

Variable Replacements and Other Graphing Techniques

Relation Replacements			
Variable	Replacement	Description of Replacement Effect on any Equation Relating x and y	
1	x	$x + k$	horizontal shift; direction is the opposite of k 's sign
2	y	$y + k$	vertical shift; direction is the opposite of k 's sign
3	x	$-x$	reflect all parts over the y -axis
4	y	$-y$	reflect all parts over the x -axis
5	x	kx	$0 < k < 1$; horizontal expansion by a factor of $\frac{1}{k}$
6	x	kx	$k > 1$; horizontal contraction by a factor of $\frac{1}{k}$
7	y	ky	$0 < k < 1$; vertical expansion by a factor of $\frac{1}{k}$
8	y	ky	$k > 1$; vertical contraction by a factor of $\frac{1}{k}$
9	x	$ x $	No effect where $x \geq 0$; omit where $x < 0$; reflect over y -axis where $x > 0$
10	y	$ y $	No effect where $y \geq 0$; omit where $y < 0$; reflect over x -axis where $y > 0$

Function Replacements	
11	Changing $y = f(x)$ to the equation $y = f(x)$ is a variable replacement as described in 9 above
12	Changing $y = f(x)$ to the equation $y = f(x) $ is not a variable replacement, but the effect is to leave any parts where $y \geq 0$ alone, and it reflects any parts where $y < 0$ up across the x -axis

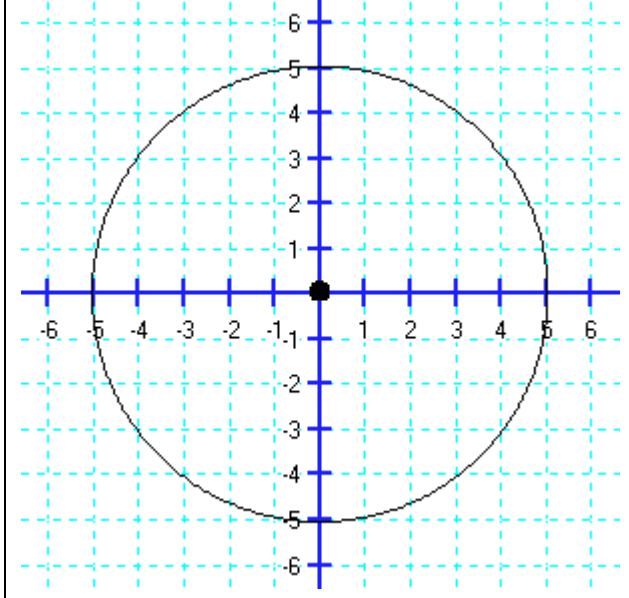
The problems given in the table below have been made so that students may practice what is described in the above table. You should try making each graph before looking at the solution graphs.

Sample Problems			
#	Original Equation	New Equation	Replacement Type (see above table)
1	$x^2 + y^2 = 25$	$(x + 3)^2 + (y - 2)^2 = 25$	1 and 2
2	$y = \sqrt[3]{x}$	$y = \sqrt[3]{8 - x}$	1 and 3
3	$y = x^2$	$y = -(x - 2)^2 + 4$	1 and 2 and 4
4	$y = x$	$y = \frac{1}{3}x$	5
5	$x^2 + y^2 = 1$	$(3x)^2 + y^2 = 1$	6
6	$x^2 + y^2 = \frac{1}{9}$	$x^2 + (\frac{1}{3}y)^2 = \frac{1}{9}$	7
7	$x^2 + y^2 = 16$	$x^2 + (2y)^2 = 16$	8
8	$y = \sqrt[3]{x + 3}$	$y = \sqrt[3]{ x + 3}$	9
9	$y = x^3$	$ y = x^3$	10
10	$y = (x - 2)^2 - 4$	$ y = (x - 2)^2 - 4$	11
11	$y = (x - 2)^2 - 4$	$y = (x - 2)^2 - 4 $	12
12	$(x - 4)^2 + (y - 4)^2 = 25$	$(x - 4)^2 + (y - 4)^2 = 25$	9 and 10

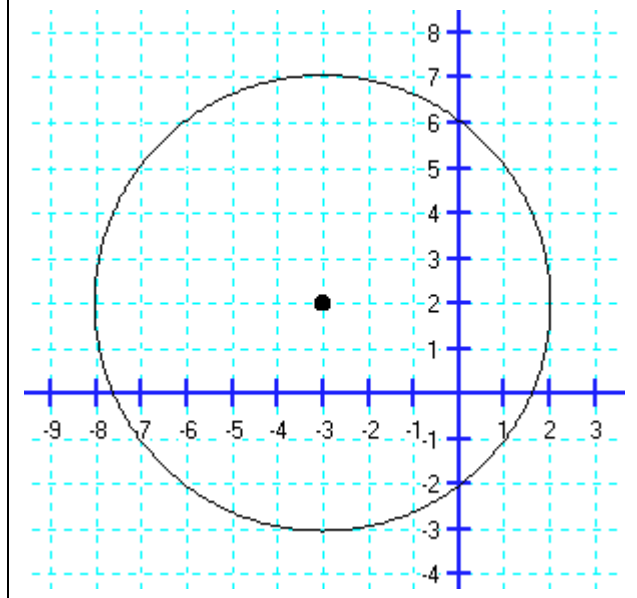
Solutions to the above problems appear on the following pages.

Problem #1**Original Equation**

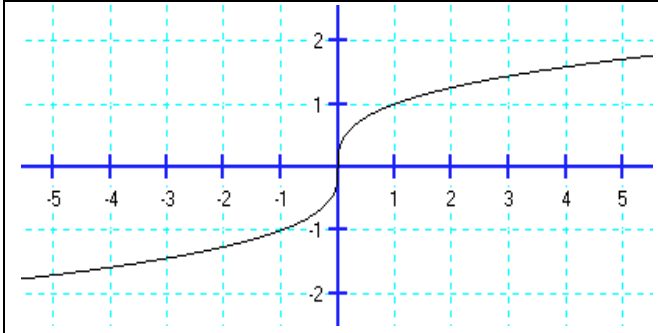
$$x^2 + y^2 = 25$$

**New Equation**

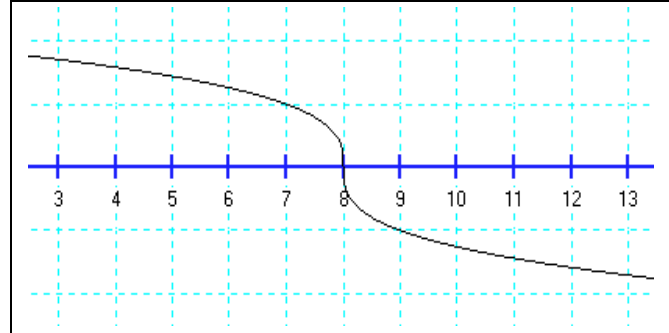
$$(x + 3)^2 + (y - 2)^2 = 25$$

**Problem #2****Original Equation**

$$y = \sqrt[3]{x}$$

**New Equation**

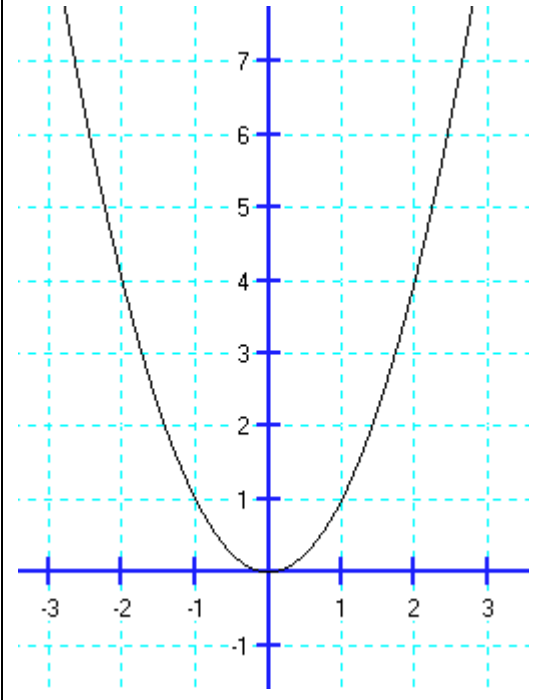
$$y = \sqrt[3]{8 - x}$$



Problem #3

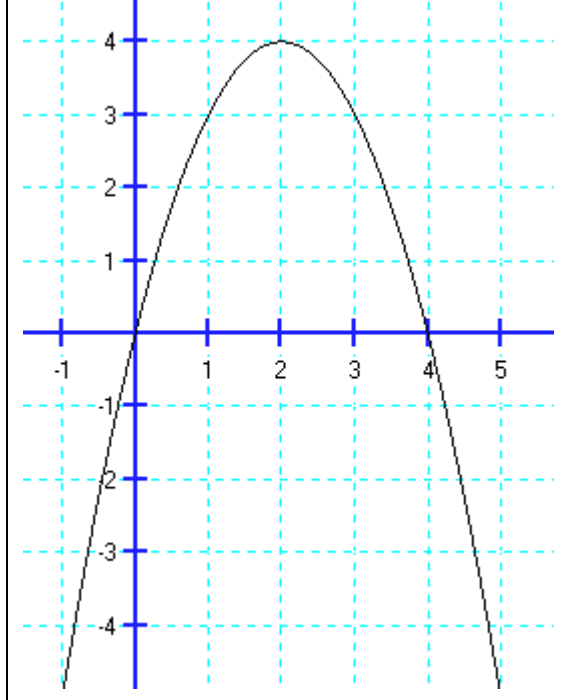
Original Equation

$$y = x^2$$



New Equation

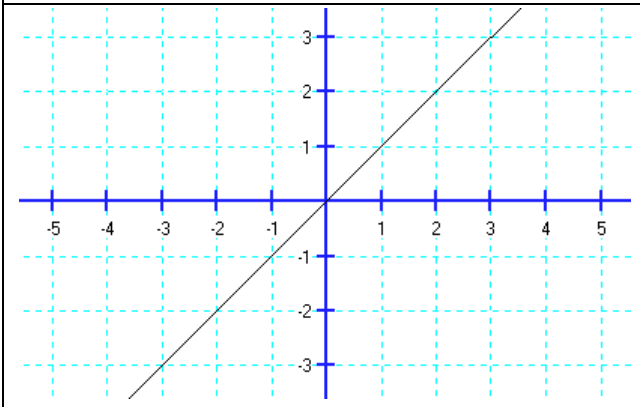
$$y = -(x - 2)^2 + 4$$



Problem #4

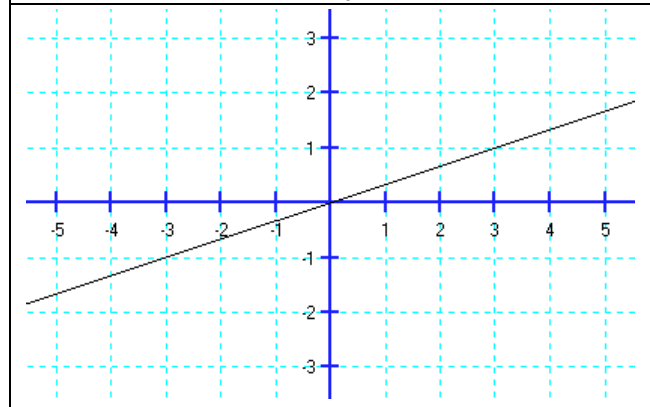
Original Equation

$$y = x$$



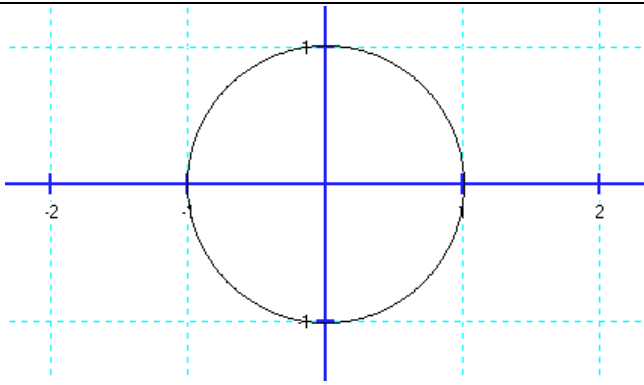
New Equation

$$y = \frac{1}{3}x$$

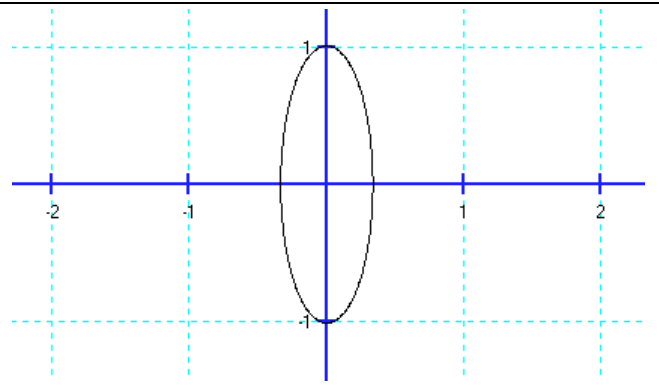


Problem #5**Original Equation**

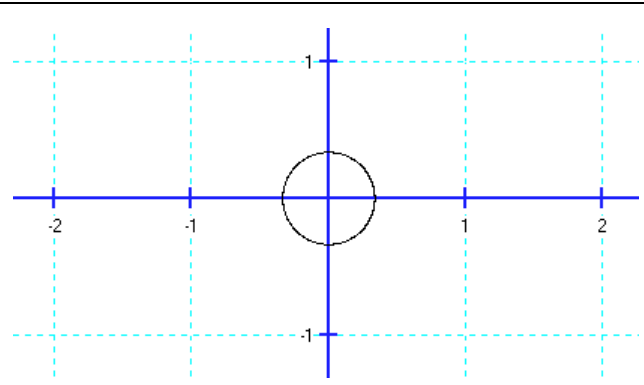
$$x^2 + y^2 = 1$$

**New Equation**

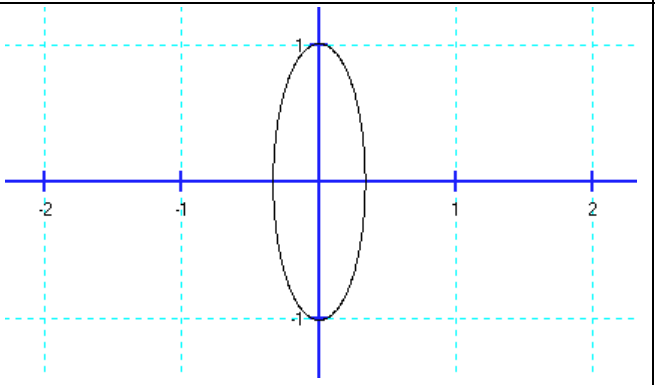
$$(3x)^2 + y^2 = 1$$

**Problem #6****Original Equation**

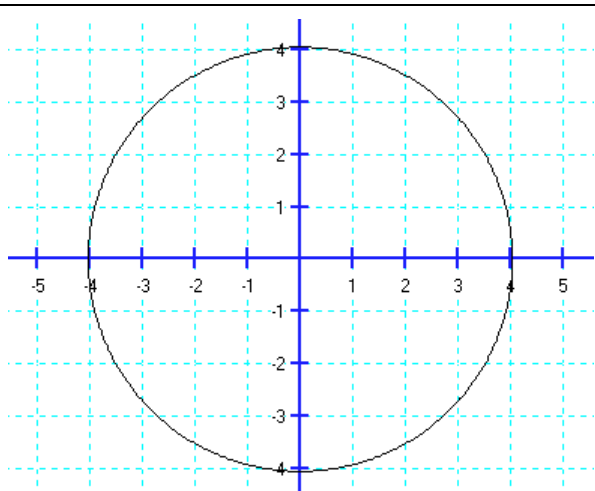
$$x^2 + y^2 = \frac{1}{9}$$

**New Equation**

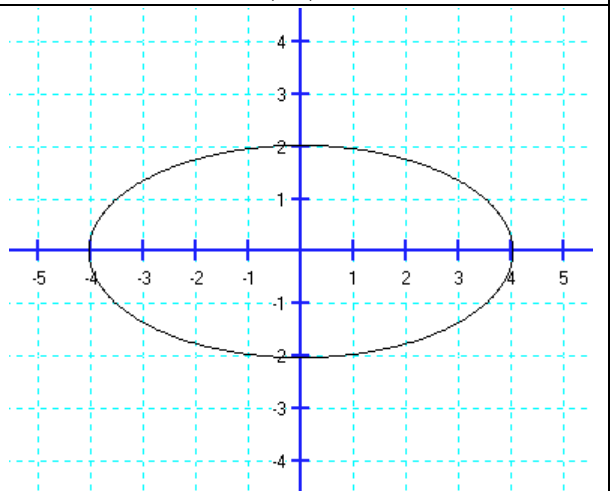
$$x^2 + \left(\frac{1}{3}y\right)^2 = \frac{1}{9}$$

**Problem #7****Original Equation**

$$x^2 + y^2 = 16$$

**New Equation**

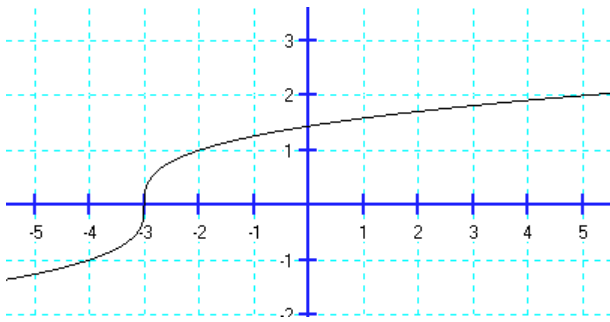
$$x^2 + (2y)^2 = 16$$



Problem #8

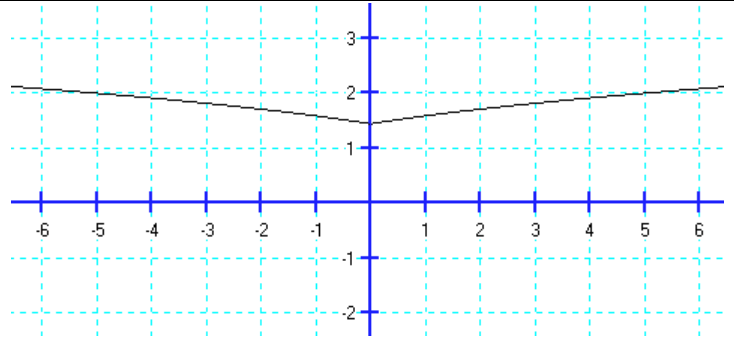
Original Equation

$$y = \sqrt[3]{x + 3}$$



New Equation

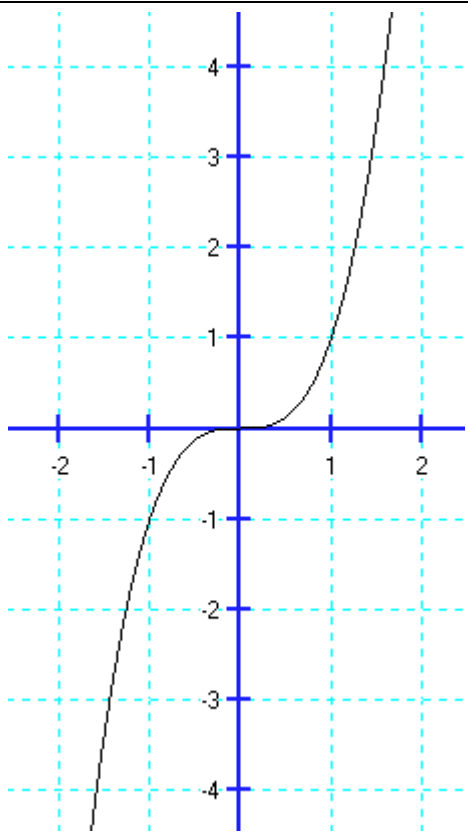
$$y = \sqrt[3]{|x| + 3}$$



Problem #9

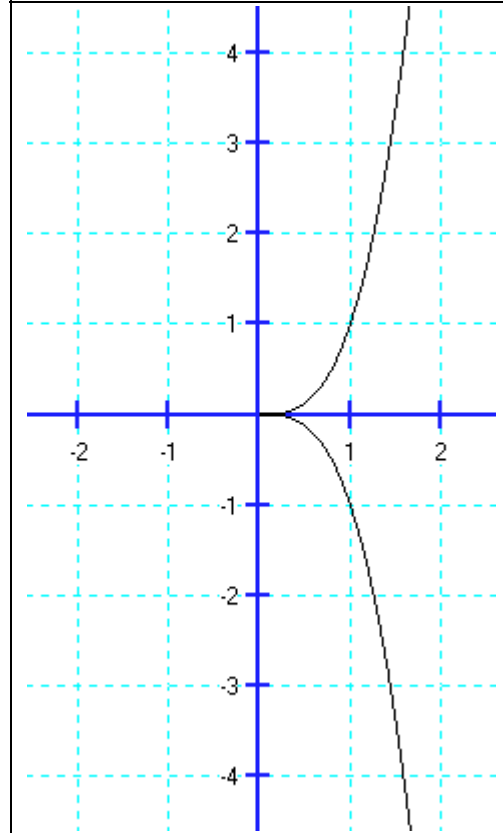
Original Equation

$$y = x^3$$



New Equation

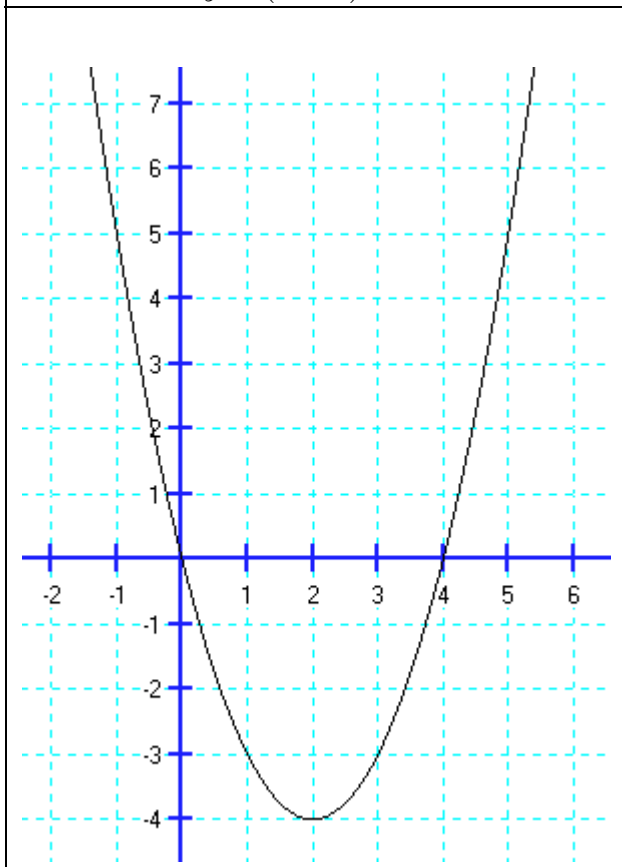
$$|y| = x^3$$



Problem #10

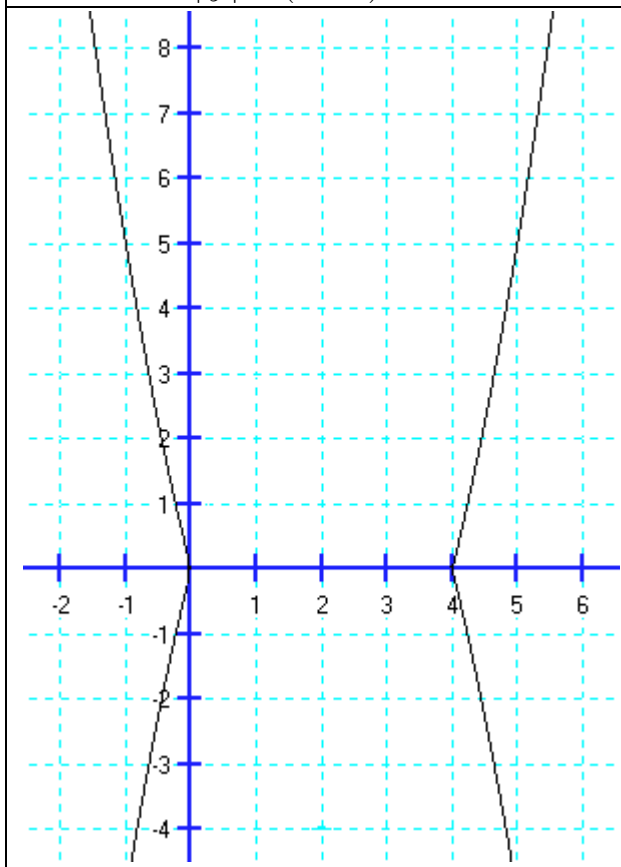
Original Equation

$$y = (x - 2)^2 - 4$$



New Equation

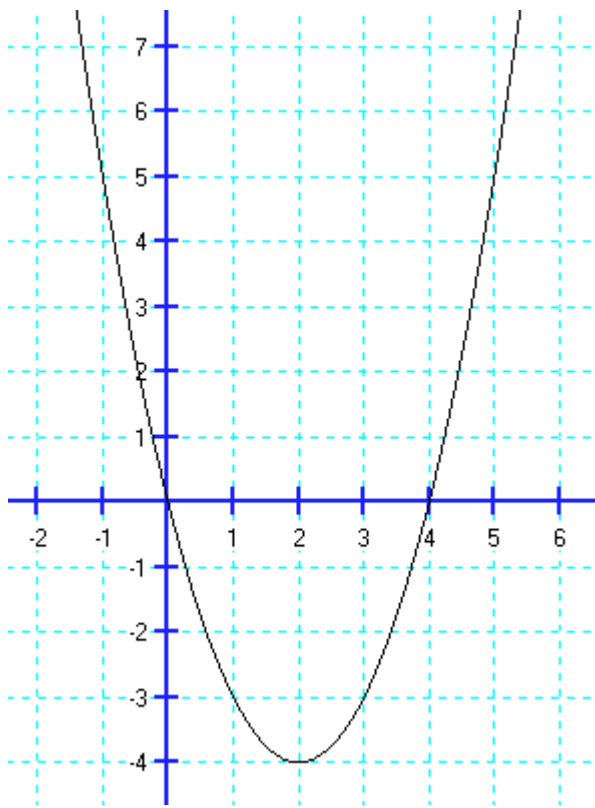
$$|y| = (x - 2)^2 - 4$$



Problem #11

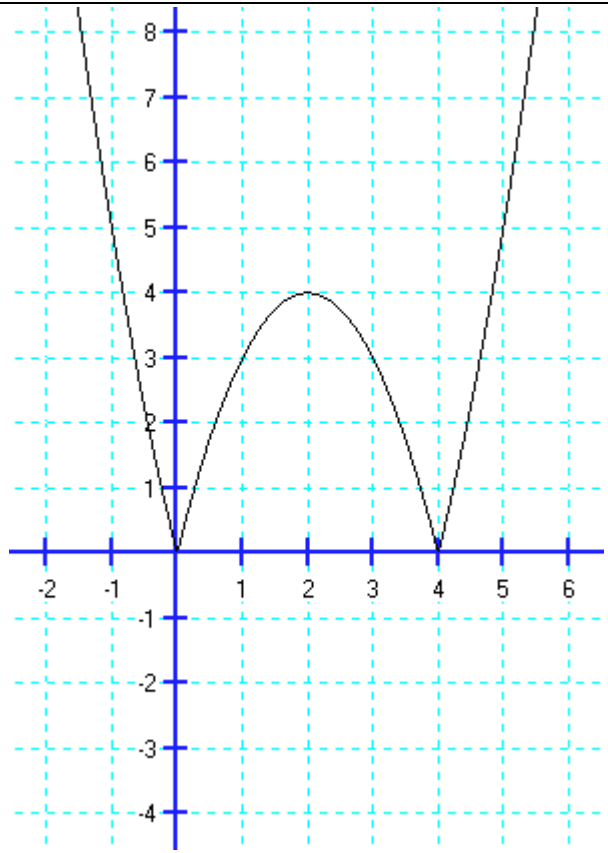
Original Equation

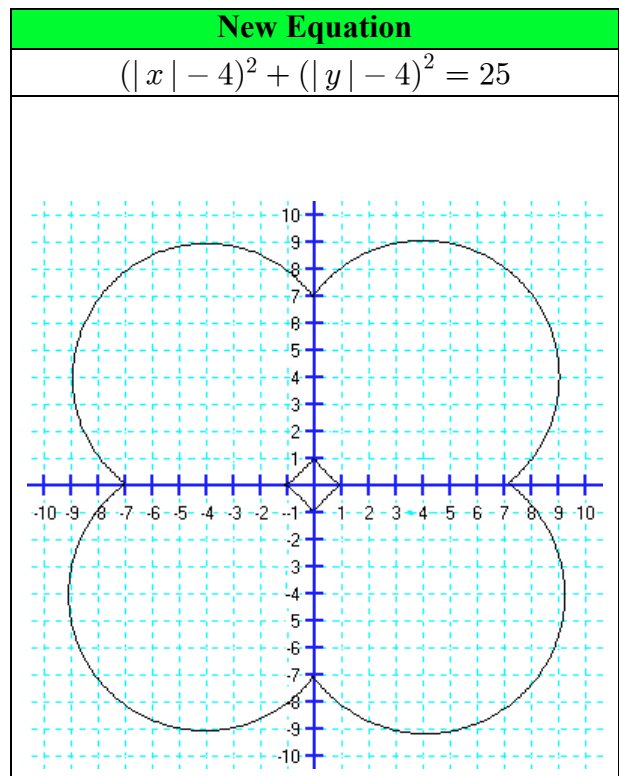
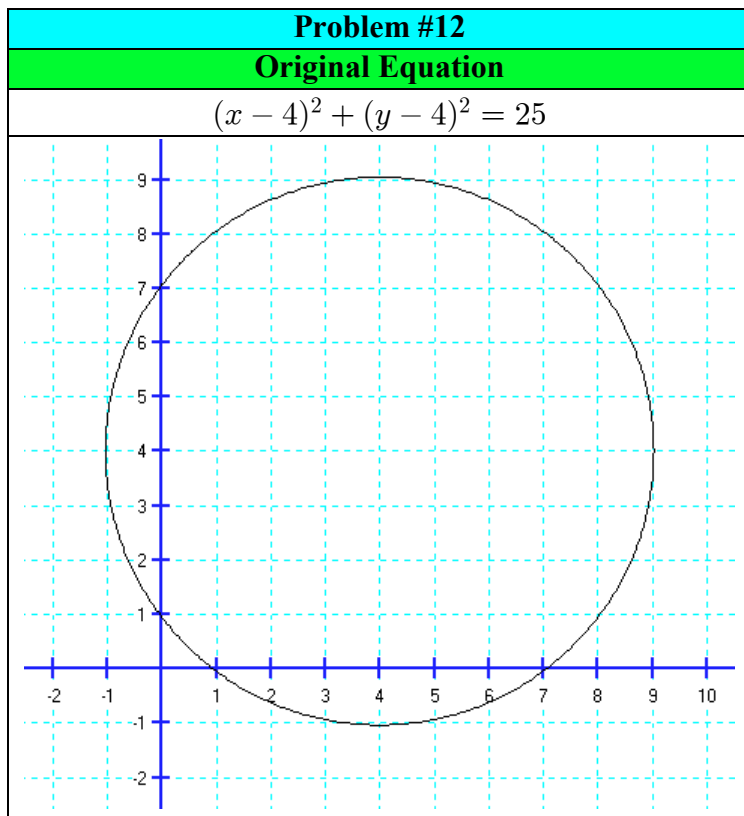
$$y = (x - 2)^2 - 4$$



New Equation

$$y = |(x - 2)^2 - 4|$$





There may be several different sequences of replacement operations that all lead to the same equation or graph. For sample problem #8 you could perform either of the following two sequences.

	operation description	1st sequence
1.	original equation	$y = \sqrt[3]{x}$
2.	replace x with $x + 8$	$y = \sqrt[3]{x + 8}$
3.	replace x with $-x$	$y = \sqrt[3]{-x + 8}$
4.	addition is commutative	$y = \sqrt[3]{8 + (-x)}$
5.	convert to subtraction	$y = \sqrt[3]{8 - x}$

	operation description	2nd sequence
1.	original equation	$y = \sqrt[3]{x}$
2.	replace x with $-x$	$y = \sqrt[3]{-x}$
3.	replace x with $x - 8$	$y = \sqrt[3]{-(x - 8)}$
4.	rewrite and simplify	$y = \sqrt[3]{8 - x}$

The following shows two different sequences of operations for problem #3. Other sequences than these two are also possible.

	operation description	1st sequence
1.	original equation	$y = x^2$
2.	replace y with $-y$	$-y = x^2$
3.	replace x with $x - 2$	$y = -(x - 2)^2$
4.	replace y with $y - 4$	$y - 4 = -(x - 2)^2$
5.	add 4 to both sides	$y = -(x - 2)^2 + 4$

	operation description	2nd sequence
1.	original equation	$y = x^2$
2.	replace x with $x - 2$	$y = (x - 2)^2$
3.	replace y with $y + 4$	$y + 4 = (x - 2)^2$
4.	subtract 4 on both sides	$y = (x - 2)^2 - 4$
5.	replace y with $-y$	$-y = (x - 2)^2 - 4$
6.	multiply both sides by -1	$y = -(x - 2)^2 + 4$