

Hypothesis Testing Example Problems:

1. The Medassist Pharmaceutical Company makes a pill intended to reduce the chance of seizures among children. The pill is supposed to contain 20.0mg of Phenobarbital. A simple random sample of 20 pills yielded the amounts (in mg) listed here. Are these pills acceptable at the 0.01 level of significance?

27.5 26.0 22.9 23.4 23.0 23.9 32.6 20.9 22.9 24.3
24.8 16.1 24.3 17.3 18.9 20.7 33.0 15.6 24.3 23.3

(What is the p-value? What is a type I error? A type II error?)

2. Shown are the birth weights (in kilograms) of male babies born to mothers taking a special vitamin supplement (based on data from the New York State Department of Health). Test the claim that this sample comes from a population with a standard deviation less than 0.88kg, which is the standard deviation for male birth weights in general. Does the vitamin supplement appear to affect the variation among male birth weights? Use a 0.05 significance level. (What is the p-value? What is a type I error? What is a type II error?)

3.73 4.37 3.73 4.33 3.39 3.68 4.68 3.52
3.02 4.09 2.47 4.13 4.47 3.22 3.43 2.54

3. In 1990, 5.8% of job applicants who were tested for drugs failed the test. At the 0.01 level of significance, test the claim that the failure rate is now lower. If a random sample of 1520 current job applicants results in 58 failures. Does the result suggest that fewer job applicants now use drugs? (What is the p-value? What is a type I error? What is a type II error?)

4. The Orange County Supervisor boasts that full-time salaried employees in her county have a mean annual income that is greater than \$30,000. If she based that claim on a sample of 150 such employees who had a mean of \$30,122, and a standard deviation of \$14,276 is her claim justified? Use a 0.01 level of significance. (What is the p-value, what is a type I error? a type II error?)

Solutions to the Examples

1. The Medassist Pharmaceutical Company makes a pill intended to reduce the chance of seizures among children. The pill is supposed to contain 20.0mg of Phenobarbital. A simple random sample of 20 pills yielded the amounts (in mg) listed here. Are these pills acceptable at the 0.01 level of significance?

27.5 26.0 22.9 23.4 23.0 23.9 32.6 20.9 22.9 24.3
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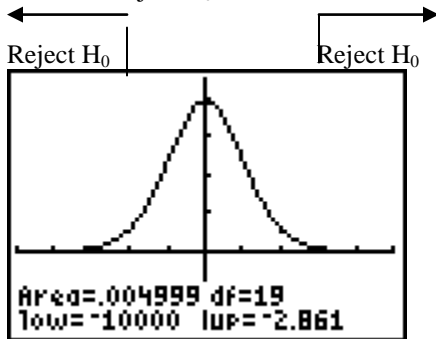
(What is the p-value? What is a type I error? A type II error?)

Step 1:

$H_0 : \mu = 20$ claim $H_1 : \mu \neq 20$

Step 2: Reject H_0 if the test statistic is ≤ 2.861 or

Reject H_0 if the test statistic is ≥ 2.861



Step 3:

$$t = \frac{23.285 - 20}{4.534 / \sqrt{20}} = 3.24 \quad (\text{get the mean and standard deviation by putting the data into the calculator and using the 1-var stats option})$$

using the 1-var stats option)

Step 4: Reject H_0 , the evidence does not support the claim that the average Phenobarbital contains 20 mg, the mean appears to be different from 20 mg.

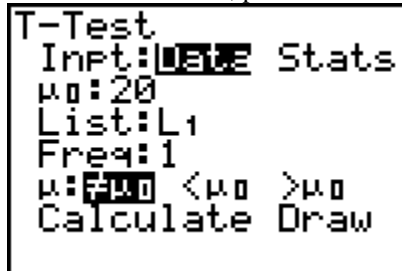
The p-value = twice the area to the right of the test statistic $t = 3.24$.

p-value = .0043 (from calculator)

Type I error: Reject H_0/H_0 true : To state that the Phenobarbital pills do not weigh 20mg when in fact they do weigh 20mg (that is, say the pills are unacceptable when in fact they would have been fine to use).

Type II error: Accept H_0/H_0 false: To state that the Phenobarbital pills weigh 20 mg when in fact they do not weigh 20mg (that is, say the pills are acceptable when in fact they are not and we would be either over or under dosing the patients).

To use the calculator, put the data in L1 and use the T-test under the stats TEST menu as follows:



2. Shown are the birth weights (in kilograms) of male babies born to mothers taking a special vitamin supplement (based on data from the New York State Department of Health). Test the claim that this sample comes from a population with a standard deviation less than 0.88kg, which is the standard deviation for male birth weights in general. Does the vitamin supplement appear to affect the variation among male birth weights? Use a 0.05 significance level (What is the p-value? What is a type I error? What is a type II error?)

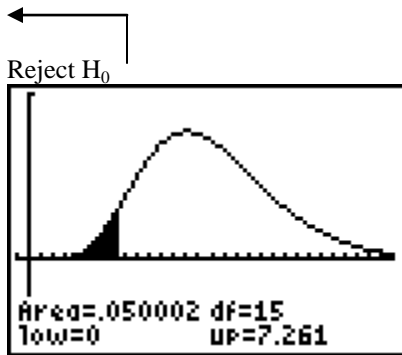
3.73	4.37	3.73	4.33	3.39	3.68	4.68	3.52
3.02	4.09	2.47	4.13	4.47	3.22	3.43	2.54

Step 1:

$H_0 : \sigma = 0.88$

$H_1 : \sigma < 0.88$ claim

Step 2: Reject H_0 if the test statistic is less equal 7.261



Step 3: $\chi^2 = \frac{(16-1)(.657)^2}{(0.88)^2} = 8.36$ (get the mean and standard deviation by putting the data into the calculator and using the 1-var stats option)

Step 4: Accept H_0 , the evidence does not support the claim that that the male babies born to mothers who received the supplement is less than the male birth weight of the male babies born to mothers who did not receive the supplement.

The p-value = the area to the left of the test statistic $\chi^2 = 8.36$, $0.05 < \text{p-value} < 0.10$

Type I error: Reject H_0/H_0 true : To state that the birth weights of the babies born to women who took the supplement is less than the birth weight of the male babies born to mother who did not take the supplement, when in fact the weight of the male babies born to women who took the supplement is at least the same as the birth weight of the babies born to mothers who did not take the supplement.

Type II error: Accept H_0/H_0 false: To state that the birth weights of the babies born to women who took the supplement is at least the same as the birth weight of the male babies born to mother who did not take the supplement, when in fact the weight of the male babies born to women who took the supplement is less than the birth weight of the babies born to mothers who did not take the supplement.

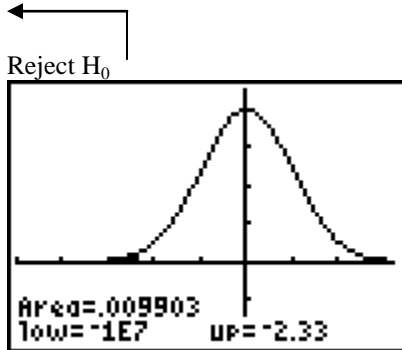
Note: we cannot do this one on the calculator.

3. In 1990, 5.8% of job applicants who were tested for drugs failed the test. At the 0.01 level of significance, test the claim that the failure rate is now lower. If a random sample of 1520 current job applicants results in 58 failures. Does the result suggest that fewer job applicants now use drugs? (What is the p-value? What is a type I error? What is a type II error?)

Step 1:

$$H_0 : p = 0.058 \quad H_1 : p < 0.058 \text{ claim}$$

Step 2: Reject H_0 if the test statistic is less equal -2.33



Step 3:

$$z = \frac{0.038 - 0.058}{\sqrt{\frac{(0.058)(0.942)}{1520}}} = -3.34 \quad \text{note } \hat{p} = \frac{58}{1520} \approx .038$$

Step 4: Reject H_0 , the evidence does support the claim that that percentage of job applicants that fail the drug test is now lower.

The p-value = the area to the left of the test statistic $z = -3.34$

p-value < .0001

Type I error: Reject H_0/H_0 true : To state the percentage of job applicants who fail the drug test now is lower than the percentage that failed the test in 1990 when in fact the percentage of job applicants that fail the test now is at least the same as the percentage that failed the test in 1990.

Type II error: Accept H_0/H_0 false: To state the percentage of job applicants who fail the drug test now is at least the same as the percentage that failed the test in 1990 when in fact the percentage of job applicants that fail the test now is less than the percentage that failed the test in 1990.

To use the calculator, use the 1-propztest as follows:

```

1-PropZTest
P0: .058
x: 58
n: 1520
PROP≠P0 <P0 >P0
Calculate Draw
    
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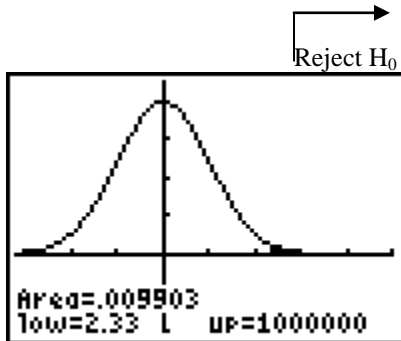
4. The Orange County Supervisor boasts that full-time salaried employees in her county have a mean annual income that is greater than \$30,000. If she based that claim on a sample of 150 such employees who had a mean of \$30,122, and a standard deviation of \$14,276 is her claim justified? Use a 0.01 level of significance. (What is the p-value, what is a type I error? a type II error?)

Step 1:

$$H_0 : \mu = 30,000$$

$$H_1 : \mu > 30,000 \text{ claim}$$

Step 2: Reject H_0 if the test statistic is greater than or equal to 2.33



Step 3:

$$z = \frac{30122 - 30000}{14276 / \sqrt{150}} = 0.105$$

Step 4: Accept H_0 , the evidence does not support the claim that the annual salary for Orange County employees is greater than \$30,000.

The p-value = the area to the right of the test statistic $z = 0.105$

p-value = .4562

Type I error: Reject H_0/H_0 true : To state that the employees of Orange County have an average salary greater than \$30,000, when in fact they employees of Orange County make an average salary that is less than or equal to \$30,000.

Type II error: Accept H_0/H_0 false: To state that the employees of Orange County have an average salary that is less than or equal to \$30,000, when in fact they employees of Orange County make an average salary that is greater than \$30,000.

To use the calculator, use the z-test as follows:

