Partial Proof of $\frac{0}{0}$ case: Suppose $f(a) = g(a) = 0$, $f'$ and $g'$ are continuous at $a$, and $g'(a) \neq 0$.

If it exists,

$$\lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$= \frac{f'(a)}{g'(a)} \quad \text{[since } f' \text{ and } g' \text{ are continuous at } a\text{]}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} \quad \text{[by definition of the derivative]}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} \quad \text{[since the quotient of limits is the limit of the quotient]}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} \quad \text{[by algebra]}$$

$$= \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)} \quad \text{[since } f(a) = 0 = g(a)\text{]}$$

Thus \(\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}\), which is the statement of L'Hopital's Rule.