EXERCISE 7-1

Things to remember:

1. AREA BETWEEN TWO CURVES
   If \( f \) and \( g \) are continuous and \( f(x) \geq g(x) \) over the interval \([a, b]\), then the area bounded by \( y = f(x) \) and \( y = g(x) \), for \( a \leq x \leq b \), is given exactly by:

\[
A = \int_a^b [f(x) - g(x)] \, dx.
\]

2. GINI INDEX OF INCOME CONCENTRATION
   If \( y = f(x) \) is the equation of a Lorenz curve, then the Gini Index = \( 2 \int_0^1 [x - f(x)] \, dx \).

1. \( A = \int_a^b g(x) \, dx \)

3. \( A = \int_a^b [-h(x)] \, dx \)

5. Since the shaded region in Figure (c) is below the \( x \)-axis, \( h(x) \leq 0 \). Thus, \( \int_a^b h(x) \, dx \) represents the negative of the area of the region.

7. \( y = x + 4; \ y = 0 \) on \([0, 4]\)
   \[
   A = \int_0^4 (x + 4) \, dx = \left( \frac{1}{2} x^2 + 4x \right)_0^4 = (8 + 16) - 0 = 24
   \]

9. \( y = -2x - 5; \ y = 0 \) on \([-1, 2]\)
   \[
   A = -\int_{-1}^2 (-2x - 5) \, dx
   = \int_{-1}^2 (2x + 5) \, dx = \left( x^2 + 5x \right)_{-1}^2 = 14 - (1 - 5) = 18
   \]
11. \( y = x^2 - 20; \ y = 0 \) on \([-3, 0]\)
   
   \[ A = -\int_{-3}^{0} (x^2 - 20) \, dx \]
   
   \[ = \int_{-3}^{0} (20 - x^2) \, dx = \left(20x - \frac{1}{3} x^3\right)\bigg|_{-3}^{0} \]
   
   \[ = 0 - (-60 + 9) = 51 \]

13. \( y = -x^2 + 10; \ y = 0 \) on \([-3, 3]\)

   \[ A = \int_{-3}^{3} (-x^2 + 10) \, dx = \left(-\frac{1}{3} x^3 + 10x\right)\bigg|_{-3}^{3} \]
   
   \[ = (-9 + 30) - (9 - 30) = 42 \]

15. \( y = x^3 + 1; \ y = 0 \) on \([0, 2]\)

   \[ A = \int_{0}^{2} (x^3 + 1) \, dx = \left(-\frac{1}{4} x^4 + x\right)\bigg|_{0}^{2} \]
   
   \[ = 4 + 2 = 6 \]

17. \( y = x(1 - x); \ y = 0 \) on \([-1, 0]\)

   \[ A = -\int_{-1}^{0} x(1 - x) \, dx = -\int_{-1}^{0} (x - x^2) \, dx \]
   
   \[ = \int_{-1}^{0} (x^2 - x) \, dx \]
   
   \[ = \left(\frac{1}{3} x^3 - \frac{1}{2} x^2\right)\bigg|_{-1}^{0} \]
   
   \[ = 0 - \left(-\frac{1}{3} - \frac{1}{2}\right) \]
   
   \[ = \frac{5}{6} \approx 0.833 \]

19. \( y = -e^x; \ y = 0 \) on \([-1, 1]\)

   \[ A = -\int_{-1}^{1} -e^x \, dx = \int_{-1}^{1} e^x \, dx \]
   
   \[ = e^x\bigg|_{-1}^{1} = e - e^{-1} \]
   
   \[ = 2.350 \]

21. \( y = \frac{1}{x}; \ y = 0 \) on \([1, e]\)

   \[ A = \int_{1}^{e} \frac{1}{x} \, dx = \ln x\bigg|_{1}^{e} \]
   
   \[ = \ln e - \ln 1 = 1 \]
23. \( y = -e^{-2x}; \ y = 0 \) on \([0, 1]\)

\[
A = -\int_0^1 e^{-2x} \, dx = \int_0^1 e^{-2x} \, dx
\]

\[
= -\frac{1}{2}e^{-2x} \bigg|_0^1 = -\frac{1}{2}e^{-2} + \frac{1}{2}
\]

\[= 0.432\]

25. \( A = \int_a^b [-f(x)] \, dx \)

27. \( a = \int_b^c f(x) \, dx + \int_c^d [-f(x)] \, dx \)

29. \( A = \int_c^d [f(x) - g(x)] \, dx \)

31. \( A = \int_a^b [f(x) - g(x)] \, dx + \int_b^c [g(x) - f(x)] \, dx \)

33. Find the x-coordinates of the points of intersection of the two curves on \([a, d]\) by solving the equation \( f(x) = g(x) \), \( a \leq x \leq d \), to find \( x = b \) and \( x = c \). Then note that \( f(x) \geq g(x) \) on \([a, b]\), \( g(x) \geq f(x) \) on \([b, c]\) and \( f(x) \geq g(x) \) on \([c, d]\).

Thus,

\[
\text{Area} = \int_a^b [f(x) - g(x)] \, dx + \int_b^c [g(x) - f(x)] \, dx + \int_c^d [f(x) - g(x)] \, dx
\]

35. \( A = A_1 + A_2 = \int_{-2}^0 -x \, dx + \int_0^1 -(x) \, dx \)

\[
= \int_{-2}^0 x \, dx + \int_0^1 x \, dx
\]

\[
= -\frac{x^2}{2} \bigg|_{-2}^0 + \frac{x^2}{2} \bigg|_0^1
\]

\[
= -\left(0 - \frac{(-2)^2}{2}\right) + \left(\frac{1^2}{2} - 0\right)
\]

\[= 2 + \frac{1}{2} = \frac{5}{2} = 2.5\]

37. \( A = A_1 + A_2 = \int_0^2 -(x^2 - 4) \, dx + \int_2^3 (x^2 - 4) \, dx \)

\[
= \int_0^2 (4 - x^2) \, dx + \int_2^3 (x^2 - 4) \, dx
\]

\[
= \left(4x - \frac{x^3}{3}\right) \bigg|_0^2 + \left(\frac{x^3}{3} - 4x\right) \bigg|_2^3
\]

\[
= \left(8 - \frac{8}{3}\right) + \left(\frac{27}{3} - 12\right) - \left(\frac{8}{3} - 8\right)
\]

\[= 13 - \frac{16}{3} = \frac{39}{3} - \frac{16}{3} = \frac{23}{3} = 7.667\]
39. \( A = A_1 + A_2 = \int_{-2}^{0} (x^2 - 3x) \, dx + \int_{0}^{2} -(x^2 - 3x) \, dx \)

\[
= \int_{-2}^{0} (x^2 - 3x) \, dx + \int_{0}^{2} (3x - x^2) \, dx \\
= \left( \frac{1}{3} x^3 - \frac{3}{2} x^2 \right) \bigg|_{-2}^{0} + \left( \frac{3}{2} x^2 - \frac{1}{3} x^3 \right) \bigg|_{0}^{2} \\
= 0 - \left( -\frac{8}{3} - 6 \right) + \left( 6 - \frac{8}{3} \right) - 0 = 12
\]

41. \( A = \int_{-1}^{-1} [12 - (-2x + 8)] \, dx = \int_{-1}^{1} (2x + 4) \, dx \)

\[
= \left( \frac{2x^2}{2} + 4x \right) \bigg|_{-1}^{1} = (x^2 + 4x) \bigg|_{-1}^{1} \\
= (4 + 8) - (1 - 4) \\
= 12 + 3 = 15
\]

43. \( A = \int_{-2}^{2} (12 - 3x^2) \, dx = \left(12x - \frac{3x^3}{3}\right) \bigg|_{-2}^{2} \)

\[
= (12x - x^3) \bigg|_{-2}^{2} \\
= (12 \cdot 2 - 2^3) - [12 \cdot (-2) - (-2)^3] \\
= 16 - (-16) = 32
\]

45. \((3, -5)\) and \((-3, -5)\) are the points of intersection.

\[
A = \int_{-3}^{3} [4 - x^2 - (-5)] \, dx \\
= \int_{-3}^{3} (9 - x^2) \, dx = \left(9x - \frac{x^3}{3}\right) \bigg|_{-3}^{3} \\
= \left(9 \cdot 3 - \frac{3^3}{3}\right) - \left(9(-3) - \frac{(-3)^3}{3}\right) \\
= 18 + 18 = 36
\]
47. \[ A = \int_{-1}^{2} \left[ (x^2 + 1) - (2x - 2) \right] dx \]
\[ = \int_{-1}^{2} (x^2 - 2x + 3) dx = \left[ \frac{x^3}{3} - x^2 + 3x \right]_{-1}^{2} \]
\[ = \left( \frac{8}{3} - 4 + 6 \right) - \left( -\frac{1}{3} - 1 - 3 \right) \]
\[ = 3 - 4 + 6 + 1 + 3 = 9 \]

49. \[ A = \int_{1}^{2} \left[ e^{0.5x} - \left( -\frac{1}{x} \right) \right] dx \]
\[ = \int_{1}^{2} \left( e^{0.5x} + \frac{1}{x} \right) dx \]
\[ = \left[ \frac{e^{0.5x}}{0.5} + \ln|x| \right]_{1}^{2} \]
\[ = 2e + \ln 2 - 2e^{0.5} = 2.832 \]

51. \[ y = \sqrt{9 - x^2}; \quad y = 0 \text{ on } [-3, 3] \]
area = \[ \int_{-3}^{3} \sqrt{9 - x^2} dx = \text{area of semicircle of radius 3} \]
\[ = \frac{9}{2} \pi = 14.137 \]

53. \[ y = -\sqrt{16 - x^2}; \quad y = 0 \text{ on } [0, 4] \]
area = \[ \int_{0}^{4} \sqrt{16 - x^2} dx = \text{area of quarter circle of radius 4} \]
\[ = \frac{1}{4} \cdot 16\pi = 4\pi = 12.566 \]

55. \[ y = -\sqrt{4 - x^2}; \quad y = \sqrt{4 - x^2}, \text{ on } [-2, 2] \]
area = \[ \int_{-2}^{2} \left[ \sqrt{4 - x^2} - (-\sqrt{4 - x^2}) \right] dx \]
\[ = 2 \int_{-2}^{2} \sqrt{4 - x^2} dx = 2 \int_{-2}^{2} \sqrt{4 - x^2} dx \]
\[ = 2 \text{ area of semi circle of radius 2} \]
\[ = \text{area of circle of radius 2} = 4\pi = 12.566 \]

57. The graphs of \[ y = 3 - 5x - 2x^2 \] and \[ y = 2x^2 + 3x - 2 \] are shown at the right. The x-coordinates of the points of intersection are: \( x_1 = -2.5, x_2 = 0.5 \).

\[ A = \int_{-2.5}^{0.5} \left[ (3 - 5x - 2x^2) - (2x^2 + 3x - 2) \right] dx \]
\[ = \int_{-2.5}^{0.5} (5 - 8x - 4x^2) dx = \left[ 5x - 4x^2 - \frac{4}{3} x^3 \right]_{-2.5}^{0.5} \]
\[ = 1.333 + 16.667 = 18 \]
59. The graphs of \( y = -0.5x + 2.25 \) and \( y = \frac{1}{x} \) are shown below.

The \( x \)-coordinates of the points of intersection are: \( x_1 = 0.5, x_2 = 4 \).

\[
A = \int_{0.5}^{4} \left[ (-0.5x + 2.25) - \left( \frac{1}{x} \right) \right] dx
\]
\[
= \left( -\frac{1}{4} x^2 + \frac{9}{4} x - \ln x \right) \bigg|_{0.5}^{4}
\]
\[
= [-4 + 9 - \ln 4] - [-0.0625 + 1.125 - \ln(0.5)]
\]
\[
= 1.858
\]

61. The graphs of \( y = e^x \) and \( y = e^{-x}, 0 \leq x \leq 4 \), are shown at the right.

\[
A = \int_{0}^{4} (e^x - e^{-x}) dx = (e^x + e^{-x}) \bigg|_{0}^{4}
\]
\[
= e^4 + e^{-4} - (1 + 1)
\]
\[
= 52.616
\]

63. The graphs are given at the right. To find the points of intersection, solve:

\[
x^3 = 4x
\]
\[
x^2 - 4x = 0
\]
\[
x(x^2 - 4) = 0
\]
\[
x(x + 2)(x - 2) = 0
\]

Thus, the points of intersection are \((-2, -8), (0, 0), \) and \((2, 8)\).

\[
A = A_1 + A_2 = \int_{-2}^{0} (x^3 - 4x) dx + \int_{0}^{2} (4x - x^3) dx
\]
\[
= \left( \frac{x^4}{4} - 2x^2 \right) \bigg|_{-2}^{0} + \left( 2x^2 - \frac{x^4}{4} \right) \bigg|_{0}^{2}
\]
\[
= 0 - \left( \frac{(-2)^4}{4} - 2(-2)^2 \right) + \left( 2(2)^2 - \frac{2^4}{4} \right) - 0
\]
\[
= -4 + 8 + 8 - 4 = 8
\]

65. The graphs are given at the right. To find the points of intersection, solve:

\[
x^3 - 3x^2 - 9x + 12 = x + 12
\]
\[
x^3 - 3x^2 - 10x = 0
\]
\[
x(x^2 - 3x - 10) = 0
\]
\[
x(x - 5)(x + 2) = 0
\]
\[
\frac{x}{x} = -2, x = 0, x = 5
\]

Thus, \((-2, 10), (0, 12), \) and \((5, 17)\) are the points of intersection.
\[ A = A_1 + A_2 \]
\[ = \int_{-2}^{0} [x^3 - 3x^2 - 9x + 12 - (x + 12)]dx \]
\[ + \int_{0}^{5} [x + 12 - (x^3 - 3x^2 - 9x + 12)]dx \]
\[ = \int_{-2}^{0} (x^3 - 3x^2 - 10x)dx + \int_{0}^{5} (-x^3 + 3x^2 + 10x)dx \]
\[ = \left( \frac{x^4}{4} - x^3 - 5x^2 \right) \bigg|_{-2}^{0} + \left( -\frac{x^4}{4} + x^3 + 5x^2 \right) \bigg|_{0}^{5} \]
\[ = -\left( \frac{(-2)^4}{4} - (-2)^3 - 5(-2)^2 \right) + \left( \frac{(-5)^4}{4} + 5^3 + 5 \cdot 5^2 \right) \]
\[ = 8 + \frac{375}{4} = \frac{407}{4} = 101.75 \]

67. The graphs are given at the right. To find the points of intersection, solve:

\[
\begin{align*}
  x^4 - 4x^2 + 1 &= x^2 - 3 \\
  x^4 - 5x^2 + 4 &= 0 \\
  (x^2 - 4)(x^2 - 1) &= 0 \\
  x &= -2, -1, 1, 2 \\
\end{align*}
\]

\[ A = A_1 + A_2 + A_3 \]
\[ = \int_{-2}^{-1} [(x^2 - 3) - (x^4 - 4x^2 + 1)]dx + \int_{-1}^{1} [(x^4 - 4x^2 + 1) - (x^2 - 3)]dx \]
\[ + \int_{1}^{5} [(x^2 - 3) - (x^4 - 4x^2 + 1)]dx \]
\[ = \int_{-2}^{-1} (-x^4 + 5x^2 - 4)dx + \int_{-1}^{1} (x^4 - 5x^2 + 4)dx + \int_{1}^{2} (-x^4 + 5x^2 - 4)dx \]
\[ = \left( -\frac{x^5}{5} + \frac{5}{3}x^3 - 4x \right) \bigg|_{-2}^{-1} + \left( \frac{x^5}{5} - \frac{5}{3}x^3 + 4x \right) \bigg|_{-1}^{1} + \left( -\frac{x^5}{5} + \frac{5}{3}x^3 - 4x \right) \bigg|_{1}^{2} \]
\[ = \left( \frac{1}{5} - \frac{5}{3} + 4 \right) - \left( \frac{32}{5} - \frac{40}{3} + 8 \right) + \left( \frac{1}{5} - \frac{5}{3} + 4 \right) - \left( \frac{1}{5} + \frac{5}{3} - 4 \right) \]
\[ + \left( -\frac{32}{5} + \frac{40}{3} - 8 \right) - \left( \frac{1}{5} + \frac{5}{3} - 4 \right) = 8 \]

69. The graphs are given below. The x-coordinates of the points of intersection are: \( x_1 = -2, x_2 = 0.5, x_3 = 2 \)

\[ A = A_1 + A_2 \]
\[ = \int_{-2}^{0.5} [(x^3 - x^2 + 2) - (-x^3 + 8x - 2)]dx \]
\[ + \int_{0.5}^{2} [(-x^3 + 8x - 2) - (x^3 - x^2 + 2)]dx \]
\[ = \int_{-2}^{0.5} (2x^3 - x^2 - 8x + 4)dx + \int_{0.5}^{2} (-2x^3 + x^2 + 8x - 4)dx \]
\[
\begin{align*}
&= \left( \frac{1}{2} x^4 - \frac{1}{3} x^3 - 4x^2 + 4x \right)\bigg|_{-2}^{0.5} + \left( -\frac{1}{2} x^4 + \frac{1}{3} x^3 + 4x^2 - 4x \right)\bigg|_{0.5}^{2} \\
&= \left( \frac{1}{32} - \frac{1}{24} - 1 + 2 \right) - (8 + \frac{8}{3} - 16 - 8) \\
&\quad + (-8 + \frac{8}{3} + 16 - 8) - \left( -\frac{1}{32} + \frac{1}{24} + 1 - 2 \right) \\
&= 18 + \frac{1}{16} - \frac{1}{12} = 17.979
\end{align*}
\]

71. The graphs are given at the right. The x-coordinates of the points of intersection are: \( x_1 \approx -1.924, \ x_2 = 1.373 \)

\[
A = \int_{-1.924}^{1.373} [(3 - 2x) - e^{-x}] \, dx
\]
\[
= (3x - x^2 + e^{-x})\bigg|_{-1.924}^{1.373}
\]
\[
= 2.487 - (-2.626) = 5.113
\]

73. The graphs are given at the right. The x-coordinates of the points of intersection are: \( x_1 \approx -2.247, \ x_2 = 0.264, \ x_3 = 1.439 \)

\[
A = A_1 + A_2 = \int_{-2.247}^{0.264} [e^x - (5x - x^3)] \, dx
\]
\[
+ \int_{0.264}^{1.439} [5x - x^3 - e^x] \, dx
\]
\[
= \left( e^x - \frac{5}{2} x^2 + \frac{1}{4} x^4 \right)\bigg|_{-2.247}^{0.264} + \left( 5x^2 - \frac{1}{4} x^4 - e^x \right)\bigg|_{0.264}^{1.439}
\]
\[
= (1.129) - (-6.144) + (-0.112) - (-1.129) = 8.290
\]

75. \( y = e^{-x}; \ y = \sqrt{\ln x}; \ 2 \leq x \leq 5 \)

The graphs of \( y_1 = e^{-x} \) and \( y_2 = \sqrt{\ln x} \) are

\[
\text{Int}(y_2-y_1; x; 2, 5) \approx 3.165511884
\]

Thus, \( A = \int_{2}^{5} (\sqrt{\ln x} - e^{-x}) \, dx = 3.166 \)
77. $y = e^{x^2}; \ y = x + 2$

The graphs of $y_1 = e^{x^2}$ and $y_2 = x + 2$ are

![Graph of $y_1 = e^{x^2}$ and $y_2 = x + 2$]

The curves intersect at $x \approx -0.588$ and $x \approx 1.057$

\[ y = 1.412 \quad \text{and} \quad y = 3.057 \]

\[ \int_{-0.588}^{1.057} (x + 2 - e^{x^2}) \, dx = 1.385 \]

79. \[ \int_5^{10} R(t) \, dt = \int_5^{10} \left( \frac{100}{t + 10} + 10 \right) \, dt = \int_5^{10} \frac{1}{t + 10} \, dt + \int_5^{10} 10 \, dt \]

\[ = 100 \ln(t + 10) \bigg|_5^{10} + 10t \bigg|_5^{10} \]

\[ = 100 \ln 20 - 100 \ln 15 + 10(10 - 5) \]

\[ = 100 \ln 20 - 100 \ln 15 + 50 = 79 \]

The total production from the end of the fifth year to the end of the tenth year is approximately 79 thousand barrels.

81. To find the useful life, set $R'(t) = C'(t)$ and solve for $t$:

\[ 9e^{-0.3t} = 2 \]

\[ e^{-0.3t} = \frac{2}{9} \]

\[-0.3t = \ln \left( \frac{2}{9} \right) \]

\[-0.3t = -1.5 \]

\[ t = 5 \text{ years} \]

\[ \int_0^5 [R'(t) - C'(t)] \, dt = \int_0^5 [9e^{-0.3t} - 2] \, dt \]

\[ = 9 \int_0^5 e^{-0.3t} \, dt - \int_0^5 2 \, dt = \frac{9}{-0.3}e^{-0.3t}\bigg|_0^5 - 2t\bigg|_0^5 \]

\[ = -30e^{-1.5} + 30 - 10 \]

\[ = 20 - 30e^{-1.5} \approx 13.306 \]

The total profit over the useful life of the game is approximately $13,306.
83. For 1935: \( f(x) = x^{2.4} \)

\[
\text{Gini Index} = 2 \int_0^1 [x - f(x)] \, dx = 2 \int_0^1 (x - x^{2.4}) \, dx \\
= 2 \left( \frac{x^2}{2} - \frac{x^{3.4}}{3.4} \right) \bigg|_0^1 \\
= 2 \left( \frac{1}{2} - \frac{1}{3.4} \right) = 0.412
\]

For 1947: \( g(x) = x^{1.6} \)

\[
\text{Gini Index} = 2 \int_0^1 [x - g(x)] \, dx = 2 \int_0^1 (x - x^{1.6}) \, dx \\
= 2 \left( \frac{x^2}{2} - \frac{x^{2.6}}{2.6} \right) \bigg|_0^1 \\
= 2 \left( \frac{1}{2} - \frac{1}{2.6} \right) = 0.231
\]

Interpretation: Income was more equally distributed in 1947.

85. For 1963: \( f(x) = x^{10} \)

\[
\text{Gini Index} = 2 \int_0^1 [x - f(x)] \, dx = 2 \int_0^1 (x - x^{10}) \, dx \\
= 2 \left( \frac{x^2}{2} - \frac{x^{11}}{11} \right) \bigg|_0^1 \\
= 2 \left( \frac{1}{2} - \frac{1}{11} \right) = 0.818
\]

For 1983: \( g(x) = x^{12} \)

\[
\text{Gini Index} = 2 \int_0^1 [x - g(x)] \, dx = 2 \int_0^1 (x - x^{12}) \, dx \\
= 2 \left( \frac{x^2}{2} - \frac{x^{13}}{13} \right) \bigg|_0^1 \\
= 2 \left( \frac{1}{2} - \frac{1}{13} \right) = 0.846
\]

Interpretation: Total assets were less equally distributed in 1983.

87. (A) Lorenz curve:

\[
f(x) = 0.3125x^2 + 0.7175x - 0.015.
\]

(B) Gini Index:

\[
2 \int_0^1 [x - f(x)] \, dx = 0.104
\]
89. \( W(t) = \int_0^{10} W'(t) \, dt = \int_0^{10} 0.3e^{0.1t} \, dt = 0.3 \int_0^{10} e^{0.1t} \, dt \)

\[ = 0.3 \left. \frac{e^{0.1t}}{0.1} \right|_0^{10} = 3e^{0.1t} \bigg|_0^{10} = 3e - 3 \approx 5.15 \]

Total weight gain during the first 10 hours is approximately 5.15 grams.

91. \( V = \int_2^4 \frac{15}{t} \, dt = 15 \int_2^4 \frac{1}{t} \, dt = 15 \ln t \bigg|_2^4 \)

\[ = 15 \ln 4 - 15 \ln 2 = 15 \ln \left( \frac{4}{2} \right) = 15 \ln 2 \approx 10 \]

Average number of words learned during the second 2 hours is 10.

**EXERCISE 7-2**

**Things to remember:**

1. **PROBABILITY DENSITY FUNCTION**
   
   A function \( f \) which satisfies the following three conditions:
   
   a. \( f(x) \geq 0 \) for all real \( x \).
   
   b. The area under the graph of \( f \) over the interval \( (-\infty, \infty) \) is exactly 1.
   
   c. If \([c, d]\) is a subinterval of \( (-\infty, \infty) \), then the probability that the outcome \( x \) of an experiment will be in the interval \([c, d]\), denoted Probability \( (c \leq x \leq d) \), is given by

   \[ \text{Probability} \ (c \leq x \leq d) = \int_c^d f(x) \, dx \]

   ![probability density function graph]

   \[ \int_c^d f(x) \, dx = \text{Probability} \ (c \leq x \leq d) \]

2. **TOTAL INCOME FOR A CONTINUOUS INCOME STREAM**

   If \( f(t) \) is the rate of flow of a continuous income stream, then the TOTAL INCOME produced during the time period from \( t = a \) to \( t = b \) is:

   \[ \text{Total income} = \int_a^b f(t) \, dt \]

   ![total income graph]