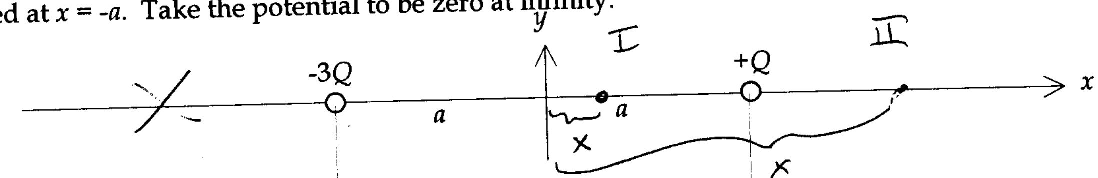
Name KEY Date 10/21/09

Fall 2009

Physics 22
Exam 2 - Chapters 23, 24, and 25

Do all your work in the space provided. If the work is not shown, then credit will not be given. Express your answer to three significant figures, unless indicated otherwise. Please underline or draw a box around your final answer, and include the correct units with your answers.

1. Two point charges are located on the x-axis. A charge $+Q$ is located at $x = +a$, and a charge $-3Q$ is located at $x = -a$. Take the potential to be zero at infinity.



a. Find a position (or positions) on the x-axis where the electric potential is zero. (6)

I. $\Sigma V = \frac{k(-3Q)}{x+a} + \frac{kQ}{a-x} = 0$, $\frac{-3}{x+a} = \frac{-1}{a-x}$

$-3a + 3x = -x - a$, $4x = 2a$, $\boxed{x = \frac{a}{2}}$

II. $\Sigma V = \frac{k(-3Q)}{x+a} + \frac{kQ}{x-a} = 0$, $\frac{-3}{x+a} = \frac{-1}{x-a}$

$-3x + 3a = -x + a$, $2x = 4a$, $\boxed{x = 2a}$

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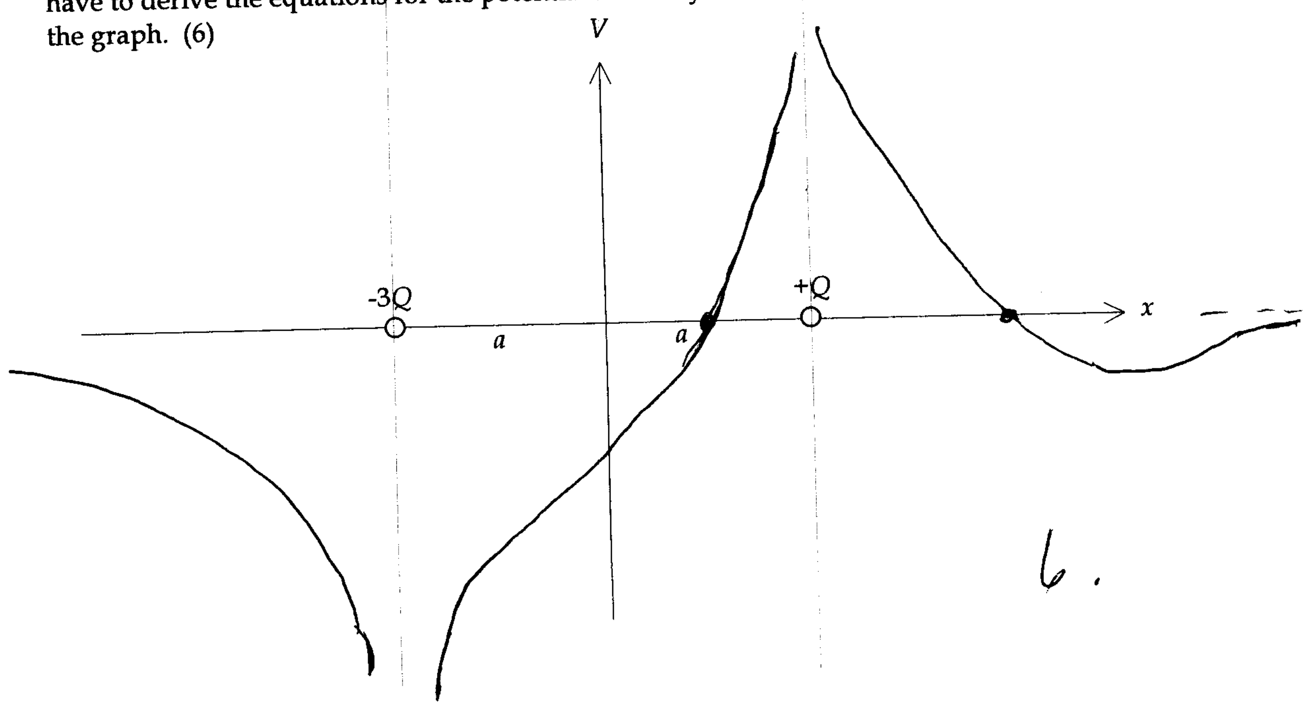
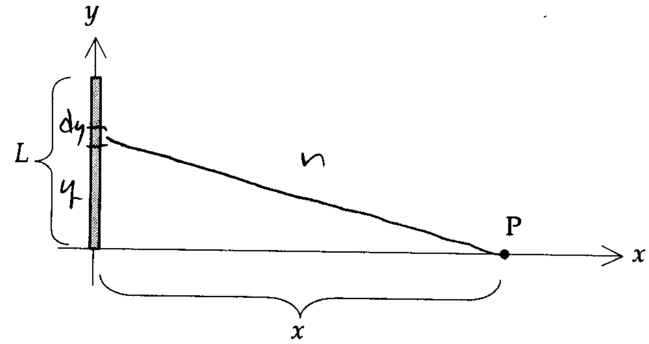
b. Draw a graph that shows how the electric potential varies on the x-axis. You do not actually have to derive the equations for the potentials unless you want to. You can deduce the shape of the graph. (6)



6.

2. A thin rod of length L lies on the y-axis with one end at origin. The linear charge density λ of the rod varies along the y-axis according to the expression $\lambda = Cy$, where C is a constant and y is the y-coordinate. (The charge on the rod is increasing as you move upward from the origin.) The potential is zero at infinity.



a. Find the potential at point P in terms of C, L, x, and constants. (8)

$$dV = \frac{k\,dq}{r} = \frac{k\lambda\,dy}{r} = \frac{kCy\,dy}{\sqrt{x^2+y^2}}$$

$$V = kC\int \frac{y\,dy}{\sqrt{x^2+y^2}}, \quad \text{let } u^2 = x^2+y^2, \; u\,du = y\,dy$$

$$V = kC\int \frac{u\,du}{u} = kCu\Big|_x^{\sqrt{x^2+L^2}} = kC\left[\sqrt{x^2+L^2} - x\right]$$

8

b. Find the x-component of the electric field at point P. (4) (You can still earn two point if you simply explain to me how the find the x-component of the electric field.

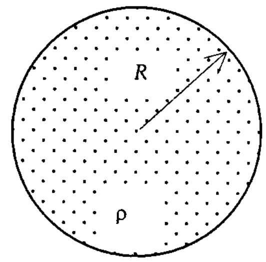
$$E_x = -\frac{dV}{dx} = -kC\left[\frac{1}{2}\frac{2x}{\sqrt{x^2+L^2}} - 1\right]$$

$$E_x = kC\left[1 - \frac{x}{\sqrt{x^2+L^2}}\right]$$

4

12

3. A sphere with a radius R has a *uniform* volume charge density ρ. The electric field outside the sphere ($r > R$) is $E = \frac{\rho R^3}{3\varepsilon_0 r^2}$, and the electric field inside the sphere ($r < R$) is $E = \frac{\rho r}{3\varepsilon_0}$, where r is measured from the center of the sphere. Find the electric potential both outside the sphere and inside the sphere if the potential is chosen to be zero ($V = 0$) at the surface of the sphere ($r = R$). (Be careful – we've done a problem like this but the potential was chosen to be zero at infinity.)



a. outside the sphere ($r > R$): (6)

$$V_A - V_B = \int \vec{E} \cdot d\vec{\ell} = \int_{r_A}^{r_B} \frac{\rho R^3}{3\epsilon_0 r^2}\, dr = \frac{\rho R^3}{3\epsilon_0}\left(-\frac{1}{r}\right|_{r_A}^{r_B}$$

$$V_A - V_B = \frac{\rho R^3}{3\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

Let $r_B = R$, $V_B = 0$: $r_A \to r$, $V_A \to V$

$$V - 0 = \frac{\rho R^3}{3\epsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$$

$$V = -\frac{\rho R^3}{3\epsilon_0}\left(\frac{1}{R} - \frac{1}{r}\right)$$

6

b. inside the sphere ($r < R$): (6)

$$V_A - V_B = \int \vec{E} \cdot d\vec{\ell} = \int \frac{\rho r}{3\epsilon_0}\, dr = \frac{\rho}{3\epsilon_0} \frac{r^2}{2}\Big|_{r_A}^{r_B}$$

$$V_A - V_B = \frac{\rho}{6\epsilon_0}(r_B^2 - r_A^2)$$

Let $r_B = R$, then $V_B = 0$: $r_A \to r$, $V_A \to V$

$$V - 0 = \frac{\rho}{6\epsilon_0}(R^2 - r^2)$$

$$V = \frac{\rho}{6\epsilon_0}(R^2 - r^2)$$

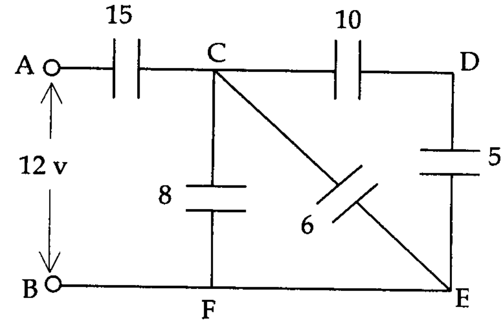
6

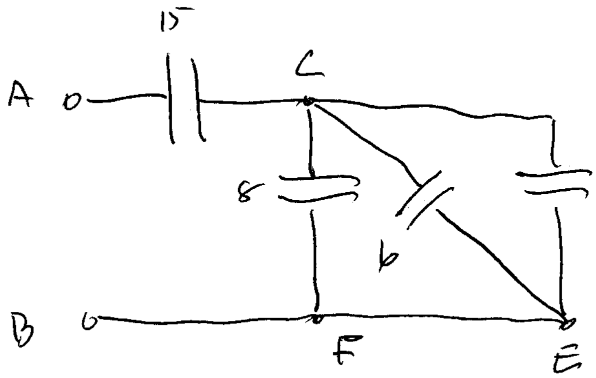
12

4. Five capacitors are connected as shown in the circuit diagram. The capacitance values are in microfarads (µF).

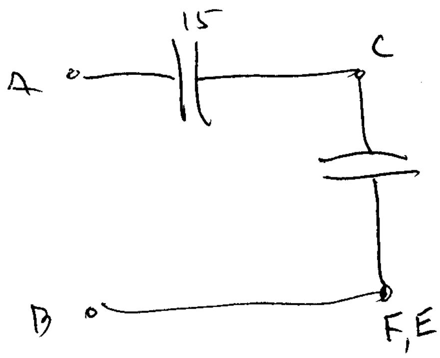
   a. Find the equivalent capacitance of the combination between points A and B. (6)

   b. Find the voltage between points A and D, V_{AD}. (6)

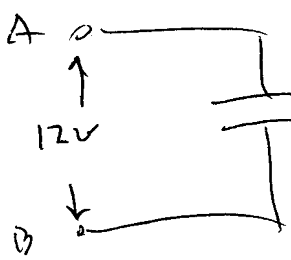




$\frac{10 \cdot 5}{10+5} = 3.33$



$8+6+3.33 = 17.33$



$\frac{15 \times 17.33}{15+17.33} = 8.04$

Voltage is 12 v, charge $Q = CV = 8.04(12) = 96.5\,\mu C$

charge is 96.5 µC

$V_{AC} = \frac{Q}{C} = \frac{96.5}{15} = 6.43\,v$

$V_{CF} = \frac{Q}{C} = \frac{96.5}{17.33} = 5.57\,v$

note = 12 v

Voltage is 5.57 v

$Q = CV = 3.33(5.57) = 18.5\,\mu C$

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charge on 10 µF, $Q = 18.5\,\mu C$

$V_{CD} = \frac{Q}{C} = \frac{18.5}{10} = 1.85\,v$

$V_{AD} = V_{AC} + V_{CD} = 6.43 + 1.85$

$V_{AD} = 8.28\,v$

5. A parallel plate capacitor has plates of area A and a distance between plates of d. It is connected to a battery, and the plates gain a charge of Q_o. The battery is then disconnected from the capacitor leaving a charge Q_o on the plates of the capacitor. In terms of given quantities A, d, Q_o, and constants:

a. write the value of the electric field between the plates. (2)

$$E_o = \frac{\sigma_o}{\epsilon_0} = \frac{Q_o/A}{\epsilon_0} = \frac{Q_o}{A\epsilon_0}$$

b. write the value of the voltage across the plates. (2)

$$V_o = E_o d = \frac{Q_o d}{A\epsilon_0}$$

c. write the value of the energy stored between the plates. (2)

$$U_o = \frac{1}{2} Q_o V_o = \frac{1}{2} Q_o \left(\frac{Q_o d}{A\epsilon_0}\right) = \frac{1}{2} \frac{Q_o^2 d}{A\epsilon_0}$$

A dielectric with dielectric constant κ is slid between the plates, completely filling the space between the plates. In terms of given quantities A, d, Q_o, κ, and constants:

d. write the value of the electric field between the plates. (2)

$$E = E_o - E_i = \frac{Q_o}{A\epsilon_0} - \frac{Q_i}{A\epsilon_0}, \quad Q_i = Q_o\left(1 - \frac{1}{K}\right)$$

$$= \frac{Q_o}{A\epsilon_0} - \frac{Q_o}{A\epsilon_0}\left(1 - \frac{1}{K}\right) = \frac{Q_o}{AK\epsilon_0}$$

e. write the value of the voltage across the plates. (2)

$$V = Ed = \frac{Q_o d}{AK\epsilon_0}$$

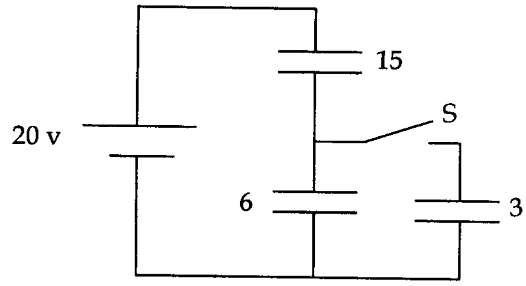
f. write the value of the energy stored between the plates. (2)

$$U = \frac{1}{2} Q_o V = \frac{1}{2} Q_o \left(\frac{Q_o d}{AK\epsilon_0}\right)$$

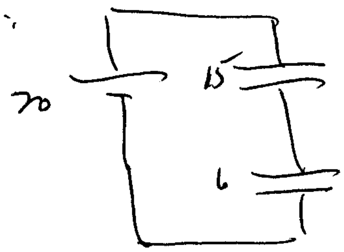
$$U = \frac{1}{2} \frac{Q_o^2 d}{AK\epsilon_0}$$

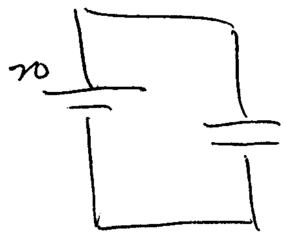
12

6. A 20 volt battery is connected to a circuit containing capacitors, as shown. The switch S is initially open. You then close the switch. Find the amount of charge that passed through the switch. The capacitance values are in terms of microfarads (μF). (12)



before:



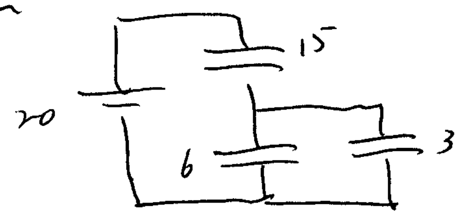


$\frac{15\cdot 6}{15+6} = 4.29$

$Q = CV = 4.29(20)$
$= 85.7\,\mu C$

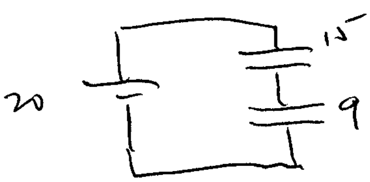
charge on each capacitor before switch is closed.

after



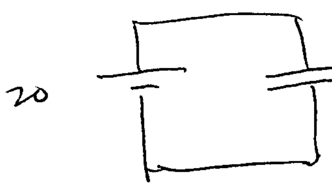
$V = 12.5\,v$

$Q_3 = CV = 3(12.5) = 37.5\,\mu C.$



$Q = 113\,\mu C$

$V_9 = \frac{Q}{C} = \frac{113}{9} = 12.5\,v.$



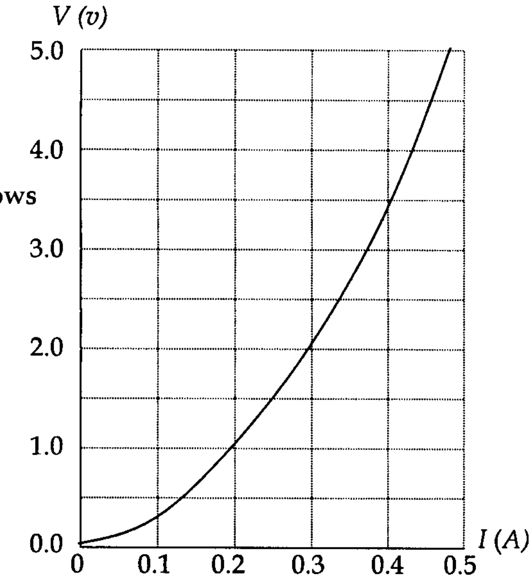
$\frac{15\cdot 9}{15+9} = 5.63$

$Q = CV = 5.63(20)$
$= 113\,\mu C.$

The 3 μF capacitor started out with no charge. It now has 37.5 μC. So, 37.5 μC must have passed through the switch.

12

7. A circuit element with resistance is connected to a power supply. The voltage of the power supply is varied and the voltage and current are recorded. The graph to the right shows the variation of the current with the voltage.



a. Does the circuit element obey Law? Explain. (4)

No. V is not proportional to I

4

b. Does this element have a positive, negative, or a ~~constant~~ zero temperature coefficient of resistivity? Explain. (4)

positive. As the current increases, the temperature increases, and the resistance increases.
(The slope of the graph represents resistance.)

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c. Explain how to design a resistor whose resistance does not change as even though the temperature changes. (4)

Combine two materials in series, one with a positive temperature coefficient, and one with a negative temperature coefficient. The increase in resistance of one is offset by the decrease of resistance of the other.

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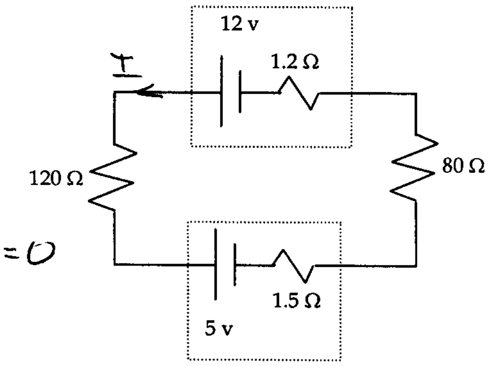
12

8. Two batteries with internal resistance and two other resistors are shown in the circuit to the right.



a. Find the magnitude and direction of current flow. (4)

$+120I + 5 + 1.5I + 80I + 1.2I - 12 = 0$

$202.7 I = 7$

$I = 0.0345\ A$

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b. Find the power supplied by the 12 v battery. (4)

$P = IV = I(\mathcal{E} - Ir) = I\mathcal{E} - I^2 r$

$= (.0345)(12) - (.0345)^2(1.2) = 0.414 - 0.001$

$P = 0.413\ W$

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c. Find the power delivered to the battery that is being charged. (4)

$P = IV = I(\mathcal{E} + Ir) = I\mathcal{E} + I^2 r$

$= (.0345)(5) + (.0345)^2(1.5) = 0.1725 + .0018$

$P = 0.174\ W$

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d. Find the power dissipated in the 120 ohm resistor. (4)

$P = I^2 R = (.0345)^2(120)$

$P = 0.142 W$

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16