

Mean & Standard Deviation of a Probability Distribution and Expected Values on the TI 83

1. Mean and Standard Deviation of a Probability Distribution

Suppose we have a probability distribution that describes the number of dogs a randomly selected person owns, the distribution is as follows:

Let x = number of dogs a person owns

Let $P(x)$ = the probability that a randomly selected person owns that many dogs.

x	P(x)
0	0.45
1	0.40
2	0.10
3	0.04
4	0.01

Find the mean and standard deviation for the number of dogs a randomly selected person owns. So we need to find mean = $\mu = \sum xP(x)$ and

standard deviation = $\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$

x	P(x)	xP(x)	x ² P(x)
0	0.45	0	0
1	0.40	0.40	0.40
2	0.10	0.20	0.40
3	0.04	0.12	0.36
4	0.01	0.04	0.16
		.76	1.32

So the mean number of dogs a randomly selected person owns is $\mu = \sum xP(x) = 0.76$

and the standard deviation = $\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{1.32 - .76^2} = 0.8616$

To find this using the calculator, enter the x's into L1 and the P(x)'s into L2 as follows:

L1	L2	L3	2
0	.45	-----	
1	.4		
2	.1		
3	.04		
4	.01		
-----	-----		
L2(1) = .45			

Then press the STATS key and under the CALC menu use the 1-var stats key as follows:

```
1-Var Stats L1,L
2
```

press ENTER to get

```
1-Var Stats
x̄=.76
Σx=.76
Σx²=1.32
Sx=
σx=.8616263691
↓n=1
█
```

Notice the values that correspond to the mean and standard deviation we already found with the formulas. Also notice that the $n = 1$, this is the sum of the L2 column which are probabilities, and recall that for this to be a valid probability distribution the sum of the probabilities should be one.

2. Expected Value

The expected value is what you expect to happen in the long run. Suppose you plan to play the game of roulette. This is a game where a wheel is spun and a small ball can land on one of 38 different numbers. There are 18 red numbers, 18 black numbers and 2 green numbers (0 and 00). There are several types of bets you can make, take the case of a STRAIGHT bet, here you are picking one of the 38 different outcomes and hoping that your number wins, so you have a $1/38$ chance of winning and a $37/38$ chance of losing. A typical casino will give pay you at 35 to 1 for each dollar you bet if you win, that means if you bet \$1 and win you will receive a \$35 profit (the casino will actually give you \$36 = \$35 profit + \$1 bet), if you bet \$2 and win, you will receive \$70 profit (the casino will actually give you \$72 = \$70 profit + \$2 bet).

Find the expected value of a \$1 STRAIGHT bet in roulette, that is, what can you expect to win or lose in the long run of playing this game.

By definition, we need to find $\sum xP(x)$, but what is x and what is $P(x)$?

Set up an expected value problem as follows:

Let x = the amount of money you win and lose

Let $P(x)$ = the probability of winning and losing

Event	x	P(x)	xP(x)
Win	\$35	1/38	35/38
Lose	-\$1	37/38	-37/38
			$\sum xP(x) = -2/38$

So the expected value is $-2/38 = -.0526$, which means in the long run if you continue to make the same \$1 STRAIGHT bet, you can expect to lose about 5 cents for each dollar you bet. (this is an average so you won't lose 5 cents on every bet, but if you take the amount won on each bet and the amount lost on each bet, in the long run you can expect to lose on average 5 cents per bet).

To do this on the calculator put the x's in the L1 column and the P(x)'s on the L2 column as follows:

L1	L2	L3	Z
35	0.026315789	-----	
-1	.97368		
-----	-----		
L2(*) = .0263157894...			

Notice $1/38 = .0263157895$

To get the expected value, press the STAT key and under the CALC select the 1-var stats choice as follows:

1-Var Stats L1,L
2

Press ENTER to get

1-Var Stats
$\bar{x} = -.0526315789$
$\sum x = -.052631579$
$\sum x^2 = 33.2105263$
Sx =
$\sigma x = 5.762617134$
$\downarrow n = 1$

Notice that the mean and sum of the x's both give us $-.0526$ which is the expected value for a \$1 STRAIGHT bet.