Math 7 - 3.1/3.2 - Rates of Change and the Definition of the Derivative

Defns. The average rate of change of \( y = f(x) \) with respect to \( x \) on the interval \([a, a + h]\) is

\[
y_{av} = \frac{f(a+h) - f(a)}{h}
\]

and the instantaneous rate of change of \( f \) with respect to \( x \) at \( x = a \) is

\[
\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
\]

if this limit exists.

Defns. The derivative of \( f \) at an interior point \( a \) of its domain is the number

\[
f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}
\]

if this limit exists.

The derivative of \( f \) at a left endpoint \( a \) of its domain is the number

\[
f'(a) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}
\]

This is called the right-hand derivative of \( f \) at \( a \).

The derivative of \( f \) at a right endpoint \( a \) of its domain is the number

\[
f'(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}
\]

This is called the left-hand derivative of \( f \) at \( a \).

This defines a new function, \( f'(x) \), called the derivative of \( f \).

If \( f' \) exists at \( x = a \), then we say \( f \) is differentiable at \( a \).

If \( f' \) exists at all points of the domain of \( f \), then we say \( f \) is a differentiable function.

Two immediate applications:

\( f'(a) \) is the slope of the tangent line of the graph of \( y = f(x) \) at \( x = a \)

and \( f'(a) \) is the instantaneous rate of change of \( f(x) \) at \( x = a \)

Notation: Denote the derivative of \( y = f(x) \) by \( f'(x) \) "f prime of x"

and by \( y' \) "y prime"