# TRIGONOMETRIC IDENTITIES

There are many relationships among the trigonometric functions. The following list contains some of the more important identities.

Pythagorean Relationships:

$$sin^2(\theta) + cos^2(\theta) = 1$$
$$1 + cot^2(\theta) = csc^2(\theta)$$
$$tan^2(\theta) + 1 = sec^2(\theta)$$

Negative, Supplementary, and Complementary Angle Relationships:

$$sin(-\theta) = -sin(\theta) \qquad sin(\pi - \theta) = sin(\theta) \qquad sin\left(\frac{\pi}{2} - \theta\right) = cos(\theta)$$
$$cos(-\theta) = cos(\theta) \qquad cos(\pi - \theta) = -cos(\theta) \qquad cos\left(\frac{\pi}{2} - \theta\right) = sin(\theta)$$
$$tan(-\theta) = -tan(\theta) \qquad tan(\pi - \theta) = -tan(\theta) \qquad tan\left(\frac{\pi}{2} - \theta\right) = cot(\theta)$$

Quarter Circle and Half-Circle Relationships:

$$sin\left(\theta \pm \frac{\pi}{2}\right) = \pm\, cos(\theta) \qquad sin(\theta \pm \pi) = -sin(\theta)$$
$$cos\left(\theta \pm \frac{\pi}{2}\right) = \mp\, sin(\theta) \qquad cos(\theta \pm \pi) = -cos(\theta)$$
$$tan\left(\theta \pm \frac{\pi}{2}\right) = -cot(\theta) \qquad tan(\theta \pm \pi) = tan(\theta)$$

Angle Addition Formulas:

$$sin(\alpha \pm \beta) = sin(\alpha) \cdot cos(\beta) \pm cos(\alpha) \cdot sin(\beta)$$

$$cos(\alpha \pm \beta) = cos(\alpha) \cdot cos(\beta) \mp sin(\alpha) \cdot sin(\beta)$$

$$tan(\alpha \pm \beta) = \frac{tan(\alpha) \pm tan(\beta)}{1 \mp tan(\alpha) \cdot tan(\beta)}$$

Double Angle Formulas:

$$sin(2 \cdot \theta) = 2 \cdot sin(\theta) \cdot cos(\theta) \qquad tan(2 \cdot \theta) = \frac{2 \cdot tan(\theta)}{1 - tan^2(\theta)}$$

$$cos(2 \cdot \theta) = cos^2(\theta) - sin^2(\theta) = 1 - 2 \cdot sin^2(\theta) = 2 \cdot cos^2(\theta) - 1$$

Half-Angle Formulas:

$$sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - cos(\theta)}{2}}$$

$$cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + cos(\theta)}{2}}$$

$$tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - cos(\theta)}{1 + cos(\theta)}} = \frac{1 - cos(\theta)}{sin(\theta)} = \frac{sin(\theta)}{1 + cos(\theta)}$$

Product Identities:

$$2 \cdot sin(\alpha) \cdot cos(\beta) = sin(\alpha + \beta) + sin(\alpha - \beta)$$
$$2 \cdot cos(\alpha) \cdot sin(\beta) = sin(\alpha + \beta) - sin(\alpha - \beta)$$
$$2 \cdot cos(\alpha) \cdot cos(\beta) = cos(\alpha + \beta) + cos(\alpha - \beta)$$
$$2 \cdot sin(\alpha) \cdot sin(\beta) = cos(\alpha - \beta) - cos(\alpha + \beta)$$

Factoring Identities:

$$sin(\alpha) + sin(\beta) = 2 \cdot cos\left(\frac{\alpha - \beta}{2}\right) \cdot sin\left(\frac{\alpha + \beta}{2}\right)$$
$$sin(\alpha) - sin(\beta) = 2 \cdot cos\left(\frac{\alpha + \beta}{2}\right) \cdot sin\left(\frac{\alpha - \beta}{2}\right)$$
$$cos(\alpha) + cos(\beta) = 2 \cdot cos\left(\frac{\alpha + \beta}{2}\right) \cdot cos\left(\frac{\alpha - \beta}{2}\right)$$
$$cos(\alpha) - cos(\beta) = 2 \cdot sin\left(\frac{\alpha + \beta}{2}\right) \cdot sin\left(\frac{\alpha - \beta}{2}\right)$$

Law of Sines:

$$\frac{sin(\alpha)}{a} = \frac{sin(\beta)}{b} = \frac{sin(\gamma)}{c}$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot cos(\alpha)$$
$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot cos(\beta)$$
$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot cos(\gamma)$$

Analytic Relationships With the Unit Circle:
Figure 1. ⇓

Laws of Sines & Cosines
Figure 2. ⇓