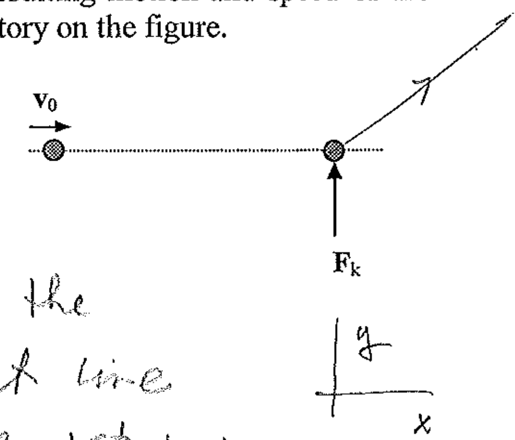
1. Provide short answers to the following questions.

a) When a stationary hockey puck on frictionless ice is given a sharp kick of force $\mathbf{F}_k$ it is found that the puck attains speed v_k. Suppose the puck is traveling at speed v_0 and it is given the *same kick* perpendicular to its velocity $\mathbf{v}_0$ as shown in the figure. Describe the resulting motion and speed of the puck. Explain your reasoning and provide a sketch of the resulting trajectory on the figure.



the x-component of velocity remains constant since $F_x = 0$. From the information given $\Delta V_y = V_k$. After the kick the puck travels in a straight line with velocity $\vec{v} = v_0\hat{i} + v_k\hat{j}$ by the 1st law.

b) A physics student is troubled by a paradox. When they start running they accelerate forward. However, to accelerate forward requires a net force. They believe this implies the force of friction on them must be greater than the frictional force they exert on the ground – in contradiction to Newton's 3rd Law. What is wrong with this reasoning?

the student is incorrectly adding forces that act on the floor on themself. For the purpose of assessing the motion of the student only the forces that act on the student must be considered.

3rd Law act. reaction pairs always act on differant bodis.

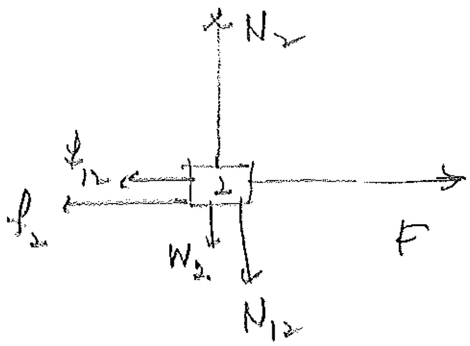
c) Someone in a car rounding a corner feels a force pulling them outward. Similarly, when one speeds up or slows down they feel forces pulling them backward and forward. What is the cause of these apparent forces? Do such forces have a reaction force? Your explanation should make reference to Newton's Laws.

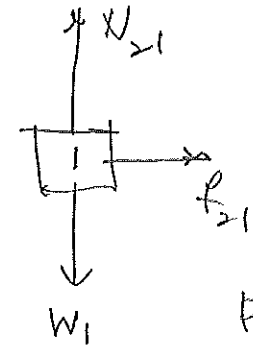
the cause of these forces is the tendency of a body to move with constant velocity (1st law)

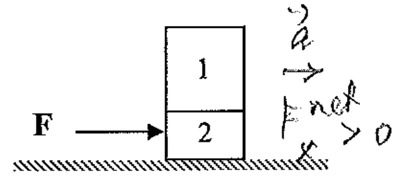
An accelerated frame of reference constantly accelerats away from bodis at rest within it – the tendency to continue at constant velocity is perceived as a force. Such forces do not have a 3rd law reaction force.

2. A person pushes two blocks, one stacked on the other and not slipping, along a rough horizontal surface with increasing speed as shown in the figure. Assume the larger block is twice as massive as the smaller block and that all surfaces have the same coefficients of friction with $\mu_k = \mu_s = \mu$.

a) Draw a free-body diagram for each block and rank the magnitudes of all the horizontal forces in the problem.







$F > f_2 + f_{12}$ since $a_2 > 0$ ⇒

$F > f_2 > f_{12} = f_{21}$

b) For each of the forces acting on block 2, identify the respective reaction force. For each reaction force state *in words* the type of force acting, the body that exerts this force, and the body this force is exerted upon.

W_2: Gravitational field attraction of block 2 on earth

N_2: Contact normal force of block 2 on surface

f_{12}: Contact friction force of block 2 on block 1.

N_{12}: Contact normal force of block 2 on block 1

f_2: Contact friction force of block 2 on surface.

c) If the two blocks are pushed with sufficient force block 1 will begin to slip off from block 2. Assume block 2 has mass m and that the static and kinetic coefficients of friction are equal and given by μ. Determine the minimum magnitude of the force **F** that will cause block 1 to slip off block 2.

$f_{21} \le \mu N_{21} = \mu(2mg)$

Max. acceleration: $(f_{21})_{max} = 2mg\mu = (2m)a_{max}$

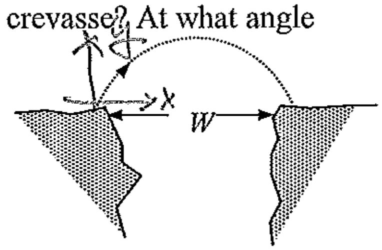
⇒ $a_{max} = \mu g$

Treat both blocks as a system:

$F - (3mg)\mu = (3m)a \Rightarrow F = 3m(a + \mu g) = 6mg\mu$

3. A person throws rocks over a crevasse of width $W = 20\text{m}$ as shown in the figure.

a) What minimum speed must the person throw a rock so that it just clears the crevasse? At what angle must such a rock be thrown?



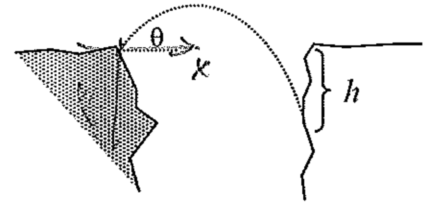
Minimum Speed ⇒ optimal angle

$\Rightarrow \theta = 45^\circ$

$$W = v_0 \cos\theta \left(\frac{2 v_0 \sin\theta}{g}\right) = \frac{v_0^2}{g}$$

$$\Rightarrow v_0 = \sqrt{gW} = 14\ \text{m/s}$$

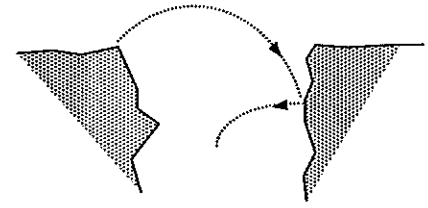
b) Suppose that the rocks are thrown with speed $v_0 = 14\text{m/s}$ at an angle $\theta = 30°$ above the horizontal. In such case the rocks do not make it to the other side. Determine the distance h below ground level that the rocks strike the opposite cliff.

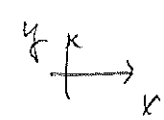


The motion is limited by the x-direction.

$$t = \frac{W}{v_0 \cos\theta} \qquad h = \frac{g t^2}{2} = \frac{g W^2}{2 v_0^2 \cos^2\theta} = 2.1\ \text{m}$$

c) What are the components of the rock's velocity when it strikes the side of the cliff in part (b)? Suppose upon striking the cliff the rock bounces off horizontally with speed $v' = 0.5v_0$. If the rock is in contact with the cliff for 50ms determine the average acceleration of the rock while bouncing off the cliff.





$$v_x = v_0 \cos\theta = 12\ \text{m/s}$$

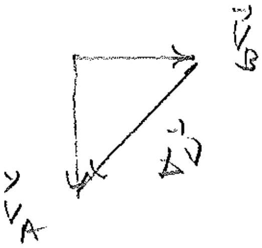
$$v_y = v_0 \sin\theta - g t = -9.5\ \text{m/s}$$

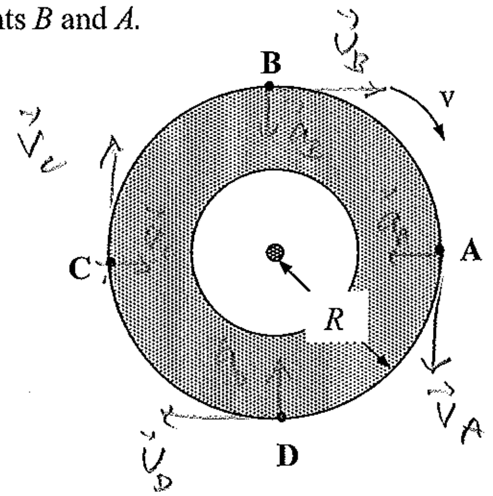
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\tau}\hat{i} + \frac{\Delta v_y}{\tau}\hat{j} \qquad \tau = 5 \cdot 10^{-2}\ \text{s}$$

$$= -\frac{\left(\frac{1}{2}v_0 + v_0\cos\theta\right)}{\tau}\hat{i} + \frac{(0 - v_y)}{\tau}\hat{j} = -380\hat{i} + 190\hat{j}\ \text{m/s}^2$$

4. Consider the motion of a rotating car wheel shown in the figure. Assume that a point on the tire surface is a distance of $R = 25\text{cm}$ from the wheel center and rotates at constant speed $v = 15\text{m/s}$.

a) Assume the car is stationary while the wheel is spinning (the car is on a lift). On the figure sketch the instantaneous velocity and acceleration vectors corresponding to points *A, B, C, D*. As a point on the surface of the tire moves from positions *B* to *A* what is its *average* acceleration *vector*? Show how the direction of this vector is obtained by considering the velocities at points *B* and *A*.





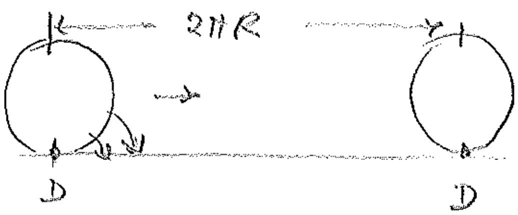
From the vector diagram

$\Delta V = \sqrt{2}\, V$. Since $\Delta t = \dfrac{\pi R}{2V}$

$a_{av} = \dfrac{2\sqrt{2}}{\pi} \cdot \dfrac{V^2}{R} = 810\, m/s^2$

$\vec{a}_{av}$ is parallel to $\Delta \vec{V}$.

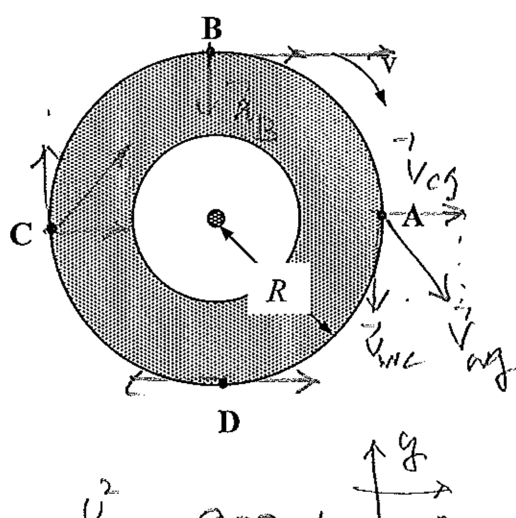
b) Suppose the wheel is spinning as in part (a) but now is in contact with the ground and rolls with no slipping. Assuming the tire spins as shown above, which way does the car go? Explain why the speed of the car is 15m/s.



the car moves to the right. Since the wheel rolls without slipping it must track down a full circumference after one revolution.

$V_A = \dfrac{\Delta x}{\Delta t} = \dfrac{2\pi R}{T} = V$

c) Assuming the wheel is rolling as in part (b) determine the velocity **vectors** *relative to the ground* of a point on the wheel's surface when it is at positions *A, B, C,* and *D*. Sketch these velocity vectors on the figure and show how they are obtained from the car velocity and the vectors you sketched in part (a). Is any point on the wheel at instantaneously rest with respect to the ground? What is the acceleration vector relative to the ground corresponding to point *B*?

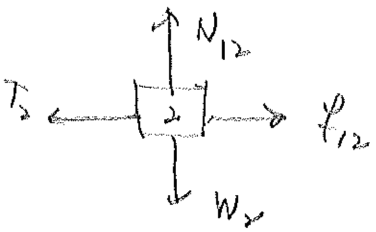


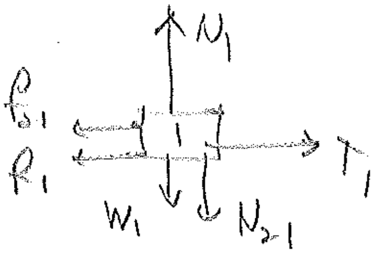
the velocity of a point on the wheel relative to the ground obeys the usual expression: $\vec{V}_{wg} = \vec{V}_{wc} + \vec{V}_{cg}$

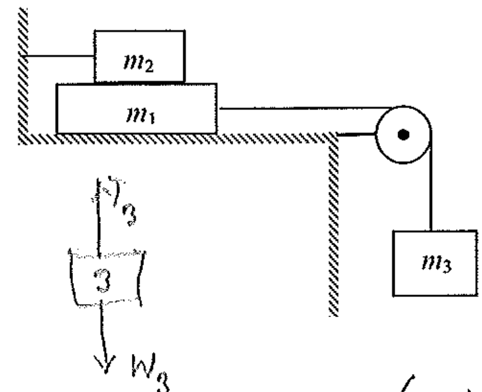
From part (b) $\vec{V}_{cg} = V\hat{i}$. Point D is instantaneously at rest. Since $\vec{V}_{cg}$ is constant $\vec{a}_B$ is the same as in (a) $a_B = \dfrac{v^2}{R} = 900\, m/s^2$

5. Consider three blocks situated as shown in the figure. Assume the cords and pulley are massless and that $m_1 = 3\text{kg}$ and $m_2 = 2\text{kg}$. *All* the surfaces in contact are identical and have static friction coefficient $\mu_S = 0.3$ and kinetic friction coefficient $\mu_k = 0.2$.

a) Draw a free-body diagram for each block and determine the maximum value of m_3 that will allow the system to remain at rest.







$f_{12} = \mu_s m_2 g = f_{21}$

$f_1 = (m_1 + m_2)\mu_s g$

$T_1 = T_3 = f_1 + f_{21}$

$m_3 g = \mu_s (m_1 + 2m_2) g \Rightarrow m_3 = \mu_s(m_1 + 2m_2) = 2.1\text{ kg}$

b) If $m_3 = 4\text{kg}$ determine the acceleration of m_1 and m_3, and the tension in each cord.

$T_1 - f_1 - f_{21} = m_1 a$

$m_3 g - T_1 = m_3 a$

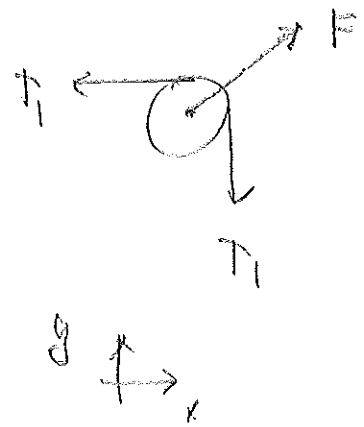
$[m_3 - \mu_k(m_1 + 2m_2)] g = (m_1 + m_3) a$

$$a = \frac{[m_3 - \mu_k(m_1 + 2m_2)]g}{m_1 + m_3} = 3.7\text{ m/s}^2$$

$T_2 = \mu_k m_2 g = 4\text{N}$

$T_1 = m_3(g - a) = 25\text{N}$

c) For m_3 as in part (b) what force does the axle exert on the pulley?

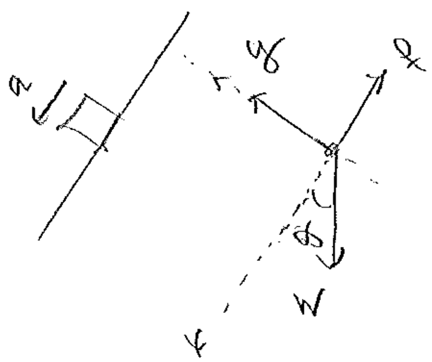


$\vec{F} = T_1(\hat{i} + \hat{j}) = 25(\hat{i} + \hat{j})\text{ N}$

or $F = 36\text{N}$ at 45° above horizontal

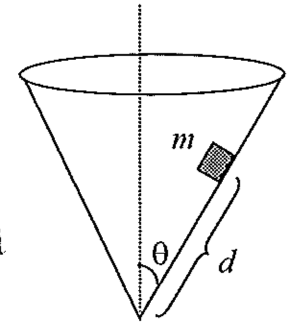
6. Consider a small cube of mass *m* placed inside an inverted cone of apex half-angle θ = 37°.

a) Suppose the cube is released from rest and slides down the rough stationary cone wall. Determine the acceleration of the cube. Assume the kinetic friction coefficient is μ = 0.5.



(x) $W\cos\theta - f = ma$

(y) $N - W\sin\theta = 0$

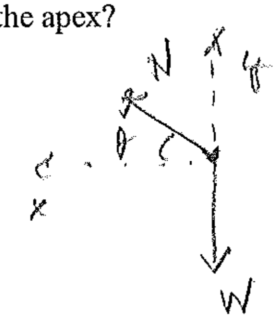


$f = \mu N \Rightarrow a = g(\cos\theta - \mu\sin\theta) = 5\ m/s^2$

b) Now suppose the cone is *frictionless.* With what rotational frequency must the cube spin around the inside of the stationary cone so that it maintains the same distance *d* = 20cm from the apex?

(x) $N\cos\theta = ma$

(y) $N\sin\theta - mg = 0$

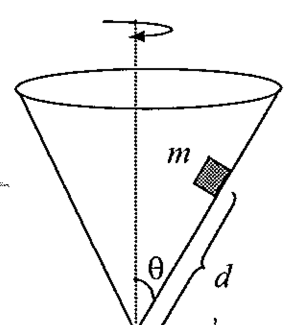


$a/g = \cot\theta$, $a = \omega^2 r$, $r = d\sin\theta$

$\omega^2 = \frac{g}{r}\cot\theta$

$f = \frac{1}{2\pi}\sqrt{\frac{g\cos\theta}{d\sin^2\theta}} = 1.7\ Hz$

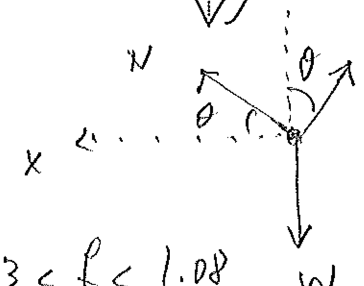
c) Suppose the cone is rough with static friction coefficient μ = 0.5. The cone is now spun around its axis at a sufficient rate so that the cube does not slip. For what range of rotational frequencies will this be possible?



the friction can be up or down the cone depending on whether the frequency is min or max.

(x) $N\cos\theta - f\sin\theta = ma$

(y) $N\sin\theta + f\cos\theta - W = 0$



$\Rightarrow f = \frac{1}{2\pi}\sqrt{\left(\frac{g}{d\sin\theta}\right)\left(\frac{\cos\theta \pm \mu\sin\theta}{\sin\theta \mp \mu\cos\theta}\right)} \Rightarrow 1.03 < f < 1.08\ Hz$