REAL NUMBERS A LA GREEN!

**Addition Rules:**
A) When adding any two positive real numbers, the sum is positive. \( 7 + 8 = 15 \)
B) When adding any two negative real numbers, the sum is negative. \(-4 + (-12) = -16\)
C) When adding one of each, one positive real number and one negative real number, take the absolute value of each and subtract. The sign of the answer is the sign of the real number which has the larger absolute value. \(-122 + 79 = -43\)

**Subtraction Rules:**
When subtracting, use the rule \( x - y = x + (-y) \). Leave the first real number alone, change the operation to addition, and change the second real number to its opposite. \( 12 - (-14) = 12 + 14 = 26 \)

**Multiplication Rules:**
A) When the signs of the real numbers are like, the product is positive. \((8)(6) = 48\) or \((-7)(-8) = 56\)
B) When the signs of the real numbers are unlike, the product is negative. \((-3)(12) = -36\)
C) When the problem has an even number of negative factors, the product is positive. \((-4)(-2)(-3)(-10) = 240\)
D) When the problem has an odd number of negative factors, the product is negative. \((-3)(-5)(-7) = -105\)

**Division Rules:**
A) When the signs of the real numbers are like, the quotient is positive. \(\frac{20}{4} = 5\) or \(\frac{-100}{-10} = 10\)

B) When the signs of the real numbers are unlike, the quotient is negative. \(\frac{-48}{4} = -12\)

C) When 0 is in the numerator and any real number other than 0 is in the denominator, the quotient is 0. \(\frac{0}{4} = 0\) or algebraically speaking, \(\frac{0}{k} = 0\) where \(k\) is any real number except 0.

D) When 0 is in the denominator or is the divisor, the quotient is undefined. \(\frac{4}{0}\) is undefined, or algebraically speaking, \(\frac{k}{0}\) is undefined where \(k\) is any real number.

**Exponentiation:**
A) If the base is a negative real number, it is enclosed in parentheses. \((-5)^2 = (-5)(-5) = 25\)
B) When no grouping symbols are present, the base is a positive real number. \(8^2 = 8 \cdot 8 = 64\)
or \(-12^2 = -(12 \cdot 12) = -144\)

**Order of Operations:**
When grouping symbols are used, evaluate within the innermost pair of grouping symbols using the following steps.
A) Do exponentiations.
B) Do any multiplications or divisions as they occur from left to right.
C) Do any additions or subtractions as they occur from left to right.
\[-5 \cdot \{12 + 2 \cdot [-32 \div (3 - 5)^2]\} = -5 \cdot \{12 + 2 \cdot [-32 \div (-2)^2]\} = -5 \cdot \{12 + 2 \cdot [-32 \div 4]\} = -5 \cdot \{12 + 2 \cdot (-8)\} = -5 \cdot \{12 + (-16)\} = -5 \cdot \{-4\} = 20\]

**Opposites:**

\(-5\) is read as the opposite of \(-5\) and equals 5.

\(-5\) is read the opposite of 5 and equals \(-5\).

**THE GREAT EIGHT RULES OF EXPONENTS!**

**#1) THE ADDITION RULE FOR EXPONENTS:** \(b^m \cdot b^n = b^{m+n}\)

EXAMPLE: \(c^5 \cdot c^4 \cdot c^{11} = c^{20}\)

**#2) THE SUBTRACTION RULE FOR EXPONENTS:** \(\frac{b^m}{b^n} = b^{m-n}\)

EXAMPLE: \(\frac{c^{17}}{c^{12}} = c^{17-12} = c^5\)

**#3) THE ZERO EXPONENT RULE:** \(b^0 = 1\) provided that \(b \neq 0\).

EXAMPLES: \(z^0 = 1;\) \(10^0 = 1;\) \((-5)^0 = 1;\) \((-7)^0 = -(1) = -1\)

**#4) THE NEGATIVE EXPONENT RULE:** \(b^{-n} = \frac{1}{b^n}\)

EXAMPLES: \(x^{-5} = \frac{1}{x^5};\) \(2^{-6} = \frac{1}{2^6} = \frac{1}{64}\)

**#5) A PRODUCT RAISED TO A POWER RULE:** \((a \cdot b)^n = a^n \cdot b^n\)

EXAMPLE: \((3 \cdot c \cdot h)^4 = 3^4 \cdot c^4 \cdot h^4 = 81c^4h^4\)

**#6) A FRACTION RAISED TO A POWER RULE:** \(\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\)

EXAMPLE: \(\left(\frac{4}{9}\right)^3 = \frac{4^3}{9^3} = \frac{64}{729}\)

**#7) THE NEGATIVE POWER OF A FRACTION RULE:** \(\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\)

EXAMPLE: \(\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = \frac{16}{9}\)

**#8) THE POWER TO A POWER RULE:** \((b^m)^n = b^{mn}\)

EXAMPLES: \((x^5)^3 = x^{5 \cdot 3} = x^{15};\) \((2^3)^2 = 2^6 = 64\)
PROPERTIES OF REAL NUMBERS:

1) COMMUTATIVE PROPERTY OF ADDITION: \( a + b = b + a \)

EXAMPLE: \( 4 + 6 = 6 + 4 \) We can add any two real numbers in any order we wish.

2) COMMUTATIVE PROPERTY OF MULTIPLICATION: \( a \cdot b = b \cdot a \)

EXAMPLE: \( 4 \cdot 6 = 6 \cdot 4 \) We can multiply any two real numbers in any order we wish.

3) ASSOCIATIVE PROPERTY OF ADDITION: \( (a + b) + c = a + (b + c) \)

EXAMPLE: \((4 + 6) + 10 = 4 + (6 + 10)\) We can group any two numbers of three or more where only addition is involved and add those two numbers first without changing the result.

4) ASSOCIATIVE PROPERTY OF MULTIPLICATION: \( (ab)c = a(bc) \)

EXAMPLE: \((4 \cdot 6) \cdot 10 = 4 \cdot (6 \cdot 10)\) We can group any two numbers of three or more where only multiplication is involved and multiply those two numbers first without changing the result.

5) DISTRIBUTIVE PROPERTY OF MULTIPLICATION WITH RESPECT TO ADDITION:
\[ a(b + c) = ab + ac \text{ or } a(b + c + d) = ab + ac + ad \text{ or } ba + ca = (b + c)a \]

EXAMPLE: \(4 \cdot (6 + 10) = 4 \cdot 6 + 4 \cdot 10\) Multiply the first number times each of the numbers or variables within the parentheses.

6) ADDITIVE IDENTITY: \( a + 0 = a \) or \( 0 + a = a \)

EXAMPLE: \(4 + 0 = 4\) We can add any real number and 0 and the sum is the real number we added to 0.

7) ADDITIVE INVERSE PROPERTY OR THE PROPERTY OF OPPOSITES: \( a + (−a) = 0 \)

EXAMPLE: \(4 + (−4) = 0\) When we add opposites, the sum is 0.

8) MULTIPLICATIVE IDENTITY: \( 1 \cdot a = a \) or \( a \cdot 1 = a \)

EXAMPLE: \(1 \cdot 4 = 4\) We can multiply any real number by 1 and the product is the real number we multiplied by 1.

9) MULTIPLICATIVE INVERSE PROPERTY OR THE PROPERTY OF RECIPROCALS:
\( a \cdot \left(\frac{1}{a}\right) = 1 \)

EXAMPLE: \(4 \cdot \left(\frac{1}{4}\right) = 1\) When we multiply reciprocals, the product is 1.

10) MULTIPLICATION PROPERTY OF 0: \( a \cdot 0 = 0 \) or \( 0 \cdot a = 0 \)

EXAMPLE: \(4 \cdot 0 = 0\) When we multiply any number by 0, the product is 0.