Some Uses of Capacitors

- **Defibrillators**
  - When fibrillation occurs, the heart produces a rapid, irregular pattern of beats
  - A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern

- In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse
Capacitors with Dielectrics

- A *dielectric* is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance.
  - Dielectrics include rubber, plastic, and waxed paper.

- For a parallel-plate capacitor, $C = \kappa C_0 = \kappa \varepsilon_0 (A/d)$
  - The capacitance is multiplied by the factor $\kappa$ when the dielectric completely fills the region between the plates.
Dielectrics, cont

- In theory, $d$ could be made very small to create a very large capacitance.

- In practice, there is a limit to $d$:
  - $d$ is limited by the electric discharge that could occur though the dielectric medium separating the plates.

- For a given $d$, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the dielectric strength of the material.
Dielectrics, final

Dielectrics provide the following advantages:

- Increase in capacitance
- Increase the maximum operating voltage
- Possible mechanical support between the plates
  - This allows the plates to be close together without touching
  - This decreases $d$ and increases $C$
Dielectrics

- If we insert an insulating material (dielectric) between the two conductors the capacitance increases.

- Let $C_0 =$ capacitance with no dielectric (vacuum).
- Let $C =$ capacitance with dielectric.
- Definition of dielectric constant ($K$)

$$K = \frac{C}{C_0} > 1$$
<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant $\kappa$</th>
<th>Dielectric Strength$^a$ $(10^6 \text{ V/m})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (dry)</td>
<td>1.000 59</td>
<td>3</td>
</tr>
<tr>
<td>Bakelite</td>
<td>4.9</td>
<td>24</td>
</tr>
<tr>
<td>Fused quartz</td>
<td>3.78</td>
<td>8</td>
</tr>
<tr>
<td>Mylar</td>
<td>3.2</td>
<td>7</td>
</tr>
<tr>
<td>Neoprene rubber</td>
<td>6.7</td>
<td>12</td>
</tr>
<tr>
<td>Nylon</td>
<td>3.4</td>
<td>14</td>
</tr>
<tr>
<td>Paper</td>
<td>3.7</td>
<td>16</td>
</tr>
<tr>
<td>Paraffin-impregnated paper</td>
<td>3.5</td>
<td>11</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.56</td>
<td>24</td>
</tr>
<tr>
<td>Polyvinyl chloride</td>
<td>3.4</td>
<td>40</td>
</tr>
<tr>
<td>Porcelain</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Pyrex glass</td>
<td>5.6</td>
<td>14</td>
</tr>
<tr>
<td>Silicone oil</td>
<td>2.5</td>
<td>15</td>
</tr>
<tr>
<td>Strontium titanate</td>
<td>233</td>
<td>8</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>60</td>
</tr>
<tr>
<td>Vacuum</td>
<td>1.000 00</td>
<td>—</td>
</tr>
<tr>
<td>Water</td>
<td>80</td>
<td>—</td>
</tr>
</tbody>
</table>

$^a$ The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.
Capacitor with and without dielectric

Capacitance in vacuum \( C_0 \)

Capacitance with dielectric \( C = KC_0 \)

- Suppose we put the same charge \( Q \) on the two capacitors (with and without dielectric)

- Voltages with and without \( V \) and \( V_0 \)

- \( Q = C_0 V_0 \) and \( Q = CV \)

\[
C_0 V_0 = CV = KC_0 V \quad \rightarrow \quad V = \frac{V_0}{K}
\]

For a given charge, the voltage across a capacitor is reduced in the presence of dielectric
Types of Capacitors – Tubular

- Metallic foil may be interlaced with thin sheets of paper or Mylar
- The layers are rolled into a cylinder to form a small package for the capacitor
Types of Capacitors – Oil Filled

- Common for high-voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil
Types of Capacitors – Electrolytic

- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution
Types of Capacitors – Variable

- Variable capacitors consist of two interwoven sets of metallic plates
- One plate is fixed and the other is movable
- These capacitors generally vary between 10 and 500 pF
- Used in radio tuning circuits
Capacitor types

Capacitors are often classified by the materials used between electrodes.

- Some types are air, paper, plastic film, mica, ceramic, electrolyte, and tantalum.
- Often you can tell them apart by the packaging.

Ceramic Capacitor

Plastic Film Capacitor

Electrolyte Capacitor

Tantalum Capacitor
Electric Dipole

- An electric dipole consists of two charges of equal magnitude and opposite signs.
- The charges are separated by $2a$.
- The electric dipole moment ($\mathbf{p}$) is directed along the line joining the charges from $-q$ to $+q$. 
The electric dipole moment has a magnitude of $p = 2aq$

Assume the dipole is placed in a uniform external field, $E$
- $E$ is external to the dipole; it is not the field produced by the dipole

Assume the dipole makes an angle $\theta$ with the field
Electric Dipole, 3

- Each charge has a force of $F = Eq$ acting on it.
- The net force on the dipole is zero.
- The forces produce a net torque on the dipole.
The magnitude of the torque is:
\[ \tau = 2Fa \sin \theta = pE \sin \theta \]
The torque can also be expressed as the cross product of the moment and the field:
\[ \tau = p \times E \]
The potential energy can be expressed as a function of the orientation of the dipole with the field:
\[ U_f - U_i = pE(\cos \theta_i - \cos \theta_f) \rightarrow U = -pE \cos \theta = -p \cdot E \]
Polar vs. Nonpolar Molecules

- Molecules are said to be *polarized* when a separation exists between the average position of the negative charges and the average position of the positive charges.

- **Polar molecules** are those in which this condition is always present.

- Molecules without a permanent polarization are called **nonpolar molecules**.
Water Molecules

- A water molecule is an example of a polar molecule.
- The center of the negative charge is near the center of the oxygen atom.
- The x is the center of the positive charge distribution.
Polar Molecules and Dipoles

- The average positions of the positive and negative charges act as point charges
- Therefore, polar molecules can be modeled as electric dipoles
Induced Polarization

- A symmetrical molecule has no permanent polarization (a)
- Polarization can be induced by placing the molecule in an electric field (b)
- Induced polarization is the effect that predominates in most materials used as dielectrics in capacitors
Dielectrics – An Atomic View

- The molecules that make up the dielectric are modeled as dipoles.
- The molecules are randomly oriented in the absence of an electric field.

(a)
Dielectrics – An Atomic View,

2

- An external electric field is applied
- This produces a torque on the molecules
- The molecules partially align with the electric field
The degree of alignment of the molecules with the field depends on temperature and the magnitude of the field.

In general,

- the alignment increases with decreasing temperature
- the alignment increases with increasing field strength
If the molecules of the dielectric are nonpolar molecules, the electric field produces some charge separation.

This produces an induced dipole moment.

The effect is then the same as if the molecules were polar.
Dielectrics – An Atomic View, final

- An external field can polarize the dielectric whether the molecules are polar or nonpolar.
- The charged edges of the dielectric act as a second pair of plates producing an induced electric field in the direction opposite the original electric field.
Induced Charge and Field

- The electric field due to the plates is directed to the right and it polarizes the dielectric.
- The net effect on the dielectric is an induced surface charge that results in an induced electric field.
- If the dielectric were replaced with a conductor, the net field between the plates would be zero.
Example

\[ E = \frac{E_0}{K}, \quad K = \frac{C}{C_0} \quad \Rightarrow \quad E = E_0 - E_{\text{ind}} \quad \Rightarrow \quad E = \frac{E_0}{K} = \frac{\sigma}{K \varepsilon_0} \]

\[ E_0 = \frac{\sigma}{\varepsilon_0} \quad \Rightarrow \quad \frac{\sigma}{K \varepsilon_0} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_{\text{ind}}}{\varepsilon_0} \]

- Charge density induced on the dielectric

\[ \sigma_{\text{ind}} = \left( \frac{K - 1}{K} \right) \sigma \]
Example 26.10

- Find capacitance of device

\[ C_1 = \frac{K \varepsilon_0 A}{d/3} \]
\[ C_2 = \frac{K \varepsilon_0 A}{2d/3} \]

- Equivalent capacitance is equal \( C_0 \) when \( k=1 \)

\[ \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{K \varepsilon_0 A} + \frac{2d/3}{\varepsilon_0 A} = \frac{d}{3\varepsilon_0 A} \left( \frac{1}{K} + 2 \right) \]

\[ C = \frac{3K}{2k+1} \frac{K \varepsilon_0 A}{d} \]
Example

- Find capacitance of device

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{\varepsilon_0 A}{d-a} + \frac{\varepsilon_0 A}{d-b-a} = \frac{\varepsilon_0 A}{d-a}
\]

- Equivalent capacitance is independent of the value of \( b \)
The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

\[ C_p = \frac{\varepsilon_0 \left( A/2 \right)}{d} + \frac{K \varepsilon_0 \left( A/2 \right)}{d} = \left( \frac{K + 1}{2} \right) \frac{\varepsilon_0 A}{d} \]

If \( f \) is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

\[ \frac{1}{C_s} = \frac{f d}{K \varepsilon_0 A} + \frac{(1-f) d}{\varepsilon_0 A} = \left[ \frac{f + K(1-f)}{K} \right] \frac{d}{\varepsilon_0 A} \]
Problem, cont

This gives for connection in series

\[ C_s = \left[ \frac{K}{f + K(1-f)} \right] \epsilon_0 A \frac{d}{2} \]

- From condition: \( C_p = C_s \)

\[ \frac{K+1}{2} = \frac{K}{f + K(1-f)} \]

\[ (K+1)[f + K(1-f)] = 2K \]

- For \( K = 2.00 \)

\[ 3.00 \left[ 2.00 - (1.00) f \right] = 4.00 \]

\[ f = \frac{2}{3} \]
Problem

Due to symmetry potential difference across $3C$ is zero and the equivalent connection is

\[
\begin{align*}
\text{Equivalent capacitance is} \\
C_{eq} &= \left( \frac{1}{2C} + \frac{1}{4C} \right)^{-1} = \frac{8}{6} C = \frac{4}{3} C
\end{align*}
\]
Problem

Find a) equivalent capacitance b) potential across capacitor c) charge on capacitor d) total stored energy

- a) Equivalent capacitance is

\[ C = \left( \frac{1}{3.00} + \frac{1}{6.00} \right)^{-1} + \left( \frac{1}{2.00} + \frac{1}{4.00} \right)^{-1} = 3.33 \, \mu F \]
b) potential across each capacitor

\[
\Delta V_3 = \frac{Q_3}{C_3} = \frac{180 \ \mu C}{3.00 \ \mu F} = 60.0 \ \text{V}
\]

\[
\Delta V_6 = \frac{Q_6}{C_6} = \frac{180 \ \mu C}{6.00 \ \mu F} = 30.0 \ \text{V}
\]

\[
\Delta V_2 = \frac{Q_2}{C_2} = \frac{120 \ \mu C}{2.00 \ \mu F} = 60.0 \ \text{V}
\]

\[
\Delta V_4 = \frac{Q_4}{C_4} = \frac{120 \ \mu C}{4.00 \ \mu F} = 30.0 \ \text{V}
\]
c) charge on each capacitor

\[ Q_{ac} = C_{ac} \Delta V_{ac} = (2.00 \ \mu F)(90.0 \ \text{V}) = 180 \ \mu \text{C} \]

\[ Q_3 = Q_6 = 180 \ \mu \text{C} \]

\[ Q_{df} = C_{df} \Delta V_{df} = (1.33 \ \mu F)(90.0 \ \text{V}) = 120 \ \mu \text{C} \]

- d) Stored energy

\[ U_T = \frac{1}{2} C_{eq} (\Delta V)^2 = \frac{1}{2} (3.33 \times 10^{-6}) (90.0 \ \text{V})^2 = 13.4 \ \text{mJ} \]
Problem

Show that

\[ U = \frac{k_e Q^2}{2R} \]

- Stored energy

\[ U = \frac{1}{2} C (\Delta V)^2 \quad C = 4\pi \varepsilon_0 \quad R = \frac{R}{k_e} \quad \Delta V = \frac{k_e Q}{R} - 0 = \frac{k_e Q}{R} \]

\[ U = \frac{1}{2} \left( \frac{R}{k_e} \right) \left( \frac{k_e Q}{R} \right)^2 = \frac{k_e Q^2}{2R} \]
Problem:  \( C = 3.00 \ \mu F \)

- Stored energy for \( V=12V \) and \( V= 6V \)

(a)  
\[
U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} (3.00 \ \mu F)(12.0 \ V)^2 = 216 \ \mu J
\]

(b)  
\[
U = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} (3.00 \ \mu F)(6.00 \ V)^2 = 54.0 \ \mu J
\]
Problem

- Find equivalent capacitor for infinite number of compartments

\[
\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C + C_0} + \frac{1}{C_0} = \frac{C + C_0 + C_0 + C + C_0}{C_0 \left( C + C_0 \right)}
\]
Problem, cont

Find equivalent capacitor for infinite number of compartments

\[ C_0 C + C_0^2 = 2C^2 + 3C_0 C \]
\[ 2C^2 + 2C_0 C - C_0^2 = 0 \]
\[ C = \frac{-2C_0 \pm \sqrt{4C_0^2 + 4(2C_0^2)}}{4} \]
\[ C = \frac{C_0}{2} \left( \sqrt{3} - 1 \right) \]
Problem

- Find equivalent capacitor.
- Potential difference across the capacitor must be zero by symmetry
Problem

- Find equivalent capacitor for infinite number of compartments
## Geometry of Some Capacitors

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Capacitance</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isolated sphere of radius $R$ (second spherical conductor assumed to have infinite radius)</td>
<td>$C = 4\pi\epsilon_0 R$</td>
<td>26.2</td>
</tr>
<tr>
<td>Parallel-plate capacitor of plate area $A$ and plate separation $d$</td>
<td>$C = \epsilon_0 \frac{A}{d}$</td>
<td>26.3</td>
</tr>
<tr>
<td>Cylindrical capacitor of length $\ell$ and inner and outer radii $a$ and $b$, respectively</td>
<td>$C = \frac{\ell}{2\epsilon_0 \ln(b/a)}$</td>
<td>26.4</td>
</tr>
<tr>
<td>Spherical capacitor with inner and outer radii $a$ and $b$, respectively</td>
<td>$C = \frac{ab}{\epsilon_0 (b - a)}$</td>
<td>26.6</td>
</tr>
</tbody>
</table>
Chapter 26

Capacitance and Dielectrics
A capacitor stores charge $Q$ at a potential difference $\Delta V$. If the voltage applied by a battery to the capacitor is doubled to $2\Delta V$:

(a) the capacitance falls to half its initial value and the charge remains the same

(b) the capacitance and the charge both fall to half their initial values

(c) the capacitance and the charge both double

(d) the capacitance remains the same and the charge doubles
Quick Quiz 26.1

Answer: (d). The capacitance is a property of the physical system and does not vary with applied voltage. According to Equation 26.1, if the voltage is doubled, the charge is doubled.
Many computer keyboard buttons are constructed of capacitors, as shown in the figure below. When a key is pushed down, the soft insulator between the movable plate and the fixed plate is compressed. When the key is pressed, the capacitance

(a) increases

(b) decreases

(c) changes in a way that we cannot determine because the complicated electric circuit connected to the keyboard button may cause a change in $\Delta V$. 
Quick Quiz 26.2

Answer: (a). When the key is pressed, the plate separation is decreased and the capacitance increases. Capacitance depends only on how a capacitor is constructed and not on the external circuit.
Quick Quiz 26.3

Two capacitors are identical. They can be connected in series or in parallel. If you want the *smallest* equivalent capacitance for the combination, you should connect them in

(a) series

(b) parallel

(c) Either combination has the same capacitance.
Answer: (a). When connecting capacitors in series, the inverses of the capacitances add, resulting in a smaller overall equivalent capacitance.
Consider the two capacitors in question 3 again. Each capacitor is charged to a voltage of 10 V. If you want the largest combined potential difference across the combination, you should connect them in

(a) series

(b) parallel

(c) Either combination has the same potential difference.
Answer: (a). When capacitors are connected in series, the voltages add, for a total of 20 V in this case. If they are combined in parallel, the voltage across the combination is still 10 V.
Quick Quiz 26.5

You have three capacitors and a battery. In which of the following combinations of the three capacitors will the maximum possible energy be stored when the combination is attached to the battery?

(a) series
(b) parallel
(c) Both combinations will store the same amount of energy.
Quick Quiz 26.5

Answer: (b). For a given voltage, the energy stored in a capacitor is proportional to \( C \): \( U = C(\Delta V)^2/2 \). Thus, you want to maximize the equivalent capacitance. You do this by connecting the three capacitors in parallel, so that the capacitances add.
Quick Quiz 26.6

You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires connected to the plates from touching each other. When you pull the plates apart to a larger separation, do the following quantities increase, decrease, or stay the same? (a) $C$; (b) $Q$; (c) $E$ between the plates; (d) $\Delta V$; (e) energy stored in the capacitor.
Quick Quiz 26.6

Answer: (a) $C$ decreases (Eq. 26.3). (b) $Q$ stays the same because there is no place for the charge to flow. (c) $E$ remains constant (see Eq. 24.8 and the paragraph following it). (d) $\Delta V$ increases because $\Delta V = Q/C$, $Q$ is constant (part b), and $C$ decreases (part a). (e) The energy stored in the capacitor is proportional to both $Q$ and $\Delta V$ (Eq. 26.11) and thus increases. The additional energy comes from the work you do in pulling the two plates apart.
Quick Quiz 26.7

Repeat Quick Quiz 26.6, but this time answer the questions for the situation in which the battery remains connected to the capacitor while you pull the plates apart.
Quick Quiz 26.7

Answer: (a) $C$ decreases (Eq. 26.3). (b) $Q$ decreases. The battery supplies a constant potential difference $\Delta V$; thus, charge must flow out of the capacitor if $C = Q / \Delta V$ is to decrease. (c) $E$ decreases because the charge density on the plates decreases. (d) $\Delta V$ remains constant because of the presence of the battery. (e) The energy stored in the capacitor decreases (Eq. 26.11).
Quick Quiz 26.8

If you have ever tried to hang a picture or a mirror, you know it can be difficult to locate a wooden stud in which to anchor your nail or screw. A carpenter’s stud-finder is basically a capacitor with its plates arranged side by side instead of facing one another, as shown in the figure below. When the device is moved over a stud, the capacitance will:

(a) increase
(b) decrease
Quick Quiz 26.8

Answer: (a). The dielectric constant of wood (and of all other insulating materials, for that matter) is greater than 1; therefore, the capacitance increases (Eq. 26.14). This increase is sensed by the stud-finder's special circuitry, which causes an indicator on the device to light up.
Quick Quiz 26.9

A fully charged parallel-plate capacitor remains connected to a battery while you slide a dielectric between the plates. Do the following quantities increase, decrease, or stay the same? (a) $C$; (b) $Q$; (c) $E$ between the plates; (d) $\Delta V$. 
Quick Quiz 26.9

Answer: (a) $C$ increases (Eq. 26.14). (b) $Q$ increases. Because the battery maintains a constant $\Delta V$, $Q$ must increase if $C$ increases. (c) $E$ between the plates remains constant because $\Delta V = Ed$ and neither $\Delta V$ nor $d$ changes. The electric field due to the charges on the plates increases because more charge has flowed onto the plates. The induced surface charges on the dielectric create a field that opposes the increase in the field caused by the greater number of charges on the plates (see Section 26.7). (d) The battery maintains a constant $\Delta V$. 
The positive charge is the end view of a positively charged glass rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod’s electric field positive, negative or zero?

1. Positive
2. Negative
3. Zero
The positive charge is the end view of a positively charged glass rod. A negatively charged particle moves in a circular arc around the glass rod. Is the work done on the charged particle by the rod’s electric field positive, negative or zero?

1. Positive
2. Negative
3. Zero

The correct answer is 3. Zero.
Rank in order, from largest to smallest, the potential energies $U_a$ to $U_d$ of these four pairs of charges. Each + symbol represents the same amount of charge.

1. $U_a = U_b > U_c = U_d$
2. $U_a = U_c > U_b = U_d$
3. $U_b = U_d > U_a = U_c$
4. $U_d > U_b = U_c > U_a$
5. $U_d > U_c > U_b > U_a$
Rank in order, from largest to smallest, the potential energies $U_a$ to $U_d$ of these four pairs of charges. Each + symbol represents the same amount of charge.

1. $U_a = U_b > U_c = U_d$
2. $U_a = U_c > U_b = U_d$
3. $U_b = U_d > U_a = U_c$
4. $U_d > U_b = U_c > U_a$
5. $U_d > U_c > U_b > U_a$
A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton

1. moves toward A with an increasing speed.
2. moves toward A with a steady speed.
3. remains at rest at B.
4. moves toward C with a steady speed.
5. moves toward C with an increasing speed.
A proton is released from rest at point B, where the potential is 0 V. Afterward, the proton

1. moves toward A with an increasing speed.
2. moves toward A with a steady speed.
3. remains at rest at B.
4. moves toward C with a steady speed.
5. moves toward C with an increasing speed.
Rank in order, from largest to smallest, the potentials $V_a$ to $V_e$ at the points a to e.

1. $V_a = V_b = V_c = V_d = V_e$
2. $V_a = V_b > V_c > V_d = V_e$
3. $V_d = V_e > V_c > V_a = V_b$
4. $V_b = V_c = V_e > V_a = V_d$
5. $V_a = V_b = V_d = V_e > V_c$
Rank in order, from largest to smallest, the potentials $V_a$ to $V_e$ at the points a to e.

1. $V_a = V_b = V_c = V_d = V_e$
2. $V_a = V_b > V_c > V_d = V_e$
3. $V_d = V_e > V_c > V_a = V_b$
4. $V_b = V_c = V_e > V_a = V_d$
5. $V_a = V_b = V_d = V_e > V_c$
Rank in order, from largest to smallest, the potential differences $\Delta V_{12}$, $\Delta V_{13}$, and $\Delta V_{23}$ between points 1 and 2, points 1 and 3, and points 2 and 3.

1. $\Delta V_{12} > \Delta V_{13} = \Delta V_{23}$
2. $\Delta V_{13} > \Delta V_{12} > \Delta V_{23}$
3. $\Delta V_{13} > \Delta V_{23} > \Delta V_{12}$
4. $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$
5. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$
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4. $\Delta V_{13} = \Delta V_{23} > \Delta V_{12}$
5. $\Delta V_{23} > \Delta V_{12} > \Delta V_{13}$