1. Provide clear and concise answers to questions (a) through (c).

a) Consider a charged conducting sphere. It is desired to determine the electric field of the sphere by measuring the force \( F_0 \) on a test charge \( q_0 \) that is brought adjacent to it. Will the electric field determined from \( E = F_0 / q_0 \) yield a field too large, too small, or exactly equal to that due to the charged sphere when it is isolated? Explain.

Assume \( q_0 > 0 \). If the sphere is positively charged, there will be a repulsive force on \( q_0 \). However, the polarization of the sphere will lead to a force smaller than expected \( \Rightarrow \) underestimated.

In the case of a negative sphere, polarization will increase the attraction between \( q_0 \) and the test charge \( \Rightarrow \) overestimated.

b) The figure shows the field lines in the vicinity of a conducting cylindrical shell of outer radius \( R = 10 \text{cm} \). Assume all lines are equally spaced perpendicular to the figure. If \( E_0 = 10^3 \text{ N/C} \) just outside the shell, determine the linear charge densities of the shell and the rod at its center.

Taking a cylindrical Gaussian surface,
the flux through surfaces \( S_1, S_2 \) obey
\[ e_0 \Phi_1 = \Lambda_1 \ell, \quad e_0 \Phi_2 = (\Lambda_1 + \Lambda_2) \ell. \]

But \( \frac{\Phi_2}{\Phi_1} = -\frac{8}{16} = \frac{\Lambda_2}{\Lambda_1} \Rightarrow \Lambda_2 = -\frac{3}{2} \Lambda_1 \nabla \]
\[ E_b = -\frac{2 k}{\ell} (\Lambda_1 + \Lambda_2) = -\frac{k \Lambda_1}{\ell} \Rightarrow \Lambda_1 = 1.1 \times 10^{-6} \text{ C/m}, \quad \Lambda_2 = -1.7 \times 10^{-6} \text{ C/m}. \]

c) Consider three identical metal spheres. One sphere has charge \( Q > 0 \) while the other two are neutral. By only touching the spheres among themselves, explain how two of the spheres can be given opposite charges.

Bring the two neutral spheres close so the charged sphere and touch the neutral spheres together. The polarization induced by the charged sphere will cause the spheres to be oppositely charged.
2. Consider four charges located on the vertices of a square as shown in the figure. Assume \( q > 0 \) and that the charges on the \( x \) and \( y \) axes are located respectively at \( x = \pm a \), and \( y = \pm a \).

a) Determine the electric potential \( V(x) \) along the \( x \)-axis. You may assume that the point \( x \) is to the right of the \(-q\) charges (\( x > a \)). Use your result for \( V(x) \) to determine the electric field for \( x > a \).

\[
V = kq \left[ \frac{q}{(x-a)^2} - \frac{1}{(x-a)^2} + \frac{1}{(x+a)^2} \right]
\]

\[
\mathcal{E} = -\frac{dV}{dx} = kq \left[ \frac{2x}{(x^2+a^2)^{3/2}} + \frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right]
\]

b) Determine the potential energy of the charge distribution.

\[
\mathcal{U} = \sum_{i,j} \frac{kq_i q_j}{r_{ij}} = -\frac{kq^2}{a} (2 \sqrt{2} - 1)
\]

c) Determine the force acting on the \(-q\) charge at \( x = a \). Suppose this charge is released from rest while the others are held in place. Which way will this charge move? It happens that this charge will oscillate between the points \( x = a \) and \( x = x_1 \). Write down an equation satisfied by \( x_1 \); do not attempt to solve this equation.

\[
F_x = \frac{kq^2}{(2a)^2} - \frac{2kq^2}{(\sqrt{2}a)^2} \cos \phi_{\infty} = -\frac{kq^2}{4a^2} (2 \sqrt{2} - 1) < 0.
\]

Charge moves \( \rightarrow \) left.

\[
\Delta U_i = 0 \Rightarrow \Delta V_i = 0 \Rightarrow 0 = -\frac{kq}{a + r_1} + \frac{2kq}{\sqrt{a^2+r_1^2}} \Rightarrow \frac{2}{(a^2+r_1^2)} = \frac{1}{a + r_1} + \frac{1}{2a} = 0
\]

\[
(a^2 + r_1^2) = \frac{a + r_1}{2a}
\]

\[
\Rightarrow \frac{2}{a^2 + r_1^2} = \frac{1}{a + r_1} + \frac{1}{2a} = 0
\]

d) For points along the \( x \)-axis with \( x \gg a \), it is found that \( V(x) \sim 1/x^\alpha \). Determine the value of \( \alpha \). It suffices to explain your reasoning – at detailed calculation is unnecessary.

\[
\text{Q.net} = 0 \quad \text{and} \quad \mathcal{E} = 0, \quad \text{expected system to behave like a quadrupole} \quad V \sim \frac{1}{x^3} \quad \text{or} \quad \alpha = 3
\]
3. This question concerns various continuous charge distributions.

a) Consider a semi-infinite rod of uniform charge density $\lambda$ as shown in the figure. Determine an expression for the net $y$-component of the electric field $E_y$ at point $P$ that is a distance $R$ from the rod.

$$dE_y = -dE \sin \theta$$

$$dE = \frac{k \lambda \, dq}{r^2}$$

$$E_y = -k \lambda \int \frac{dx}{(x^2 + R^2)^{3/2}}$$

b) Consider a semi-infinite plate of surface charge density $\eta$. Determine an expression for the $y$-component of the electric field at the point $P$ located a distance $y$ above the plate edge by breaking the plate up into a series of rods. Explain how $E_y$ may be guessed based on symmetry, superposition and the field due to an infinite plate.

$$dE_y = dE \sin \theta$$

$$dE = \frac{2k \eta \, dx}{r}$$

$$E_y = \int \frac{2k \eta \, dx}{x^2 + y^2}$$

$$E_y = \frac{2k \eta \tan^{-1} \frac{y}{x}}{r}$$

For $r = \sqrt{x^2 + y^2}$, that for all sheet

$$E_y = \frac{\eta}{r}$$

$$E_y = 0$$

Assume the shell has charge uniformly distributed throughout it with charge density $\rho$. Determine the electric field for all points with $r < b$. What value must $Q$ have so that $E = 0$ outside the shell?

Using Gauss’s law:

For $r < a$:

$$\oint \phi E \cdot dS = \phi (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2}$$

For $a < r < b$:

$$\oint \phi E \cdot dS = \phi (4\pi r^2) = \frac{1}{\epsilon_0} \frac{dQ}{dr} (r^2 - a^2)$$

$$E = \frac{\rho (r^2 - a^2)}{2\epsilon_0 r^2}$$

For $r > b$ when $E = 0$ then $Q_{net} = 0$
4. This question concerns aspects of charged plates. In each case assume the plates are very large with top and bottom surfaces each of area $A$. Express your answers in terms of $Q$, $A$, and $\varepsilon_0$.

a) Suppose the plate has total charge $Q$ spread uniformly over it. Use Gauss’s Law to determine the field around the plate. For full credit, show all steps and reasoning involved in the analysis.

By symmetry, $\varepsilon$ is the same above and below the sheet and does not vary along it.

$$\oint \mathbf{E} \cdot d\mathbf{a} = \Phi = \frac{1}{\varepsilon_0} \left( \frac{Q}{A} \right), \quad \varepsilon = \frac{Q}{2A\varepsilon_0}$$

b) Suppose the plate of part (a) is made of metal. Determine the field around and within it and the charge that resides on its top and bottom surfaces. On the figure sketch the arrangement of the charges.

By symmetry, $Q_A = Q_B = \frac{Q}{2}$

Inside the plate $\varepsilon = 0$

The field outside is that of part (a).

The charge is on the plate's surface.

C) Suppose another identical metal plate with the same charge is introduced next to the one of part (b). Determine the electric field everywhere and indicate how the charges are now distributed over the plates.

By symmetry, $Q_1 = Q_4, Q_2 = Q_3$

$$\Phi_1 = \oint \mathbf{E} \cdot d\mathbf{a} = 0 \Rightarrow Q_2 + Q_3 = 0$$

$\Rightarrow Q_2 = Q_3 = 0 \quad Q_1 = Q_4 = Q$

For $S_1$:

$$\oint \mathbf{E} \cdot d\mathbf{a} = \Phi_A \cdot a = \left( \frac{Q}{A} \right) \cdot \frac{a}{\varepsilon_0} \Rightarrow \varepsilon = \frac{Q}{A\varepsilon_0}$$

Outfield between plates.

D) Suppose an insulating sheet with total charge $-Q$ is introduced between the plates of part (c). Determine the electric field and the distribution of charges on the plates. Indicate the charge arrangement on the figure.

By symmetry $Q_1 = Q_4, Q_2 = Q_3$

$$\Phi_1 = \oint \mathbf{E} \cdot d\mathbf{a} = 0 \Rightarrow Q_2 + Q_3 - Q = \Phi$$

$\Rightarrow Q_2 = Q_3 = \frac{Q}{2} \quad Q_1 = Q_4 = \frac{Q}{2}$

Between plates:

$$\oint \mathbf{E} \cdot d\mathbf{a} = -\Phi = \left( \frac{Q}{A} \right) \cdot \frac{d}{\varepsilon_0} \Rightarrow \varepsilon = \frac{Q}{2A\varepsilon_0}$$

The field has the same value outside plates.