Math 20 Practice Problems for Exam #3
1) Factor $3a^4 - 3$.
2) Factor $x^3 + 3x^2 - 25x - 75$.
3) Factor $64a^3 - 125b^6$.
4) Factor $18y^3 - 3y^2 - 10y$.
5) Factor $4x^5 + 256x^2$.
6) Factor $4x^2 + 4x + 1 - 4y^2$.
7) Factor $z^2 - 25z^2 + 10x - 1$.
8) Use factoring to solve $(x + 3)(x - 2) = 50$.
9) Use factoring to solve $x^3 + 3x^2 - x - 3 = 0$.
10) The length of a rectangular ice-skating rink is 20 meters greater than twice its width. Find the dimensions of the ice-skating rink if its area is 6,000 square meters.
11) A model rocket is launched from the top of a cliff 144 feet above sea level. The function $s(t) = -16t^2 + 128t + 144$ describes the rocket's height above the water, $s(t)$ in feet, $t$ seconds after it was launched. The rocket misses the edge of the cliff on its way down and eventually lands in the ocean. How long will it take for the rocket to hit the water?
12) Simplify: $\frac{3x^2 - 10xy - 8y^2}{4y^2 - xy}$
13) Multiply: $\frac{m^2 - n^2}{2x^2 + 3x - 2} \cdot \frac{3x^2 + 5x - 3}{n^2 - m^2}$
14) Multiply: $\frac{p^2 - q^2}{q^2 - p^2} \cdot \frac{q^2 + pq}{p^2 + p^2q + pq}$
15) Subtract: $\frac{2x^4 + 4}{x^2 + 13x + 12} - \frac{x + 3}{x^2 + 13x + 12}$
16) Add: $\frac{3 - x}{2x} + \frac{x - 1}{x - 2}$
17) Simplify: $\frac{3}{x + 1} - \frac{2}{x - 1} + \frac{x + 3}{x^2 - 1}$
18) Simplify: $\frac{x - 1}{x^2 - y^2}$
19) Simplify: $\frac{1 + 6x - 8x^2}{1 + x^2 + 12x^2}$
20) Solve: $\frac{x^2 + 10}{x^2 - 1} + \frac{3x}{x - 1} = \frac{2x}{x + 1}$
21) Solve: $\frac{x}{x - 3} + \frac{x}{x - 3} = \frac{x - 3}{x - 1}$
22) Solve: $\frac{8x}{x + 1} = 4 - \frac{8}{x + 1}$
23) Solve for $x$: $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$
24) You can design a website in 9 hours. If your friend, Larry, works with you, the website can be designed in 3 hours. How long would it take Larry to design the website working alone?
25) In still water, a boat averages 8 miles per hour. It takes the same amount of time to travel 15 miles downstream, with the current, as 9 miles upstream, against the current. What is the rate of the current?
26) The distance that an object falls varies directly as the square of the time it has been falling. An object falls 144 feet in 3 seconds. Find how far it will fall in 7 seconds.
27) The loudness of a stereo speaker, measured in decibels, varies inversely as the square of your distance from the speaker. When you are 8 feet away from the speaker, the loudness is 28 decibels. What is the loudness when you are 4 feet from the speaker?
28) Simplify:
29) Expand: $(x + 2)^3$
30) Divide: $(8x^3 + 1) \div (2x + 1)$
31) Use synthetic division:
32) Find the domain of $f(x) = \frac{3x}{x^2 - 13x + 36}$.
Bonus 1) A rectangular garden measures 16 feet by 12 feet. A path of uniform width is to be added so as to surround the entire garden. The landscape artist doing the work wants the garden and path to cover an area of 320 square feet. How wide should the path be? See page 354 for a helpful diagram!

Bonus 2) Show that $x - 3$ is a factor of $2x^3 - 3x^2 - 11x + 6$ using both the remainder theorem and synthetic division.

And Now The Solutions!

1) $3a^4 - 3$
   $= 3(a^4 - 1)$
   $= 3(a^2 - 1)(a^2 + 1)$
   $= 3(a - 1)(a + 1)(a^2 + 1)$

2) $x^3 + 3x^2 - 25x - 75$
   $= x^2(x + 3) - 25(x + 3)$
   $= (x + 3)(x^2 - 25)$
   $= (x + 3)(x - 5)(x + 5)$

3) Use the difference of cubes rule:
   $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
   $64a^3 - 125b^6$
   $= (4a)^3 - (5b^2)^3$
   $= (4a - 5b^2)(16a^2 + 20ab^2 + 25b^4)$

4) $18y^3 - 3y^2 - 10y$
   $= y(18y^2 - 3y - 10)$
   $= y(3y + 2)(6y - 5)$

5) $4x^5 + 256x^2$
   $= 4x^2(x^3 + 64)$ ← sum of cubes!
   $= 4x^2(x^3 + 4^3)$
   $= 4x^2(x + 4)(x^2 - 4x + 16)$

6) $4x^2 + 4x + 1 - 4y^2$
   $= (4x^2 + 4x + 1) - 4y^2$
   $= (2x + 1)(2x + 1) - 4y^2$
   $= (2x + 1)^2 - (2y)^2$
   $= (2x + 1 - 2y)(2x + 1 + 2y)$

7) $z^2 - 25x^2 + 10x - 1$
   $= z^2 - (25x^2 - 10x + 1)$
   $= z^2 - (5x - 1)(5x - 1)$
   $= z^2 - (5x - 1)^2$
   $= (z - (5x - 1))(z + (5x - 1))$
   $= (z - 5x + 1)(z + 5x - 1)$

8) $(x + 3)(x - 2) = 50$
   $x^2 - 2x + 3x - 6 - 50 = 0$
   $x^2 + x - 56 = 0$
   $(x + 8)(x - 7) = 0$
   either $x + 8 = 0$ or $x - 7 = 0$
   $x = -8$ or $x = 7$
   $\{-8, 7\}$

9) $x^3 + 3x^2 - x - 3 = 0$
   $x^2(x + 3) - 1(x + 3) = 0$
   $(x + 3)(x^2 - 1) = 0$
   $(x + 3)(x - 1)(x + 1) = 0$
   $x + 3 = 0$ or $x - 1 = 0$ or $x + 1 = 0$
   $x = -3$ or $x = 1$ or $x = -1$
   $\{-3, -1, 1\}$

10) 1) Let $x$ = the width
     2) Then $2x + 20$ = the length
     3) width $\cdot$ length = area of rectangle
     4) $x \cdot (2x + 20) = 6,000$
         $2x^2 + 20x = 6,000$
\[2x^2 + 20x - 6,000 = 0\]
\[\frac{2x^2 + 20x - 6,000}{2} = 0\]
\[x^2 + 10x - 3,000 = 0\]
\[(x + 60)(x - 50) = 0\]
either \(x + 60 = 0\) or \(x - 50 = 0\)
\[x = -60\] impossible! Discard!
or \(x = 50\)
5) Check : \[x \cdot (2x + 20) = 6,000\]
\[50 \cdot (2 \cdot 50 + 20) = 6,000\]
\[50 \cdot 120 = 6,000\] \(\checkmark\)
width = 50 m, length = 120 m.

11) 1) Let \(t\) = time in seconds
2) the height \(s(t) = 0\) when the model rocket hits the water...
3) \(s(t) = -16t^2 + 128t + 144\)
4) \(0 = -16t^2 + 128t + 144\)
\[16t^2 - 128t - 144 = 0\]
\[16(t^2 - 8t - 9) = 0\]
\[16(t - 9)(t + 1) = 0\]
either \(t - 9 = 0\) or \(t + 1 = 0\)
\[t = 9\] or \(t = -1\) (impossible)
5) It will take 9 seconds.

12) \[\frac{3x^2 - 10xy - 8y^2}{4y^2 - xy}\]
\[= \frac{(x-4y)(3x+2y)}{y(4y-x)}\]
\[= \frac{-1(3x+2y)}{y}\]
\[= -3x-2y\] \(\blacktriangleleft\)

13) \[\frac{m^2-n^2}{2x^2+3x-2} \cdot \frac{2x^2+5x-3}{n^2-m^2}\]
\[= \frac{(m-n)(m+n)(x+3)(2x-1)}{(x+2)(2x-1)(n-m)(n+m)}\]
\[= \frac{-1(x+3)}{x+2}\]
\[= \frac{-x-3}{x+2}\] \(\blacktriangleleft\)

14) \[\frac{p^3-q^3}{q^2-p^2} \cdot \frac{q^2+pq}{p^3+p^2q+pq^2}\]

15) \[\frac{2x+4}{x^2+13x+12} - \frac{x+3}{x^2+13x+12}\]
\[= \frac{2x+4-x-3}{x^2+13x+12}\]
\[= \frac{x+1}{x^2+13x+12}\]
\[= \frac{1}{x+1}\]

16) \[\frac{3-x}{2-x} + \frac{x-1}{x-2}\]
\[= \frac{-1(3-x)}{x-2} + \frac{x-1}{x-2}\]
\[= \frac{x-2}{x-2}\]
\[= 2\] \(\blacktriangleleft\)

17) \[\frac{3}{x+1} - \frac{2}{x-1} + \frac{x+3}{x^2-1}\]
\[= \frac{3}{x+1} + \frac{-2}{x-1} + \frac{x+3}{(x-1)(x+1)}\]
\[= \frac{3(x-1)}{(x+1)(x-1)} + \frac{-2(x+1)}{(x-1)(x+1)} + \frac{x+3}{(x-1)(x+1)}\]
\[= \frac{3x-3-2x-2+x+3}{(x+1)(x-1)}\]
\[= \frac{2x-2}{(x+1)(x-1)}\]
\[= \frac{2(x-1)}{(x-1)(x+1)}\]
\[= \frac{2}{x+1}\] \(\blacktriangleleft\)

18) \[\frac{x^{-1} + y^{-1}}{x^2 - y^2}\]
\[= \frac{1}{x} + \frac{1}{y}\]
\[= \frac{1}{x^2} - \frac{1}{y^2}\]
\[
\frac{x^2 y^2}{\frac{1}{x}} + \frac{x^3 y}{\frac{1}{y}} = \frac{xy^2}{y^2 - x^2}
\]

\[
= \frac{xy(y + x)}{(y - x)(y + x)}
\]

\[
= \frac{xy}{y - x}
\]

\[19) \frac{1 + 6x - 1 + 8x^2}{1 + x^1 + 12x^2} = 1 + \frac{6x}{x^2 + \frac{8}{x^2}}
\]

\[
= \frac{x^2}{1} + \frac{x^2}{1} \cdot \frac{6}{x} + \frac{x^2}{1} \cdot \frac{8}{x^2}
\]

\[
= \frac{x^2 + 6x + 8}{x^2 + x - 12}
\]

\[
= \frac{(x + 2)(x + 4)}{(x + 4)(x - 3)}
\]

\[= \frac{x + 2}{x - 3}
\]

\[20) \frac{-x^2 + 10}{x^2 - 1} + \frac{3x}{x - 1} = \frac{2x}{x + 1}, x \neq 1 \text{ or } -1
\]

\[= \frac{-x^2 + 10 + 3x(x + 1)}{(x-1)(x+1)} + \frac{2x}{x + 1}
\]

\[= \frac{(x-1)(x+1) + (x-1)(x+3) + x}{x + 1}
\]

\[= \frac{x^2 + 10 + 3x(x + 1) = 2x(x - 1)}{x - 1}
\]

\[= \frac{-x^2 + 10 + 3x^2 + 3x}{x^2 - 2x}
\]

\[= \frac{2x^2 + 3x + 10 = 2x^2 - 2x}{3x + 10 = -2x}
\]

\[= \frac{3x + 2x = -10}{5x = -10}
\]

\[= \frac{x = -2}{x = -2}
\]

Because the restricted values were
\[x = 1 \text{ or } x = -1, -2 \text{ is a valid solution. } \{-2\}
\]

\[21) \frac{x - 4}{x - 3} + \frac{x - 2}{x - 3} = \frac{x - 3}{1}, x \neq 3
\]

\[
\frac{(x-3)(x-4)}{x-3} + \frac{(x-3)(x-2)}{x-3} = \frac{(x-3)(x-3)}{1}
\]

\[x - 4 + x - 2 = x^2 - 6x + 9
\]

\[2x - 6 = x^2 - 6x + 9
\]

\[0 = x^2 - 6x - 2x + 9 + 6
\]

\[0 = x^2 - 8x + 15
\]

\[0 = (x - 3)(x - 5)
\]

either \(x - 3 = 0 \) or \( x - 5 = 0
\]

\[x = 3 \text{ or } x = 5
\]

Because 3 is a restricted value, the only valid solution is \( x = 5! \) \{5\}

\[22) \frac{8x}{x+1} = 4 - \frac{8}{x+1}; x \neq -1
\]

\[= \frac{(x+1)\cdot 8x}{x+1} = \frac{(x+1)\cdot 4}{x+1} - \frac{(x+1)\cdot 8}{x+1}
\]

\[8x = 4x + 4 - 8
\]

\[8x = 4x - 4
\]

\[4x = -4
\]

\[x = -1
\]

Because \( x = -1 \) is a restricted value,
this equation has no solution. \( \emptyset \)

\[23) \frac{\frac{1}{x} + \frac{1}{y}}{z} = \frac{\frac{xy+1}{x}}{z}
\]

\[yz + xz = xy
\]

\[yz = xy - xz
\]

\[yz = x(y - z)
\]

\[\frac{yz}{y - z} = \frac{(y - z)}{x}
\]

\[x = \frac{yz}{y - z}
\]

\[24) \text{1) Let } x = \text{ the # of hours for Larry to design the website}
\]

\[3) \text{part of job part of job part of job}
\]

\[\text{you } + \text{ Larry } = \text{ two of you}
\]

\[\text{do in does in do together}
\]

\[1 \text{ hour } 1 \text{ hour } \text{ in } 1 \text{ hour}
\]

\[\frac{1}{9} + \frac{1}{x} = \frac{1}{3}
\]
4) \(\frac{9x-1}{9} + \frac{9x-1}{x} = \frac{9x-1}{3}\)
\[x + 9 = 3x\]
\[9 = 3x - x\]
\[9 = 2x\]
\[\frac{9}{2} = \frac{2x}{2}\]
\[x = \frac{9}{2} = 4\frac{1}{2} \text{ hours}\]

5) It would take Larry 4\(\frac{1}{2}\) hours to design the website working alone.

25) 1) Let \(x\) = the speed of the current
2) 

<table>
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<tr>
<th>trip</th>
<th>distance</th>
<th>rate</th>
<th>time</th>
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</thead>
<tbody>
<tr>
<td>downstream</td>
<td>15</td>
<td>(8 + x)</td>
<td>(\frac{15}{8+x})</td>
</tr>
<tr>
<td>upstream</td>
<td>9</td>
<td>(8 - x)</td>
<td>(\frac{9}{8-x})</td>
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</tbody>
</table>

3) Because it says the two trips are in the same amount of time, then the time downstream = time upstream.

4) \(\frac{15}{8+x} = \frac{9}{8-x}\)
\[15(8 - x) = 9(8 + x)\]
\[120 - 15x = 72 + 9x\]
\[120 - 72 = 9x + 15x\]
\[48 = 24x\]
\[\frac{48}{24} = \frac{24x}{24}\]
\[x = 2\]

5) Check:
\(\frac{15}{8+x} = \frac{9}{8-x}\)
\[\frac{15}{8+2} = \frac{9}{8-2}\]
\[\frac{15}{10} = \frac{9}{6}\]
\[\frac{3}{2} = \frac{3}{2} \checkmark\]

The rate of the current is 2 miles per hour.

Part I) Find the constant of variation, \(k\).

We know that \(y\) varies directly as the square of \(x\) is expressed as 
\(y = kx^2\)
By changing the letters, we get that distance \((s)\) varies directly as the square of time \((t)\)
\(s = kt^2\)

Insert the initial values from the problem into the formula...
\[144 = k \cdot 3^2\]
\[9k = 144\]
\[\frac{9k}{9} = \frac{144}{9}\]
\[k = 16\]

Part II) Use the formula again inserting the known quantities, \(k = 16\) and \(t = 7\).
\[s = kt^2\]
\[s = 16 \cdot 7^2\]
\[s = 16 \cdot 49\]
\[s = 784 \text{ feet}\]

The object will fall 784 feet in 7 seconds.

26) This is a direct variation problem where the distance \((s)\) an object falls is directly proportional to the square of the time \((t)\) it has been falling.
27) This is an inverse variation problem where the loudness of the speakers varies inversely with your distance from the speakers.

Part I) Find the constant of variation, \( k \).

We know that \( y \) varies inversely with the square of \( x \) is expressed as
\[
y = \frac{k}{x^2}
\]
By changing the letters, we get that loudness \( l \) varies inversely with the square of the distance \( d \)
\[
l = \frac{k}{d^2}
\]
Insert the initial values from the problem into the formula...

28 \[
\begin{align*}
28 &= \frac{k}{s^2} \\
28 &= \frac{k}{32} \\
k &= 1,792
\end{align*}
\]

Part II) Use the formula again inserting the known quantities, \( k = 1,792 \) and \( d = 4 \).

\[
l = \frac{k}{d^2}
\]
\[
l = \frac{1,792}{16}
\]
\[
l = 112
\]
The loudness increases to 112 decibels when you are 4 feet from the speakers.

29) \[
(x + 2)^3
= (x + 2)((x + 2)(x + 2))
= (x + 2)(x^2 + 4x + 4)
= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8
= x^3 + 6x^2 + 12x + 8
\]

30) \[
\begin{align*}
4x^2 - 2x + 1 &
= (x + 1)8x^3 + 0x^2 + 0x + 1
\end{align*}
\]
\[
\begin{array}{c|cccc}
& 8x^3 & 0x^2 & 0x & 1 \\
\hline
2x + 1 & \phantom{0}8x^3 & \phantom{0}0x^2 & \phantom{0}0x & \phantom{0}1 \\
\hline
& \phantom{0}8x^3 & \phantom{0}0x^2 & \phantom{0}0x & \phantom{0}1 \\
& \phantom{0}4x^2 & \phantom{0}2x & \phantom{0}2x & \phantom{0}1 \\
& \phantom{0}2x + 1 & \phantom{0}2x & \phantom{0}2x & \phantom{0}1 \\
& \phantom{0}0 & \phantom{0}0 & \phantom{0}0 & \phantom{0}0 \\
\end{array}
\]

31) \[
\begin{array}{c|cccc}
-2 & 4 & 5 & 0 & -1 \\
\hline
& -8 & 6 & -12 & \\
4 & -3 & 6 & -13
\end{array}
\]

\[
= 4x^2 - 3x + 6 + \frac{-13}{x + 2}
\]

32) Set the denominator equal to zero to find the excluded values from the domain.

\[
x^2 - 13x + 36 = 0
\]
\[
(x - 4)(x - 9) = 0
\]
either \( x = 4 \) or \( x = 9 \)

The domain of \( f \) in set builder notation is
\[
\{ x \mid x \text{ is a real number and } x \neq 4 \text{ and } x \neq 9 \}
\]
Bonus 1)
1) Let \( x \) = the uniform width around the garden
2) Then \( 12 + 2x = \text{the width} \)
   Then \( 16 + 2x = \text{the length} \)
3) \( w \cdot l = \text{area of garden} + \text{path} \)
   \((12 + 2x)(16 + 2x) = 320\)
4) \( 192 + 24x + 32x + 4x^2 - 320 = 0 \)
   \( 4x^2 + 56x - 128 = 0 \)
   \( \frac{4x^2}{4} + \frac{56x}{4} - \frac{128}{2} = 0 \)
   \( x^2 + 14x - 32 = 0 \)
   \( (x - 2)(x + 16) = 0 \)
   either \( x - 2 = 0 \) or \( x + 16 = 0 \)
   \( x = 2 \) or \( x = -16 \) which is impossible
5) Check:
   \((12 + 2x)(16 + 2x) = 320\)
   \((12 + 2 \cdot 2)(16 + 2 \cdot 2) \neq 320\)
   \(16 \cdot 20 = 320\)

The uniform width of the garden path is 2 feet.

Bonus 2)
\[ f(3) = 2 \cdot 3^3 - 3 \cdot 3^2 - 11 \cdot 3 + 6 \]
\[ = 2 \cdot 27 - 3 \cdot 9 - 11 \cdot 3 + 6 \]
\[ = 54 - 27 - 33 + 6 \]
\[ = 0 \]

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<th></th>
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<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>-2</td>
<td>0</td>
<td></td>
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</tbody>
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Because when we use the remainder theorem or synthetic division, we get a remainder of 0, we have shown that \( x - 3 \) is a factor of \( 2x^3 - 3x^2 - 11x + 6 \).