1. Let \( f(x, y) = x^3 + 6x^2 - y^2 - 4y - 4 \).
   Find the local extrema and saddle points of the function.

2. Let \( f(x, y) = x^3 y \).
   Find the absolute maximum and minimum values of \( f \)
on the square domain \( D \) where \( D = \{(x, y) \mid |x| \leq 1, |y| \leq 1\} \).

3. Use Lagrange multipliers to find the extreme values of \( f(x, y) = y^2 + x^2 + 2 \) subject to the constraint \( x^3 + y^3 = 1 \).
   Extra Credit: Graph the level curves \( f(x, y) = k \) that touch the constraint at
   the points where the extreme values occur.

4. Consider \( \iint_{-1}^{e^{-1}} \int_{\ln(1+y)}^{0} \sqrt{e^x - x} \, dx \, dy \).
   (a) Sketch the region of integration.
   (b) Evaluate the integral by reversing the order of integration.

5. Find the surface area of the part of the sphere \( x^2 + y^2 + z^2 = 4 \) that lies within the cylinder \( x^2 + y^2 = 1 \).
   Use \( A(S) = \iint_{D} \sqrt{1 + \left( \frac{\partial z}{\partial x} \right)^2 + \left( \frac{\partial z}{\partial y} \right)^2} \, dA \)
   \( D \)

6. The figure shows the solid region of integration for
Rewrite the integral as an equivalent iterated integral in the five other orders. DO NOT EVALUATE THEM.

7. A contour map is shown for the function \( f \) on the rectangle \( R = [0, 4] \times [0, 2] \). Use the midpoint rule with \( n = 2 \) (x direction) and \( m = 1 \) (y direction) to estimate the value of \( \iint f(x,y) \, dA \).
1. \( f(x, y) = x^3 + 6x^2 - y^2 - 4y - 4 \)
\( f_x = 3x^2 + 12x \quad f_y = -2y - 4 \)
\( f_{xx} = 6x + 12 \quad f_{yy} = -2 \)
\( f_{xy} = 0 \)
\( f_x = 0 \) and \( f_y = 0 \)
\( 3x(x+4) = 0 \) and \(-2(y+2) = 0 \)
\( x = 0, x = -4 \quad y = -2 \)
\( (0, -2) \quad (-4, -2) \)
D \((-4, -2) = -12 (-2) - 0 > 0 \)
and \( f_{xx} (-4, -2) < 0 \)
Local max. at \((-4, -2) \)

2. \( D(0, -2) = 12(-2) - 0 < 0 \)
Saddle pt. at \((0, -2) \)
\( f(0, -2) = 32 \quad f(0, 2) = 0 \)

3. \( f(x, y) = y^2 + x^2 + 2 \quad x^3 + y^3 - 1 = 0 \quad g(x, y) = 0 \)
\( \nabla f = \nabla g \)
\( \langle 2x, 2y \rangle = \lambda \langle 3x, 3y \rangle \)
\( 2x - \lambda 3x^2 = 0 \) and \( 2y - \lambda 3y^2 = 0 \)
\( x(2 - 3\lambda x) = 0 \quad y(2 - 3\lambda y) = 0 \)
\( x = 0 \) or \( x = \frac{2}{3\lambda} \)
\( y = 0 \) or \( y = \frac{2}{3\lambda} \)
\( (0, 1) \quad \frac{2}{3\lambda} \)
\( \left( \frac{2}{3\lambda} \right)^3 + \left( \frac{2}{3\lambda} \right) = 1 \)
\( 16 = 27\lambda^3 \)
\( x = \frac{1}{3\sqrt{2}} \quad y = \frac{2\sqrt{2}}{3} \)

4. \( \int_{0}^{1} \int_{0}^{x} e^{-x} \, dy \, dx \)
\( \ln(1+y) \)
\( e^{-1} \)
\( 1+y < e^x < e \)
\( y = e^{x-1} \)
\( \int_{0}^{1} (e^{-x}) \ln(e^{-x}) \, dx \)
\( 1 \)
\( \int_{0}^{1} (e^{-x}) \ln(e^{-x}) \, dx = \frac{1}{2} (e-1)^{-\frac{3}{2}} \)