ex: Consider two carts repelling from each other as shown:

(a) Draw a FBD on each:

\[ m_2 = 2m_1 \]

(b) Relate the acceleration of each cart and their velocity change:

\[ \vec{F}_1 = \vec{F}_{21} = m_1 a_1 \]
\[ \text{3rd Law: } \vec{F}_{21} = -\vec{F}_{12} \Rightarrow m_1 a_1 = -2m_1 a_2 \]

\[ \vec{F}_2 = \vec{F}_{12} = 2m_1 a_2 \]
\[ \Rightarrow a_1 = -2a_2 \]

\[ \Delta \vec{V}_1 = a_1 \Delta t \]
\[ m_2 \Delta \vec{V}_2 = -m_1 \Delta \vec{V}_1 \]
\[ \Rightarrow \Delta \vec{V}_1 = -\frac{m_2}{m_1} \Delta \vec{V}_2 = -2 \Delta \vec{V}_2 \]

\[ m_1 \Delta \vec{V}_1 + m_2 \Delta \vec{V}_2 = \vec{0} \]

Define the "momentum" \[ p = m \vec{v} \]

p is a vector.

For a system of particles, total momentum \[ \vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots \]

- If no net external force acts on a system, its total momentum is constant.
ex: Consider a cart colliding and sticking with another as shown:

\[ \vec{v}_2 \quad \vec{v}_1 = 0 \]

\[ m_2 = 2m_1 \]

Formal way: \( P_{\text{before}} = P_{\text{after}} \) (Mom. Conservation)

\[
P_{\text{before}} = m_2 v_2 + 0 \quad \Rightarrow \quad m_2 v_2 = (m_1 + m_2) v' \quad \Rightarrow \quad v' = \frac{m_2 v_2}{m_1 + m_2} = \frac{2}{3} v_2
\]

Consider collision on more details: \( v_2 = 12 \text{ m/s} \)

\[
\begin{array}{c|cc}
v_2 & v_1 & \text{Acceleration}\ 
\hline
12 & 0 & \text{acceleration of 1 is twice as big as 2} \ 
11 & 2 & \text{so change in velocity is also the same in magnitude.}
10 & 4 & \text{speeding up}
9 & 6 & \text{Collision stops! (sticking)}
8 & 8 & \text{Elastic collision}
7 & 10 & 
6 & 12 & 
5 & 14 & 
4 & 16 & 
\end{array}
\]

- Momentum is naturally related to force.

2nd law: \( \vec{F} = m \vec{a} = -m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} \Rightarrow \vec{F} = \frac{d\vec{p}}{dt} \)
Chapter 9: Momentum

If the momentum changes by $\Delta p$ in $\Delta t$, then:

$$\Delta p = F \cdot \Delta t$$

For a constant force:

For constant force:

For general forces:

$$F = \frac{dp}{dt} \Rightarrow \int dp = \int F \cdot dt \Rightarrow \Delta p = I$$

"Impulse": $I = \int F \cdot dt = \text{Area under } F \text{ vs. } t$

2nd-law: $\Delta p = I$

ex: Consider a ball dropped onto a table from a height of 1.5 m above it; assume ball rises to same height.

$N_{max}$ if $m$ of ball's 200 g:

$N = \frac{1}{2} N_{max} \cdot t$

First find change in momentum:

$V_1 = \sqrt{2gh} \Rightarrow \Delta p = m \Delta V = 2m \sqrt{2gh} = 2 \times 0.2 \sqrt{50} = 2.2 \text{ kg m/s} < N_{max}$
Impulse from weight: $-\mathbf{w} \cdot \Delta t$

Final: $I = \Sigma N + \Sigma f = \Delta P \Rightarrow \Sigma N = \Delta P - I_g$

$\frac{1}{2} N_{max} \cdot \Delta t = \Delta P \Rightarrow N_{max} = 2 \frac{\Delta P}{\Delta t} = \frac{2(2.2)}{0.64} = 44.0 \text{ N}$

Note: $w = 2 \text{ N}$

Ex: Consider a projectile shot into the air and exploding into two pieces:

1. Consider the 2nd law for each object:
   - $\mathbf{F}_{\text{net}} = \mathbf{W} + \mathbf{F}_{21} = m_1 \mathbf{a}_1 = \frac{\Delta \mathbf{P}}{\Delta t}$
   - $\mathbf{F}_{21} = \mathbf{W}_2 + \mathbf{F}_{12} = m_2 \mathbf{a}_2 = \frac{\Delta \mathbf{P}}{\Delta t}$

2. $\mathbf{W} = \frac{\Delta \mathbf{P}}{\Delta t}$

Net external force on system, total momentum

Ex: Consider a constant force that pushes two carts.

Cart A starts from rest, cart B is initially moving forward. Compare the impulse delivered to each cart ($m_A = m_B$)

$\mathbf{F}_{\text{net}} = (m_1 + m_2) \left( \frac{m_1 \mathbf{a}_1^2 + m_2 \mathbf{a}_2^2}{m_1 + m_2} \right) \mathbf{a}_{cm}$

$\mathbf{F}_{\text{net}} = M \mathbf{a}_{cm}$

Since $F$ is the same the accelerations are the same $\Rightarrow (V_B)^{av} > (V_A)^{av}$

$\Delta t_A > \Delta t_B \Rightarrow \int_A = \mathbf{F} \Delta t_A > \mathbf{F} \Delta t_B = I_B$
ex: Consider a bullet of mass \( m = 10 \text{g} \) fired through a cart of mass \( M = 200 \text{g} \). The bullet has initial speed \( v_0 = 500 \text{m/s} \) and exits with \( v = 20\% \) of \( v_0 \).

(a) What's car's velocity after the impact?

A: Momentum of the cart and bullet is conserved.

\[
p = p', \quad \Rightarrow \quad mV_0 = mV_1 + MV_2
\]

\[
p = mV_0 + 0, \quad p' = mV_1 + MV_2 \quad \Rightarrow \quad V_2 = \frac{m(v_0 - v)}{M} = 20 \text{m/s}
\]

(b) Suppose the collision lasts for 0.3 ms, determine the impulse delivered to the cart and the average collision force acting on it.

- Impulse \( I_c = \Delta p_2 = M(v_2 - 0) = (0.2)(20) = 4.0 \text{ Ns} \)

- Average collision force \( F_{av} = \frac{\Delta p}{\Delta t} = \frac{I_c}{\Delta t} = \frac{4.0}{0.3 \times 10^{-3}} = 1.3 \times 10^4 \text{ N} \)

ex: Consider two cars colliding at an intersection as shown:

Cars go off together at 15 m/s at 37°. \( m_1 = 1000 \text{kg} \) and \( m_2 = 2000 \text{kg} \)

(a) Determine the velocity of each car before the impact?

A: Momentum's conserved: \( p = p' \)

\[
x: \quad p_2 = p' \cos 37° \quad \Rightarrow \quad p_2 = p_x
\]

\[
y: \quad p_1 = p' \sin 37° \quad \Rightarrow \quad p_1 = p_y
\]

\[
p_2 = m_2 V_2 = (m_1 + m_2) V' \cos 37°
\]

\[
V_2 = \frac{(m_1 + m_2)}{m_2} V' \cos 37° \quad \Rightarrow \quad V_2 = \frac{3}{2} (15) \cos 37° = 18 \text{ m/s}
\]

\[
V_1 = \frac{3}{4} (15) \sin 37° = 8.7 \text{ m/s}
\]
(b) Suppose $\mu = 0.8$, how far do the cars slide after the collision?

\[ F = \text{M} \cdot a = \mu \text{mg} \Rightarrow a = \mu \text{g} = 8 \text{ m/s}^2 \]

\[ V = V_0 + 2a \Delta x \Rightarrow 0 = V_0^2 - 2\mu \text{g} \cdot D \]

\[ \Rightarrow D = \frac{V_0^2}{2\mu \text{g}} = \frac{15^2}{2(8)} = 14 \text{ m} \]

(c) Is the momentum of the cars conserved after the collision?

\[ \vec{p}_k = \vec{0} \]

\[ \vec{F}_{\text{ext}} \neq 0 \]

Recall exploding projectile example on page 44.

If $\vec{F}_{\text{net}} = 0$ then $\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{constant}$

ex: A ball thrown at a wall and bounces off with the same speed. Is its momentum conserved?

\[ \vec{p} \]

\[ \vec{p}' \]

\[ \vec{A}: \Delta \vec{p} \neq 0 \Rightarrow \text{Impulsive force from wall present!} \]