ex: consider a cart moving as shown:

\[ V \]

\[ m_2 = 2m_1 \]

\[ \Rightarrow V_{cm} = \frac{P}{M} \]

\[ V_{an} = \left( \frac{1}{m_1 + m_2} \right) \left[ m_1 \left( \frac{dx_1}{dt} \right) + m_2 \left( \frac{dx_2}{dt} \right) \right] \Rightarrow \]

\[ \frac{dx_{cm}}{dt} = \frac{d}{dt} \left[ \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \right] \Rightarrow \]

\[ x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \]

General Result: \[ x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i} \]

ex: consider two objects \( m_2 = 2m_1 \) separated by 1m. Determine the CM location?

\[ \begin{array}{c}
\text{m}_1 \hspace{1cm} \text{X} \hspace{1cm} \text{m}_2 \\
0 \hspace{1cm} \Rightarrow \hspace{1cm} L
\end{array} \]

\[ x_{cm} = \frac{m_1(0) + m_2(L)}{m_1 + m_2} = \frac{2m_1 L}{3m_1} = \frac{2}{3} L \]

- For a uniformly distributed object the CM is @ the "center" of the object, and must lie in any axes of symmetry.
ex: Consider a planar L-shaped object as shown:

\[
X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}
\]

\[
= \frac{8m(0) + m(1.5d) + 2m(1.5d)}{5m} = 0.6d
\]

By symmetry: \(y_{CM} = 0.6d\)

Recall circular motion lab:

- Bob was regarded as being at its center.

Two particles: total momentum

\[
\vec{p} = m_1 \vec{V}_1 + m_2 \vec{V}_2 = MV_{CM}
\]

\[
\vec{V}_{CM} = \frac{\vec{p}}{M}, \quad \vec{F} = \frac{d\vec{p}}{dt} = M \frac{d\vec{V}_{CM}}{dt} = M\vec{a}_{CM}
\]

ex: Consider a projectile that explodes in mid-air:

Where does the heavier fragment land (they both land at the same time)

\[
F = (m_1 + m_2)g
\]

CM executes just a projectile motion

\[
R = \frac{v_0^2 \sin 2\theta}{g} = \frac{50^2}{10} \sin 60 = 216\ m
\]

CM is \(R - x_1 = 24\ m\) behind \(m_1 \Rightarrow m_2 = \frac{34}{2} = 17\ m\ behind\ CM\)

\[
\Rightarrow x_2 = 200\ m\]
ex: Consider a person on a raft \( L = 10 \text{ m} \), person has mass twice that of the raft: person walks across raft

- Draw a sketch of situation when half way.

Determine the displacement of the person related to the ground.

\[
\begin{align*}
\Delta x_{pg} &= \Delta x_{pr} + \Delta x_{eg} = L - \frac{2L}{3} = \frac{L}{3} = 3.3 \text{ m} \\
\Delta x_{pr} &= L \\
\Delta x_{eg} &= -\frac{2L}{3}
\end{align*}
\]

ex: Consider two masses on a massless beam:

(a) Where must a support be placed so that system balances?

A: Support should be at CM.

(b) Draw a FBD for the beam:

(c) Could support location be found from just considering \( \sum \text{net}=0 \) A: NO!

\[
\begin{align*}
\sum F &= 0 \\
\sum N &= 0 \\
\sum \tau &= 0 \\
N_1 &= r_1 N_1 \\
N_2 &= r_2 N_2 \\
m_1 gh_1 &= m_2 gh_2
\end{align*}
\]

A force's ability to rotate an object is characterized by 'torque':

\[
\tau = r \vec{F} \sin \theta = r \vec{F}_1
\]
Equilibrium Conditions:
Translation: \[ F_{net} = \sum F_i = 0 \]
\[ \dot{a}_{CM} = 0 \] \[ F_{x} = 0 \quad F_{y} = 0 \] \[ (2) \]
\[ \dot{r} = 0 \]
\[ \dot{\theta} = 0 \]
\[ \alpha = 0 \] \[ (1) \]

Ex: Consider a beam of mass \( m \) with a block on it:
(a) Draw a FBD for the beam.
(b) Taking the left end as a pivot, identify the clockwise & counterclockwise torques?
\( N_1 \Rightarrow \) doing nothing!
(c) Write down the equilibrium equations for the beam?
A: Translation: \[ \sum F_i = 0 \]
\[ (3) \quad N_1 + N_2 - N - mg = 0 \]
Rotation: \[ \sum \tau_i = 0 \]
\[ L \cdot N_2 - \frac{1}{2} \cdot mg \cdot \frac{L}{2} - \frac{X}{L} \cdot N = 0 \]
\[ N_2 = \frac{XN + \frac{mg}{2} \cdot \frac{L}{2}}{L} = \frac{X}{L} \cdot N + \frac{mg}{2} \]
Consider a car moving as shown:

\[ V \]

\[ m_2 = 2m_1 \]

Total Momentum:

\[ P = m_1 V_1 + m_2 V_2 \]

\[ = (m_1 + m_2) V \]

\[ \Rightarrow V_{CM} = \frac{P}{M} \]

\[ V_{CM} = \left( \frac{1}{m_1 + m_2} \right) \left[ m_1 \left( \frac{dx_1}{dt} \right) + m_2 \left( \frac{dx_2}{dt} \right) \right] \]

\[ \frac{d}{dt} \left[ \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \right] \]

\[ x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \]

General Result:

\[ x_{CM} = \frac{\sum m_i x_i}{\sum m_i} \]

\[ y_{CM} = \frac{\sum m_i y_i}{\sum m_i} \]

Ex: Consider two objects \( m_2 = 2m_1 \) separated by \( L \) m. Determine the CM location?

\[ L \quad \rightarrow \quad x \]

\[ x_{CM} = \frac{m_1 (0) + m_2 (L)}{m_1 + m_2} = \frac{2m_1 L}{3m_1} = \frac{2}{3} L = \frac{2}{3} \]

- For a uniformly distributed object, the CM is at the center of the object, and must lie in any axes of symmetry.
Consider a planar L-shaped object as shown:

\[ X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \]

\[ = \frac{8m(0) + m(0) + 2m(1.5d)}{5m} = 0.6d \]

By symmetry: \( y_{CM} = 0.6d \)

- Recall circular motion lab:

- Bob was regarded as being at its center.

Two particles: total momentum

\[ \vec{p} = m_1 \vec{v_1} + m_2 \vec{v_2} = \vec{M} \vec{V_{CM}} \]

\[ \vec{V_{CM}} = \frac{\vec{p}}{M}, \quad \vec{F}_{ext} = \frac{d\vec{p}}{dt} = M \frac{d\vec{V_{CM}}}{dt} = M \vec{a}_{CM} \]

Consider a projectile that explodes in mid-air:

- Where does the heavier fragment land (they both land at the same time)

\[ A: \quad F_{ext} = (m_1 + m_2) \vec{g} \]

\[ \vec{F}_{ext} = \vec{0} \]

\[ \vec{F}_{cm} = \vec{F}_{ext} \]

\[ \begin{align*}
\text{CM executes just a projectile motion} \\
R &= \frac{v_0^2 \sin 2\theta}{g} = \frac{50^2 \sin 60}{10} = 216m \\
R &= X_{CM} = 250m \\
\Rightarrow \text{CM travels} \quad \text{cm} \quad \text{m} \\
\end{align*} \]

\[ \text{CM is} \quad R - X_{1} = 24 \text{m behind} \quad m_1 \Rightarrow \quad m_2 = \frac{34}{2} = 17 \text{m behind CM} \]

\[ \Rightarrow x_2 = 200 \text{m} \]
ex: Consider a person on a raft. \( L = 10 \text{ m} \), person has mass twice that of the raft; person walks across raft.

- Draw a sketch of situation when half way.
- Determine the displacement of the person related to the ground.

\[
\Delta x_{\text{pf}} = 0 \quad \Delta x_{\text{py}} = L - \frac{2L}{3} = \frac{L}{3} = 3.3 \text{ m}
\]

ex: Consider two masses on a massless beam.

\( m_2 = 2m_1 \)

(a) Where must a support be placed so that system balances?
- A: Support should be at CM.

(b) Draw a FBD for the beam.

(c) Could support location be found from just considering \( F_{\text{net}} = 0 \)?
- A: No!

\[ \text{CM: } r_1 N_1 = r_2 N_2 \]

\[ m_1 gh_1 = m_2 gh_2 \]

\[ \tau = rF \sin \theta = rF \]

\[ \text{pivot} \]
Equilibrium Conditions:

Translation: \[ \sum \vec{F} = \vec{0} \]

Rotation: \( \sum \tau = 0 \) \( \Rightarrow \) (1)

\[ \alpha = 0 \]

(2)

Example: Consider a beam of mass \( m \) with a block on it:

(a) Draw a FBD for the beam.

(b) Taking the left end as a pivot, identify the clockwise and counterclockwise torques.

\( N_1 \rightarrow \) doing nothing!

(c) Write down the equilibrium equations for the beam.

A: Translation: \( \sum \vec{F} = \vec{0} \)

Y: \( N_1 + N_2 - N - mg = 0 \)

Rotation: \( \sum \tau = 0 \)

\[ L \cdot N_2 - \left( \frac{1}{2} \right) mg \cdot \frac{L}{2} = 0 \]

\[ N_2 = \frac{\frac{mg}{2}}{L} \cdot \frac{L}{N} = \frac{mN + \frac{mg}{2}}{L} \]
ex: Consider a beam of mass \( m = 10 \text{ kg} \) supporting a block of mass \( M = 4 \text{ kg} \). Draw a FBD for the beam.

\[
\begin{align*}
\text{FBD:} & \\
& \text{F}_H = \text{horizontal force of the hinge} \\
& \text{F}_V = \text{vertical force} \\
& \text{F}_T = \text{tension in the cord} \\
\end{align*}
\]

\( \theta = 30^\circ \), \( x = \frac{2}{3} L \)

\( K = \frac{L}{2} \)

\( T_2 = Mg \)

(b) Write down set of expressions that say the beam is at equilibrium.

A: \( \sum F_x = 0 \) \( \text{Translational equilibrium (} \vec{a}_{CM} = \vec{0} \) \)

\( \sum F_x = 0 \)

\( T_2 = Mg \)

\( T_2 \cos \theta = 0 \)

\( T_2 \sin \theta = mg \)

\( \sum z_i = 0 \) \( \text{Rotational eq. (} x = 0 \) \)

\[
(T_1 \sin \theta) \cdot L - mg \cdot \left( \frac{L}{2} \right) = T_2 x = 0
\]

L's cancel out!

& \( x = \frac{2}{3} L \)

\( \text{(c) Determine the tension in the suspending cord.} \)

\( T_1 \sin \theta = \frac{mg}{2} + Mg \cdot \left( \frac{x}{L} \right) \Rightarrow T_1 = \frac{\left( \frac{M}{2} + \frac{M(2)}{3} \right) g}{\sin \theta} = 6.30 \text{ N} \)

ex:

A block \( m \) is placed on a beam. The beam is hinged at the left end and supported on a wall. The beam is at equilibrium. (It's at equilibrium)
Ex: Consider a stick with a roller on it leaning against a wall. Draw a FBD for the rod.