#1) Find the inverse of \(-2x + y = -4\). Then graph both the function and its inverse on a single coordinate system. Find the equation of the axis of symmetry and graph it as a dashed or dotted line.

The function is \(-2x + y = -4\) or \(y = 2x - 4\) so its inverse is \(x = 2y - 4\). Solve \(x = 2y - 4\) for \(y\).

\[
\begin{align*}
x &= 2y - 4 \\
2y - 4 &= x \\
2y &= x + 4 \\
\frac{2y}{2} &= \frac{x}{2} + \frac{4}{2} \\
y &= \frac{1}{2}x + 2
\end{align*}
\]

Graph \(y = 2x - 4\) and \(y = \frac{1}{2}x + 2\) as lines and \(y = x\) as a dotted or dashed line as the line of symmetry.

#2) Find the inverse of \(y = 2x^2\). You need not solve the inverse for \(y\). Then graph both the function and its inverse on a single coordinate system. Answer two questions: 1) Is the function one-to-one? 2) Is its inverse a function?

First graph \(y = 2x^2\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

The graph of \(y = 2x^2\) is a parabola with the vertex at \((0, 0)\) that opens up. \(y = 2x^2\) is not a one-to-one function because it does not pass the horizontal line test. Another way to tell it is not a one-to-one function is to notice that in the table of values there are repeating values for \(x\). Recall that the definition of a one-to-one function is one where each input value of \(x\) in the domain determines a different output value of \(y\) in the range. No \(y\) value can repeat.

The inverse of \(y = 2x^2\) is \(x = 2y^2\). Graph \(x = 2y^2\) on the same coordinate system. As \(y\) is the independent variable and \(x\) is the dependent variable in this equation, choose values of \(y\) to determine the corresponding values of \(x\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>8</th>
<th>2</th>
<th>0</th>
<th>2</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

\(x = 2y^2\) is not a function because it does not pass the vertical line test. Also no two \(y\)-values can be associated with any one \(x\)-value. All parabolas which open either to the left or right are not functions.

#3) Graph \(x^2 + y^2 = 16\)

\(x^2 + y^2 = 4^2\) is an equation of a circle whose center is \((0, 0)\) and whose radius is 4. To graph the circle start at the center \((0,0)\) and plot points 4 units to the north, south, east, and west of the center. Then make the circle by connecting those 4 points in circular form. ★
#4) Graph \( x^2 + y^2 + 6x - 8y = -21 \).

\[
\begin{align*}
x^2 + y^2 + 6x - 8y &= -21 \\
x^2 + 6x + \frac{9}{4} + y^2 - 8y + 16 &= -21 + \frac{9}{4} + 16 \\
(x + 3)^2 + (y - 4)^2 &= 2^2
\end{align*}
\]

The center of the circle is \((-3, 4)\) and the radius is 2. To graph the circle start at the center \((-3, 4)\) and plot points 2 units to the north, south, east, and west of the center. Then make the circle by connecting those 4 points in circular form. ⭐

#5) Graph \( x = (y - 2)^2 - 4 \)

To graph \( x = (y - 2)^2 - 4 \), recall that graphs of equations in the form \( x = a(y - k)^2 + h \) are parabolas with a vertex at \((h, k)\) and open to the right if \( a \) is a positive real number. The vertex of \( x = 1(y - 2)^2 - 4 \) is \((-4, 2)\). Choose values of \( y \) to find the corresponding values of \( x \). ⭐

| \( x \) | 0 | -3 | -4 | -3 | 0 |
| \( y \) | 0 | 1 | 2 | 3 | 4 |

#6) Graph \( x = -y^2 - 6x - 8 \)

To graph \( x = -y^2 - 6y - 8 \), find the \( y \)-value of the vertex using the formula \( y = \frac{-b}{2a} \).

\[
y = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3.
\]

Then the \( x \)-value of the vertex can be found by substituting \(-3\) for \( y \) in the equation.

\[
x = -(−3)^2 - 6 \cdot (-3) - 8
\]

\[
= -9 + 18 - 8
\]

\[
= 1
\]

The vertex is \((1, -3)\) and opens to the left because the coefficient of \( x^2 \) is negative. Find more points by choosing values of \( y \) to find the corresponding values of \( x \). ⭐

| \( x \) | -3 | 0 | 1 | 0 | -3 |
| \( y \) | -5 | -4 | -3 | -2 | -1 |

#7) Graph \( y = 4^x \) and \( y = \log_4 x \) and the axis of symmetry. Give the domain and range of both functions.

To graph the exponential function, \( y = 4^x \) choose \(-2\) through \( 2 \) for values of \( x \) to find the corresponding values of \( y \). \( y = 4^x \) is a one-to-one function that is also an increasing function. Note that every exponential function \( y = b^x \) passes through \((0, 1)\) and \((1, b)\) and has a domain of \((-\infty, \infty)\) and a range of \((0, \infty)\). The \( x \)-axis is an asymptote of the graph.

| \( x \) | -2 | -1 | 0 | 1 | 2 |
| \( y \) | \( \frac{1}{16} \) | \( \frac{1}{4} \) | 1 | 4 | 16 |
To understand the logarithmic function \( y = \log_4 x \), convert the equation to exponential form \( 4^y = x \) and note that \( 4^y = x \) is the inverse of the one-to-one function \( y = 4^x \) and thus is also a function.

To graph \( 4^y = x \), choose \(-2\) through \(2\) for values of \( y \) to find the corresponding values of \( x \). Note that every logarithmic function \( y = \log_b x \) or \( b^y = x \) passes through \((1, 0)\) and \((b, 1)\) and has a domain of \((0, \infty)\) and a range of \((-\infty, \infty)\). The \( y \)-axis is an asymptote of the graph. ★

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{1}{16} )</th>
<th>( \frac{1}{4} )</th>
<th>( 1 )</th>
<th>( 4 )</th>
<th>( 16 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(-2)</td>
<td>(-1)</td>
<td>( 0 )</td>
<td>( 1 )</td>
<td>( 2 )</td>
</tr>
</tbody>
</table>

#8) An initial deposit of \$10,000 earns 8% interest, compounded monthly. How much will be in the account after 10 years? If \( P \) \$ is deposited in an account and interest is paid \( n \) times a year at an annual rate \( r \), the amount \( A \) in the account after \( t \) years can be found using the formula \( A = P \left(1 + \frac{r}{n}\right)^{nt} \).

Round your answer to the nearest penny.

\( P = 10,000 \), \( r = 8\% = 0.08 \), \( n = 12 \) compounding periods (monthly), \( t = 10 \) years

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]

\[
A = 10,000 \left(1 + \frac{0.08}{12}\right)^{12\cdot10}
\]

\[
A = 10,000 \left(1 + \frac{0.08}{12}\right)^{120}
\]

*Calculator key strokes using a Sharp Advanced D.A.L.* :

\[
(0.08 \div 12 + 1) y^x 120 \times 10,000 =
\]

\$22,196.40 to the nearest penny. ★

#9) The population of a town is currently 25,000. The town is expected to grow according to the Malthusian model, with an annual growth rate of 6%. Find the population of the town in 20 years. Use the formula \( A = P e^{rt} \) … Round to the nearest person!

\( P = 25,000 \), \( r = 6\% = 0.6 \), \( t = 20 \) years

\[
A = Pe^{rt}
\]

\[
A = 25,000e^{0.06 \cdot 20}
\]

\[
A = 25,000e^{1.2}
\]

\[
A \approx 83,003 \text{ people}
\]

*Calculator key strokes using a Sharp Advanced D.A.L.* :

\[
2nd F \ln 1.2 \times 25,000 =
\]
#10) Solve for \( x \). \( \log_5 x = -2 \).

\[
\log_5 x = -2 \quad \text{in exponential form is} \quad 5^{-2} = x. \quad \text{Therefore} \quad x = \frac{1}{5^2} = \frac{1}{25}. \quad \boxed{C} \quad \star
\]

#11) Use the properties to expand \( \log_b \frac{\sqrt[3]{x} \ y^z}{z^3} \) in terms of logarithms of \( x \), \( y \), and \( z \).

\[
\log_b \frac{\sqrt[3]{x} \ y^z}{z^3} = \log_b \left( x^{\frac{1}{3}} \ y^z \right) \quad \text{Write} \quad \frac{\sqrt[3]{x} \ y^z}{z^3} \quad \text{using exponents}. \\
= \log_b x^{\frac{1}{3}} + \log_b y^z - \log_b z^3 \quad \text{(P. 5)} \\
= \frac{1}{3} \log_b x + 2 \log_b y + (-3) \log_b z \quad \text{(P. 7)} \\
= \frac{1}{3} \log_b x + 2 \log_b y - 3 \log_b z \quad \boxed{E} \quad \star
\]

#12) Use the properties of logarithms to condense \( 3 \log_7 x - \frac{1}{3} \log_7 y \) as the logarithm of a single quantity.

\[
3 \log_7 x - \frac{1}{3} \log_7 y = \log_7 x^3 - \log_7 y^{\frac{1}{3}} \quad \text{(Prop. 7)} \\
= \log_7 \frac{x^3}{y^{\frac{1}{3}}} \quad \text{(Prop. 6)} \\
= \log_7 \frac{x^3}{\sqrt[3]{y}} \quad \text{(Convert to radical notation.)} \quad \boxed{A} \quad \star
\]

#13 Use the change of base formula \( \log_b x = \frac{\log_a x}{\log_a b} \) to find \( \log_2 9 \) correct to four places.

\[
\log_b x = \frac{\log_a x}{\log_a b} \\
\log_2 9 = \frac{\log_{10} 9}{\log_{10} 2} \quad \text{(Prop 8)}
\]

\textit{Calculator key strokes using a Sharpe Advanced D.A.L. :}

\[
\log 9 \div \log 2 \quad = \quad 3.169925001 \approx 3.1699 \ \text{correct to four places}. \quad \boxed{B} \quad \star
\]

#14) Solve \( 4^x = 3 \).

\[
4^x = 3 \\
\log 4^x = \log 3 \\
x \log 4 = \log 3 \\
\frac{x \log 4}{\log 4} = \frac{\log 3}{\log 4} \\
x = \frac{\log 3}{\log 4} \quad \boxed{B} \quad \star
\]

#15) Solve the exponential equation \( 3^{x^2 - 3x} = \frac{1}{9} \).

\[
3^{x^2 - 3x} = \frac{1}{9} \\
3^{x^2 - 3x} = 3^{-2} \\
x^2 - 3x = -2
\]
\[ x^2 - 3x + 2 = 0 \]
\[ (x - 1)(x - 2) = 0 \]

either \( x - 1 = 0 \) or \( x - 2 = 0 \)

\[ x = 1 \quad \text{or} \quad x = 2 \quad A \star \]

#16) Solve the logarithmic equation \( \log(-2x - 4) - \log(x - 1) = 0 \).

\[
\log(-2x - 4) - \log(x - 1) = 0 \\
\log(-2x - 4) = \log(x - 1) \\
-2x - 4 = x - 1 \\
-2x - x = 4 - 1 \\
-3x = 3 \\
x = -1
\]

Must Check \(-1\):

\[
\log(-2x - 4) - \log(x - 1) = 0 \\
\log(-2 \cdot (-1) - 4) - \log(-1 - 1) \neq 0 \\
\log(2 - 4) - \log(-1 - 1) \neq 0 \\
\log(-2) - \log(-2) = 0
\]

The logarithm of a negative number does not exist. Therefore, we have to discard \(-1\) as an extraneous solution. The logarithmic equation \( \log(-2x - 4) - \log(x - 1) = 0 \) has no solution. \( E \star \)

#17) Solve the logarithmic equation \( \log x + \log(x + 21) = 2 \)

\[
\log x + \log(x + 21) = 2 \\
\log x(x + 21) = 2 \quad \text{(Prop. 5)} \\
\log_{10} x(x + 21) = 2 \\
10^2 = x(x + 21) \text{ Convert to exponential notation.}
\]

\[ x(x + 21) = 10^2 \]
\[ x^2 + 21x = 100 \]
\[ x^2 + 21x - 100 = 0 \]
\[ (x + 25)(x - 4) = 0 \]
\[ x = -25 \text{ or } x = 4 \]

Must Check 4:

\[
\log x + \log(x + 21) = 2 \\
\log 4 + \log(4 + 21) \neq 2 \\
\log 4 + \log 25 \neq 2 \\
\log 100 \neq 2 \\
\log_{10} 100 \neq 2 \\
10^2 = 100 \checkmark
\]
The logarithm of a negative number does not exist so $-25$ is an extraneous solution and must be discarded. The only solution is 4. $\{4\}$

#18) Solve the logarithmic equation: $\log_3(4x + 15) - \log_3x = 2$

\[
\log_3(4x + 15) - \log_3x = 2 \\
\log_3\left(\frac{4x+15}{x}\right) = 2 \\
\frac{4x+15}{x} = 3^2 \\
\frac{4x+15}{x} = 9 \\
x = \frac{15}{5} \\
x = 3 \\
\]

$Must Check 3:$

\[
\log_3(4x + 15) - \log_3x = 2 \\
\log_3(4 \cdot 3 + 15) - \log_33 = 2 \\
\log_327 - \log_33 = 2 \\
3 - 1 = 2 \\
\]

#19) Solve the non-linear system algebraically. (You might check your work by solving the system graphically.

\[
x^2 + y^2 = 9 \\
x + y = 3 \\
\]

The first equation represents a circle, and the second equation represents a line. They could intersect at two points, 1 point, or not at all. Thus, this system could have two solutions, 1 solution or no solution.

$Solve for x in equation \#2 and substitute into equation \#1.$

\[
x = -y + 3 \\
(-y + 3)^2 + y^2 = 9 \\
(-y + 3)(-y + 3) + y^2 = 9 \\
y^2 - 6y + 9 + y^2 = 9 \\
2y^2 - 6y + 9 - 9 = 0 \\
2y^2 - 6y = 0 \\
2y(y - 3) = 0 \\
y = 0 or y = 3 \\
\]

$Substitute the y-values back into what appears to be the simpler of the two equations, in this case, x + y = 3.$

\[
x + y = 3 \\
x + 0 = 3 \\
x = 3 \\
x + 3 = 3 \\
x = 3 \\
x = 0 \\
\]

The solutions are $(3, 0)$ and $(0, 3)$. $\[D\]$
#20) Simplify \( \ln e^2 \)
\[
\ln e^2 = 2
\]
according to property 3. \( \square \)

Or...
\[
\ln e^2 = y
\]
\[
\log_e e^2 = y
\]
\[
e^y = e^2
\]
\[
\therefore y = 2
\]

#21) The function \( P(t) = 89.18e^{-0.004t} \) models the percentage of married men in the United States who are employed \( t \) years after 1959. In which year were 80% of U.S. married men employed?
\[
P(t) = 89.18e^{-0.004t}
\]
\[
\frac{80}{89.18} = e^{-0.004t}
\]
\[
e^{-0.004t} = \frac{80}{89.18}
\]
\[
\ln e^{-0.004t} = \ln \frac{80}{89.18}
\]
\[
-0.004t = \ln \frac{80}{89.18}
\]
\[
t = \frac{\ln \frac{80}{89.18}}{-0.004}
\]
\[
t \approx 27
\]

In 1986 80% of married men were employed. \((1959 + 27 = 1986) \quad \square \)

**Calculator key strokes using a Sharpe Advanced D.A.L. :**
\[
\ln (\frac{80}{89.18}) + / - 0.004 =
\]

#22) How long to the nearest tenth of a year, will it take $5,000 to grow to $10,000 at 6% annual interest compounded monthly? Use the formula \( A = P(1 + \frac{r}{n})^{nt} \).
\[
A = P(1 + \frac{r}{n})^{nt}
\]
\[
10,000 = 5,000\left(1 + \frac{0.06}{12}\right)^{12t}
\]
\[
2 = 1.005^{12t}
\]
\[
1.005^{12t} = 2
\]
\[
\ln 1.005^{12t} = \ln 2
\]
\[
12t \ln 1.005 = \ln 2
\]
\[
t = \frac{\ln 2}{12 \ln 1.005}
\]
\[
t \approx 11.6 \text{ years} \quad \square
\]

**Calculator key strokes using a Sharpe Advanced D.A.L. :**
\[
\ln 2 \div (12 \times \ln 1.005) =
\]

#23) The number of bacteria \( B(t) \) present in a culture after \( t \) minutes is given as \( B(t) = 10e^{kt} \). If there are 3,995 bacteria present after 3 minutes, find \( k \).
\[
B(t) = 10e^{kt}
\]
\[
3,995 = 10e^{k \times 3}
\]
\[
3,995 = 10e^{3k}
\]
\[
\frac{3995}{10} = e^{3k}
\]
\[
399.5 = e^{3k}
\]
\[
\ln 399.5 = \ln e^{3k}
\]
\[
\ln 399.5 = 3k
\]
\[
\ln 399.5 = 3k
\]
\[
\frac{\ln 399.5}{3} = \frac{3k}{3}
\]
\[
k = 1.997 \quad \square
\]
#24) Find the distance between \((8, 6)\) and \((13, -6)\). If necessary, round the answer to two decimal places. Use the formula \(d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\).

\[
\begin{align*}
d & = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
d & = \sqrt{(13 - 8)^2 + (-6 - 6)^2} \\
d & = \sqrt{5^2 + (-12)^2} \\
d & = \sqrt{25 + 144} \\
d & = \sqrt{169} \\
d & = 13 \quad \star
\end{align*}
\]

#25) If \(f(x) = x^3 - x\), and \(g(x) = 7x + 2\), find \((g \circ f)(-3)\).

\[
\begin{align*}
(g \circ f)(-3) & = g(f(-3)) \\
& = g(-24) \\
& = -166 \quad \star
\end{align*}
\]

I) \(f(-3) = (-3)^3 - (-3) = -27 + 3 = -24\)

II) \(g(-24) = 7 \cdot (-24) + 2 = -168 + 2 = -166\)

Bonus #1) Solve the logarithmic equation \(\frac{\log(5x+6)}{\log x} = 2\).

\[
\begin{align*}
\frac{\log(5x+6)}{\log x} & = 2 \\
\log(5x + 6) & = 2 \cdot (\log x) \\
\log(5x + 6) & = \log x^2 \\
x^2 & = 5x + 6 \\
x^2 - 5x - 6 & = 0 \\
(x - 6)(x + 1) & = 0 \\
x & = 6 \text{ or } x = -1 \quad \blacksquare
\end{align*}
\]

**Must Check 6:**

\[
\begin{align*}
\frac{\log(5x+6)}{\log x} & = \frac{2}{1} \\
\frac{\log(5x+6)}{\log 6} & = \frac{2}{1} \\
\log 36 & = 2 \cdot \log 6 \\
\log 36 & = \log 6^2 \\
\log 36 & = \log 36 \quad \checkmark
\end{align*}
\]

**Must Check -1:**

\[
\begin{align*}
\frac{\log(5x+6)}{\log x} & = \frac{2}{1} \\
\frac{\log(5x+6)}{\log (-1)} & = \frac{2}{1} \\
\log 36 & \neq 2 \cdot \log 6 \\
\log 36 & \neq \log 6^2 \\
\log 36 & \neq \log 36 \quad \checkmark
\end{align*}
\]

The log of \(-1\) does not exist!

The only solution is 6. \{6\} \star
Bonus #2) How old are the bones of a prehistoric man that retains 6% of its original carbon-14 content? Use the formula $A = A_0e^{-0.000121t}$ if $A$ is the amount of carbon-14 still present after $t$ years, $A_0$ was the amount present at $t = 0$. Round your answer to the nearest year.

\[
A = A_0e^{-0.000121t} \\
0.06A_0 = A_0e^{-0.000121t} \\
0.06 = e^{-0.000121t} \quad \text{Divide both sides by } A_0. \\
\ln 0.06 = \ln e^{-0.000121t} \\
\ln 0.06 = -0.000121t \\
t = \frac{\ln 0.06}{-0.000121} \\
t \approx 23,251 \text{ years} \star
\]

_Calculator key strokes using a Casio:_

\[
\text{ln} \quad 0.06 \quad \div \quad +/− \quad 0.000121 \quad =
\]

Bonus #3) Find the vertex of the parabola of $y = -2x^2 - 4x + 1$ by completing the square.

To find the vertex of the parabola of $y = -2x^2 - 8x - 2$, rather than using $x = \frac{-b}{a}$ to find the $x-value$ of the vertex, use completing the square to get the equation in the form $y = a(x - h)^2 + k$.

\[
y = -2x^2 - 4x + 1 \\
y = -2(x^2 + 2x) + 1 \\
y = -2(x^2 + 2x + \square) + 2 \cdot \square + 1 \\
y = -2(x^2 + 2x + 1) + 2 \cdot 1 + 1 \\
y = -2(x + 1)^2 + 2 + 1 \\
y = -2(x + 1)^2 + 3 \\
y = -2(x - (-1))^2 + 3
\]

The graph is a parabola opening down with a vertex of $(-1, 3)$. ★

The points from the table below would help you to then graph the parabola if you were asked to do so on an exam.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-5</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>-5</td>
</tr>
</tbody>
</table>