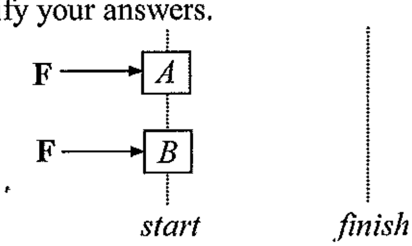
**1.** Provide short answers to the following questions (a) through (f). To receive full credit you must fully *justify* your answers. Answers with little explanation will receive little credit.

a) Identical constant forces push two identical blocks continuously from a starting line to a finish line. Assume block $A$ is initially at rest and block $B$ is initially moving to the right. Compare the impulse delivered to the blocks, and the change in kinetic energy of each block. Justify your answers.

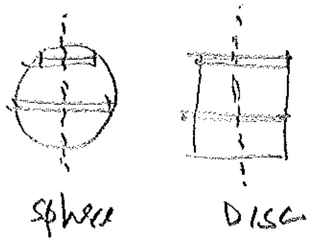



Since block B is initially moving it takes less time to cross the finish $\Delta t_B < \Delta t_A$.

But $J_A = F\Delta t_A$ and $J_B = F\Delta t_B \Rightarrow J_A > J_B$

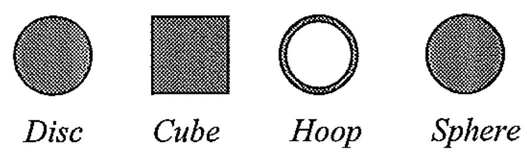
the distance covered by each block is the same and thus so is the work $W = F.d$. Since $W = \Delta K$, $\Delta K_A = \Delta K_B$

b) Consider the solids shown in the figure. All have equal mass, height, and width. Rank the respective moment of inertias about an axis through their center of mass perpendicular to the plane of the page.



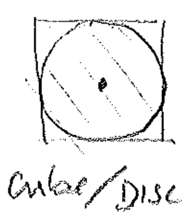

the sphere generally has $r < R \Rightarrow$

$I_S < I_D$



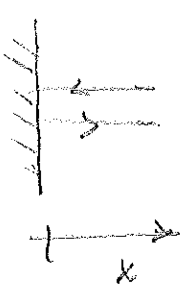

the hoop has all its mass at $r = R$

$\Rightarrow I_H > I_C > I_D > I_S$



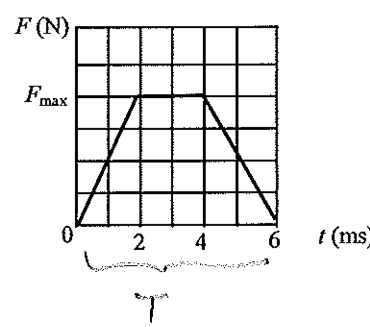

the cube has portions for which $r > R$, hence $I_C > I_D$

c) The graph below shows the approximate variation of force versus time for a 58g tennis ball bouncing elastically from a wall. If the ball strikes the wall perpendicularly with speed 32m/s determine the value of $F_{max}$ shown on the graph.




the impulse delivered to the ball from the wall is:

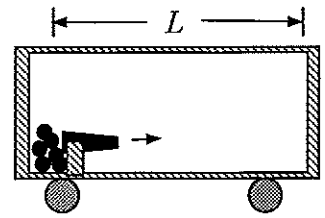
$J = mv - (-mv) = 2mv$




from the graph $J = \int F dt = F_m \left(\frac{1}{2}\right)\left(T + \frac{T}{3}\right) = \frac{2F_m T}{3}$

$\Rightarrow F_m = \frac{3mv}{T} = 930N$

d) A cannon is inside a railroad car as shown. The cannon fires balls of mass $m$ at speed $v_r$ relative to its muzzle toward the opposite wall. The balls come to rest upon hitting the wall and travel a distance $L$ relative to car. The total combined mass of the system is $M$. Describe as completely as possible the motion of the car when the first ball is fired. How far does the car move after $N$ balls have been fired?



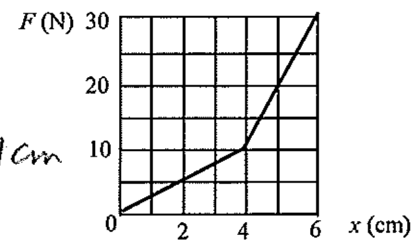

When a ball is fired the car will recoil to the left. When the ball hits the wall the system will come to rest.

Let v be the recoil speed relative to the ground, then

$(M-m)v = m(v_r - v) \Rightarrow v = \frac{mv_r}{M} \Rightarrow$ the car moves a distance

$d = \left(\frac{mv_r}{M}\right)\left(\frac{L}{v_r}\right) = \frac{mL}{M}$. After N balls are shot $D = Nd = \frac{NmL}{M}$

e) The graph below depicts the force required to stretch a spring from its relaxed ($x = 0$) position. Does the spring obey Hooke's Law? If so what is the spring constant? How may the graph be used to determine the potential energy stored in the spring? What is the spring potential energy when it is stretched 6cm from its relaxed length?




The spring does not obey Hooke's law for extensions $x > 4$cm. However, for $x < 4$cm the law is satisfied.

The change in PE $\Delta U = W^{ext}$ where $W^{ext}$ is the work required to stretch the spring. Taking $U = 0$ when $x = 0$

$U = W^{ext} = \int F\,dx = \frac{1}{2}(10)(.04) + \frac{1}{2}(10+30)(.02) = 0.6$ J

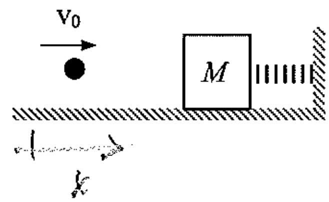
f) Consider a track consisting of two separate segments: one rough ($R$) and the other frictionless ($S$) as shown. Suppose a ball with rotational inertia $I = 0.4mr^2$ starts from rest at point $P$ and rolls down the rough portion without slipping. How does the maximum height the ball attains on the other side compare with $h$?


The ball will acquire a rotational KE in addition to its translational KE when rolling on the rough section with $K_r/K_T = 0.4$. Since no friction exists on the smooth section the ball will continue to spin - it therefore reaches a lower height.

$mgh = K_r + K_T = 1.4K_T$, $mgh' = K_T \Rightarrow h' = \frac{5}{7}h$

**2.** A projectile of mass $m = 50\text{g}$ traveling at $v_0 = 20\text{m/s}$ hits a block of mass $M = 450\text{g}$. The block rests on a frictionless horizontal surface and is attached to a spring of force constant $k = 2000\text{N/m}$. The projectile ricochets backward off the block with speed $v' = 0.6v_0$. The collision lasts for 4ms.

a) What impulse is delivered to the block and what is the average force for the collision?




the impulse delivered to the projectile

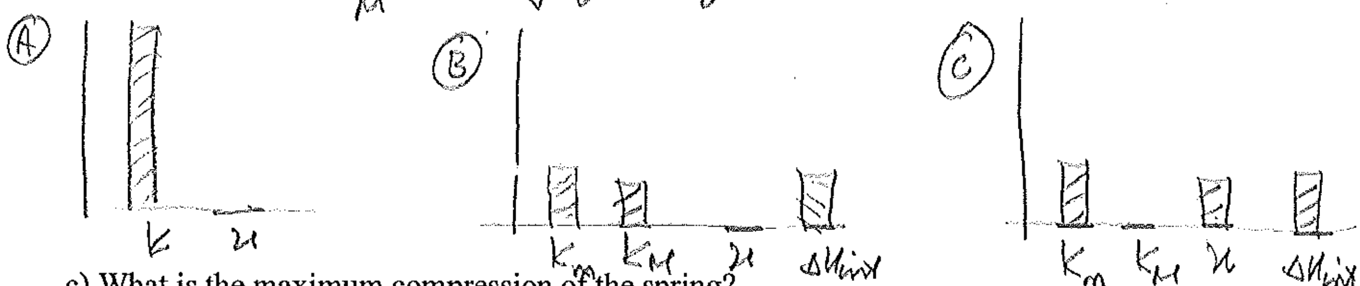
is $J_p = -0.6mv_0 - mv_0 = -1.6mv_0$

For the block, $J_B = -J_p = 1.6mv_0 = 1.6\ \text{N}\cdot\text{s}$, $F_{av} = \frac{\Delta p}{\Delta t} = \underline{400\text{N}}$

b) Is the collision elastic? Justify your answer. Draw energy bar charts corresponding to: *A* just before the collision, *B* just after the collision, and *C* at the moment the spring is maximally compressed.

the relative velocity of the blocks just after the collision

$v_r' = \left[1.6\frac{m}{M} + 0.6\right]v_0 \neq v_0 \Rightarrow$ the collision is not elastic




c) What is the maximum compression of the spring?

the speed of the block just after the collision

$v = \frac{1.6mv_0}{M}$. Energy conservation $\Rightarrow \frac{1}{2}M\left(\frac{1.6mv_0}{M}\right)^2 = \frac{1}{2}kx^2 \Rightarrow$

$x = \frac{1.6mv_0}{\sqrt{kM}} = \underline{0.053\text{m}}$

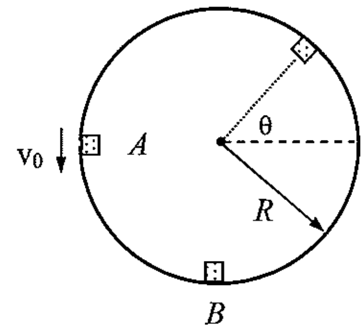
d) Suppose the surface has a "small" coefficient of friction $\mu_k = 0.1$. Estimate the total distance the block traverses from the moment it is struck to when it comes finally to rest. Why is this only an estimate?

the total energy dissipated in friction is equal to the initial energy of the block: $fD = \frac{1}{2}Mv^2$

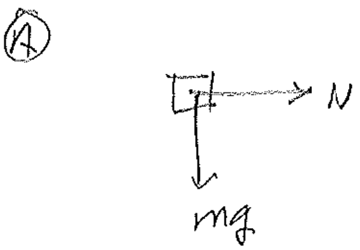
$\Rightarrow D = \frac{v^2}{2g\mu_k} = 6.3\text{m}$. the result is not exact because friction will cause some deformation of the spring when the block comes to rest.

**3.** Consider an ice cube of mass $m = 5$g sliding on a frictionless vertical circular track of radius $R = 50$cm. The cube has speed $v_0 = 2$m/s at the position $A$ shown in the figure.

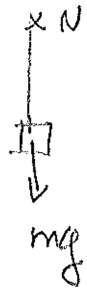
a) Draw free-body diagrams for the cube when it is in position $A$ and bottom of the track at point $B$. What are the radial and tangential components of the cube's acceleration at point $A$?




A




B




$a_t = g, \quad a_r = \frac{v_0^2}{R} = 8\,m/s^2$

b) Determine the speed of the cube when it reaches point $B$ at the bottom of the track and the normal force the track exerts on the cube at points $A$ and $B$ in the figure.

$\frac{1}{2}mv_0^2 + mgR = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{v_0^2 + 2gR}$

$= 3.7\,m/s$

A $N = ma_r$

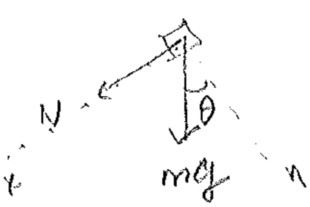
$= 0.04N$

B $N - mg = \frac{mv^2}{R}$

$N = m\left(g + \frac{v^2}{R}\right) = 0.19N$

c) Because the cube is given an initial velocity while in the horizontal position $A$ it will rise above its initial height on the other side. Determine the angle $\theta$ at which the cube loses contact with the track on its upward ascent. For what minimum value of $v_0$ will the cube remain in contact with the track?

$\frac{1}{2}mv_0^2 = mgR\sin\theta + \frac{1}{2}mv^2$




$\Rightarrow v_0^2 = 3gR\sin\theta$

$\sin\theta = \frac{v_0^2}{3gR} \Rightarrow \theta = 16^\circ$

$N + mg\sin\theta = \frac{mv^2}{R}$

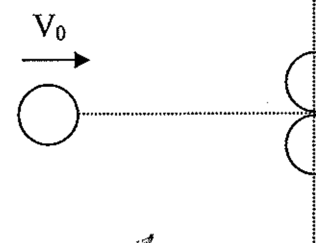
at top $\frac{v^2}{R} = g \Rightarrow$

$N = 0 \Rightarrow g\sin\theta = \frac{v^2}{R}$

$v_0^2 = 3gR \Rightarrow v_0 > \sqrt{3gR} = 3.9\,m/s$

**4.** Consider the collision between three identical billiard balls shown in the figure. One ball is incident with speed $V_0$ upon two other stationary balls along the perpendicular bisector to the line joining their centers.

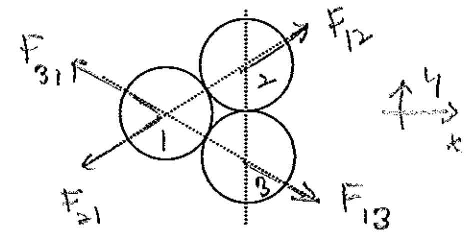
a) Describe the motion of the center of mass before and after the collision. How does the collision appear as viewed from an observer moving with a velocity equal to that of the center of mass?




the center of mass travels with a constant velocity through out the collision

$V_{cm} = \dfrac{mV_0}{3m} = \dfrac{V_0}{3}$  $\xrightarrow{2V_0/3}$  $\xleftarrow{V_0/3}$

b) Consider the direction of the collision contact forces. Assume these forces act along the line connecting the centers of the respective balls. On the diagram sketch the contact forces acting on each ball and from this determine the direction of the impulse delivered to each ball. What directions do the two initially stationary balls travel after the collision?




the impulse to balls 2, 3 are at $30^0$ to the x-axis; these balls go off at $30^0$.
the net impulse to ball 1 is along the x-axis.

c) Assume the collision is elastic. Write down three relations reflecting the conservation laws that are valid during the collision. (*Caution*: the result for relative velocity reversal is only valid for two-particle one-dimensional collisions).

Momentum: (x) $mV_0 = mV_1 + mV_2\cos\theta + mV_3\cos\theta$

(y) $0 = mV_2\sin\theta - mV_3\sin\theta$

Energy: $\frac{1}{2}mV_0^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2 + \frac{1}{2}mV_3^2$  $\theta = 30^0$

d) Use your results to part (c) to determine the final velocities of the balls after collision. Express your results in terms of $V_0$.

(y) $\Rightarrow V_2 = V_3$, (x) $\Rightarrow V_0 = V_1 + 2V_2\cos\theta$

energy $\Rightarrow V_0^2 = V_1^2 + V_2^2 + V_3^2 \Rightarrow$

$V_1^2 + 2V_2^2 = V_1^2 + 4V_1V_2\cos\theta + 4V_2^2\cos^2\theta \Rightarrow V_2 = -\dfrac{2V_1\cos\theta}{2\cos^2\theta - 1}$

$\Rightarrow V_1 = -\dfrac{(2\cos^2\theta - 1)}{2\cos^2\theta + 1}V_0 = -\dfrac{V_0}{5} \Rightarrow V_2 = \dfrac{2\sqrt{3}}{5}V_0$