Chapter 14: "Oscillations"

Consider a block attached to a spring on a frictionless surface.

- Draw an FBD for the block at location shown:

\[ F_S = ma \]
\[ N = mg \]

\[ F_S = kx \Rightarrow -kx = ma \]

Recall \[ a = \frac{dv}{dt} \quad v = \frac{dx}{dt} \]

Energy:
\[ E = \frac{1}{2} m v^2 + \frac{1}{2} kx^2 = \text{Const.} \]
\[ \frac{d}{dt} \left( \frac{1}{2} m v^2 + \frac{1}{2} kx^2 \right) = 0 \]

Let's assume that the max spring stretch is \( x = A \)

\[ E = \frac{1}{2} kA^2 \]

\[ \frac{1}{2} m v^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \]

\[ v = \pm \sqrt{\frac{k}{m} (A^2 - x^2)} \Rightarrow \int \frac{dx}{\sqrt{\frac{k}{m} (A^2 - x^2)}} = \int dt \Rightarrow \sin^{-1} \left( \frac{x}{A} \right) = \sqrt{\frac{k}{m}} t + C \]

\[ x = A \sin(\sqrt{\frac{k}{m}} t + C) \]
\[
\frac{1}{n} \sum_{i=1}^{n} X_i \to \mu \quad \text{as} \quad n \to \infty
\]
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Recall from last time: \( x = A \sin(\omega t + \phi) \)
if \( C = \frac{\pi}{2} + \phi \Rightarrow x = A \cos(\omega t + \phi) \)

- **Motion:** period of time for the motion to repeat.
- **Motion repeats argument of \( \cos() \)**
- **Extends by \( 2\pi \).**

\[ \omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \]

\( \omega = \sqrt{\frac{k}{m}} \)

**Note:** The period in independent of the amplitude—this will occur when the restoring force \( F \propto x \). Such motion is called "simple harmonic motion".

- **Recall:** \( ma = -kx \Rightarrow a + \frac{k}{m}x = 0 \Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0 \) "Differential Equation" of motion.

**Ex:** Consider a block on a frictionless surface as shown:

\[ T = 2\pi \sqrt{\frac{m}{k_{\text{eff}}}} = 2\pi \sqrt{\frac{m}{k_1 + k_2}} \]

\( \Box: -F_1 - F_2 = ma \)
- \( F_1 = k_1 x \), \( F_2 = k_2 x \)
- \( -k_1 x - k_2 x = ma \)
- \( (k_1 + k_2) x = ma \)
- \( k_{\text{eff}} x = ma \)
Ex: A certain spring stretches by 10 cm when a 2.5 N is applied to it. We attach a 500 g block to the spring pulled 20 cm and released. (a) Determine the spring constant:

\[ F = k d \]

\[ k = \frac{F}{d} = \frac{2.5}{0.1} = 25 \frac{N}{m} \]

(b) What force acts on block just when it's released?

\[ F_s = kx = 25(0.2) = 5 \text{ N} \]

(c) What's the period of oscillation?

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.5}{25}} = 0.89 \text{ s} \]

(d) Determine the amplitude, max speed and max acceleration of the block.

- Max position is at beginning: \( x_{\text{max}} = A = 20 \text{ cm} \) (max. PE)

- \( V_{\text{max}} \) occurs when \( a = 0 \) \( \Rightarrow \) \( x = 0 \)

\[ \frac{1}{2} k A^2 = \frac{1}{2} m v_0^2 \]

\[ v_0 = \sqrt{\frac{k}{m}} \cdot A = W/A = 1.4 \text{ m/s} \]

- \( a = \frac{F_x}{m} = \frac{kx}{m} \text{ max } a = \frac{kx_{\text{max}}}{m} = \frac{25(0.2)}{0.5} = 10 \text{ m/s}^2 \)

Also

\[ \frac{dx}{dt} = -W/A \sin(wt + \phi) \]

\[ \frac{d^2x}{dt^2} = -W/A \cos(wt + \phi) \]

(x = 0)

(e) What's the velocity and accel. when \( x = +10 \text{ cm} \), moving toward.

\[ V: \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \text{ } \Rightarrow \text{ } V = \frac{k}{m} (A^2 - x^2) \text{ } \Rightarrow \text{ } V = \pm W \sqrt{A^2 - x^2} = -1.2 \text{ m/s} \]

\[ a = -\frac{kx}{m} = -25(0.1) = -5 \text{ m/s}^2 \]
(a) Determine the position as a fraction of time.

\[ x = A \cos(\omega t + \phi) \quad x(0) = A \Rightarrow A \cos \phi = A \]

\[ v(0) = 0 \quad \cos \phi = 1 \Rightarrow \phi = 0 \]

\[ W = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{0.5}} = 7.1 \text{ rad/s} \]

\[ x = 0.2 \cos(7.1t) \]

(b) How long does it take for the block to reach \( x = 10 \text{ cm} \) after released.

\[ 10 \text{ cm} = \frac{A}{2} \Rightarrow x = \frac{A}{2} = A \cos(\omega t) \Rightarrow \cos(\omega t) = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{3} \Rightarrow \]

\[ t = \frac{\pi}{3W} = \frac{1}{6} \left( \frac{2\pi}{W} \right) = \frac{T}{6} \]

**Example:** Consider a block hitting a spring. Determine the average force the spring exerts on the block while bringing it to rest.

\[ F_{AV} = \frac{\Delta P}{\Delta t}, \quad \Delta P = 0 - m v_0 \]

\[ K = 20 \text{ N/m} \]

\[ v_0 = 5 \text{ m/s} \]

\[ m = 100 \text{ g} \]

\[ \Delta t = \frac{1}{v_0} T = \frac{\pi}{2} \sqrt{\frac{m}{k}} \]

\[ F_{AV} = -\frac{m v_0^2}{\pi} \sqrt{\frac{k}{m}} = -\frac{9}{\pi} v_0 \sqrt{\frac{k}{m}} = -2.5 \text{ N} \]

**Pendulum:**

\[ I = -mgL \sin \theta \quad \tau = I \alpha \]

\[ I = ml^2, \quad \alpha = \frac{d^2 \theta}{dt^2} \]

\[ ml \frac{d^2 \theta}{dt^2} = -mg \sin \theta \]

\[ \sin \theta = 0 \quad \text{small angle} \]

\[ \frac{d^2 \theta}{dt^2} + \frac{g}{L} \theta = 0 \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}} \]