that to graph \(y = \frac{1}{3}x + 2\), first plot the \(y\)-intercept \((0, 2)\) and then from that point move over horizontally \(3\) and vertically \(1\) and plot a second point. Graph the line through the two points and label it \(y = \frac{1}{3}x + 2\).
Example 6) Show that the two lines $y = \frac{1}{4}x + 1$ and $y = -4x + 1$ are perpendicular by graphing the two lines using their slopes and $y$–intercepts on the provided rectangular grid. Label each of the lines.
Ex. 5 demonstrated that two lines that have the same slopes are parallel, or vice-versa, if two lines are parallel, they have the same slope. Ex. 6 demonstrated that two lines that have slopes that are negative reciprocals and multiply to make $-1$ are perpendicular, and vice-versa that two lines that are perpendicular have slopes that are negative reciprocals. In Ex. 6, the slope of $y = \frac{1}{4}x + 1$ is $\frac{1}{4}$, and the slope of $y = -4x + 1$ is $-4$. The slopes are negative reciprocals because $\frac{1}{4} \cdot (-4) = -1$. Negative reciprocals are reciprocals where one of the reciprocals is positive and one is negative.

Example 7) Write the equation of the line passing through $(-2, 6)$ and parallel to the line $y = -4x + 3$. Then graph and label both lines.

Part I) The given line is $y = -4x + 3$.

Part II) The slope of the new line is $-4$ in order to be parallel to the given line $y = -4x + 3$. The given point of the new line is $(-2, 6)$.

Part III) We can use point-slope form to write the equation of the new line because we know the slope and one point.

Write the formula:

$y - y_1 = m(x - x_1)$

Substitute into the formula:

$y - 6 = -4(x - (-2))$

Add the opposite:

$y - 6 = -4(x + 2)$

Distribute:

$y - 6 = -4x - 8$

Isolate for $y$:

$y = -4x - 2 \star$

Check: $(-2, 6)$

$y = -4x - 2$

$6 \pm -4 \cdot (-2) - 2$

$6 \mp 8 - 2$

$6 = 6 \checkmark$
Example 8) Write the equation of the line passing through \((-3, 1)\) and perpendicular to the line \(y = -2x + 3\). Then graph and label both lines.

Part I) The given line is \(y = -2x + 3\).
Part II) The slope of the new line is \(\frac{1}{2}\) in order to be perpendicular to the given line \(y = -2x + 3\). \([-2 \cdot \frac{1}{2} = -1]\) The given point of the new line is \((-3, 1)\).
Part III) We can use point-slope form to write the equation of the new line because we know the slope and one point.

\[
\begin{align*}
\text{Write the formula:} & \quad y - y_1 = m(x - x_1) \\
\text{Substitute into the formula:} & \quad y - 1 = \frac{1}{2}(x - (-3)) \\
\text{Add the opposite:} & \quad y - 1 = \frac{1}{2}(x + 3) \\
\text{Multiply both sides by 4:} & \quad 2[y - 1] = 2[\frac{1}{2}(x + 3)] \\
\text{Distribute:} & \quad 2y - 2 = x + 3 \\
\text{Eliminate constant on left:} & \quad 2y + 2 = x + 5 \\
\text{Divide each term by 2:} & \quad \frac{2y}{2} = \frac{x}{2} + \frac{5}{2} \\
& \quad y = \frac{1}{2}x + \frac{5}{2} \star
\end{align*}
\]

Check : \((-3, 1)\)
\[y = \frac{1}{2}x + \frac{5}{2}\]
\[1 = \frac{1}{2} \cdot (-3) + \frac{5}{2}\]
\[1 = \frac{-3}{2} + \frac{5}{2}\]
\[1 = 1 \star\]
A Review Of Key Formulas!

Example 9) Any linear equation can be written in general form as \(Ax + By = C\) where \(A, B,\) and \(C\) are real numbers and both \(A\) and \(B\) cannot both be zero. When we write a linear equation in general form, we usually clear the fractions and make \(A\) positive. Graph the linear equation in general form \(2x + 3y = 6\).

We can graph \(2x + 3y = 6\) by finding the \(x-\) and \(y-\)intercepts and an insurance point. We can also convert to slope-intercept form \(y = mx + b\) and use a table of values or the slope and \(y-intercept\) to graph the line.

Example 10) Any linear equation can be written in slope-intercept form \(y = mx + b\). Determine the slope and \(y-intercept\) of \(y = 3x + 5\) and then graph the line.

The slope of \(y = 3x + 5\) is \(\frac{3}{1}\) and the \(y-intercept\) is 5. To graph \(y = 3x + 5\), locate the \(y-intercept\), \((0, 5)\) and plot the point. From that point, move horizontally 1 unit and vertically 3 units to find a second point. Plot that point and then draw the line connecting those two points and label the line \(y = 3x + 5\).

Example 11) The slope of a line passing through any two points can be found using the formula \(m = \frac{y_2 - y_1}{x_2 - x_1}\). Find the slope of the line passing through \((5, 7)\) and \((-2, -7)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 7}{-2 - 5} = \frac{-14}{-7} = 2
\]

Example 12) The equation of a horizontal line is written in the form \(y = b\). Graph \(y = 3\).

Go up 3 units on the \(y-axis\) and draw a horizontal line.

Example 13) The equation of a vertical line is written in the form \(x = a\). Graph \(x = -2\).

Go to the left 2 units on the \(x-axis\) and draw a vertical line.

Example 14) We can use point-slope form \(y - y_1 = m(x - x_1)\) to find the equation of the line if we know the slope and one point. Write the equation of the line with a slope of 2 and passing through the point \((3, 5)\).

\[
\text{Write the formula : } \\
\text{Substitute into the formula : } \\
\text{Distribute : } \\
\text{Isolate for } y : \\
\]

\[
y - 5 = 2(x - 3) \\
y - 5 = 2x - 6 \\
+ 5 + 5 \\
y = 2x - 1 \star
\]

Check : \((3, 5)\)

\[
y = 2x - 1 \\
5 = 5 \checkmark
\]