Objective: Learning To Solve Uniform Motion Problems

Imagine that you are on a trip, traveling on the highway at 55 miles per hour, using cruise control. After three hours of traveling, how far have you traveled? Because you are using cruise control, your speed remains constant. When the speed does not vary, the vehicle is in uniform motion. We can calculate the solution to the above uniform motion problem using the formula \( r \cdot t = d \), where \( r \) is the constant rate of the vehicle, \( t \) is the time and \( d \) is the distance traveled. The units used for \( d \), \( r \), and \( t \) must all be consistent. For example, if \( r \) represents miles per hour, then the units for \( t \) must be hours and the units for \( d \) must be miles.

\[
\begin{align*}
\quad \quad \quad \quad \quad r \cdot t &= d \\
55 \cdot 3 &= d \\
165 &= d
\end{align*}
\]

Substitute known values.

Example 1: Another problem about uniform motion in the opposite direction
A minivan leaves San Francisco, traveling toward Los Angeles at a speed of 60 miles per hour. At the same time, a professional bicyclist leaves Los Angeles, traveling toward San Francisco at a speed of 20 miles per hour. If Los Angeles and San Francisco are about 400 miles apart, how many hours will it take them to meet?

Solution: Assign \( t \) to be the number of hours it will take them to meet. Because the vehicles are going in opposite directions, the sum of the algebraic expressions which represent the distances each travels equals the total distance traveled.

Step 1. Let \( t \) = the time it takes the vehicles to meet
Step 2. \( 60t = \) Minivan distance and \( 20t = \) bicycle distance

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minivan</td>
<td>60</td>
<td>( t )</td>
<td>( 60t )</td>
</tr>
<tr>
<td>Bicycle</td>
<td>20</td>
<td>( t )</td>
<td>( 20t )</td>
</tr>
</tbody>
</table>

Total \( \rightarrow \) 400

Minivan's distance + Bicycle's distance = Total distance

\[
\begin{align*}
60t + 20t &= 400 \\
80t &= 400 \\
\quad t &= 5
\end{align*}
\]

Step 4.

Step 5. Check \( \checkmark \) is left for the reader.

Step 6. The vehicles meet in five hours.

WorkOut 1: Two Amtrak trains are 480 miles apart. The second train is going 10 miles per hour faster than the first. After four hours, they meet at a station. How fast is each train going?

Your Solution:
Example 2: A round trip problem

A community college earth science club goes by van on a trip from Los Angeles to Morro Bay and back. On the way to Morro Bay, they drove 50 miles per hour. On the return trip home, they drove 40 miles per hour. If the round trip took 9 hours, how far is Morro Bay from Los Angeles?

Solution: In a round trip problem, the distance to the destination is the same as the distance returning from the destination. In the problem, we assign $t$ to be the time it takes to go to Morro Bay and $(9 - t)$, the total time minus the time to Morro Bay, to be the time it takes to return. Once we find the time, we can determine the distance.

Step 1. Let $t =$ the time it takes to reach Morro Bay
Step 2. Then $9 - t =$ the time it takes to return
Step 3.

<table>
<thead>
<tr>
<th>Trip</th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>To Morro Bay</td>
<td>50</td>
<td>$t$</td>
<td>$50t$</td>
</tr>
<tr>
<td>To Los Angeles</td>
<td>40</td>
<td>$9 - t$</td>
<td>$40(9 - t)$</td>
</tr>
</tbody>
</table>

Distance to Morro Bay = Distance to Los Angeles

$50t = 40(9 - t)$
$50t = 40(9 - t)$
$50t = 360 - 40t$
$50t + 40t = 360 - 40t + 40t$
$90t = 360$
$t = 4$

Because the trip to Morro Bay took 4 hours, the distance to Morro Bay is $50t = 50 \cdot 4 = 200$ miles.

Step 5. Check √ is left for the reader.

Step 6. The distance to Morro Bay from Los Angeles is 200 miles.

WorkOut 2: A pilot takes a trip to visit relatives and travels there at a speed of 100 miles per hour. On the way home, the pilot travels at 80 miles per hour. If the round trip took nine hours, how far did the pilot travel to visit her relatives?

Your Solution:
Example 3: A problem about uniform motion in the same direction

Exactly one hour after the Carascos head north on the freeway, the Hendersons set out from the same point to catch up with them. The Carascos travel 50 miles per hour while the Hendersons travel at 60 miles per hour. How long does it take the Hendersons to catch up with the Carascos?

Solution: In a problem where both vehicles are going in the same direction and one vehicle catches up with another, the distance the first vehicle travels is equal to the distance the second vehicle travels. We assign \( t \) to be the time the Hendersons are traveling and \( t + 1 \) to be the time the Carascos are traveling because they start one hour before the Hendersons and thus travel one hour longer than the Hendersons.

Step 1. Let \( t \) = the time the Hendersons are traveling

Step 2. Then \( t + 1 \) = the time the Carascos are traveling

Step 3.

*Figure 4.11 Cars traveling in the same direction*

<table>
<thead>
<tr>
<th>Trip</th>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carascos</td>
<td>50</td>
<td>( t + 1 )</td>
<td>50(( t + 1 ))</td>
</tr>
<tr>
<td>Hendersons</td>
<td>60</td>
<td>( t )</td>
<td>60( t )</td>
</tr>
</tbody>
</table>

\[
\text{Carascos Trip} = \text{Hendersons Trip} \\
50 \ (t + 1) = 60t
\]

Step 4.

\[
50 \ (t + 1) = 60t \\
50t + 50 = 60t \\
50t + 50 - 50t = 60t - 50t \\
50 = 10t \\
5 = t
\]

Step 5. Check ✓ is left for the reader.

Step 6. It took the Hendersons 5 hours to catch up with the Carascos.
Workout 3: Darlene and Lessa are long distance runners. If Darlene can run at a speed of 7.25 miles per hour and Lessa can run at a speed of 5.5 miles per hour and Lessa gets a $\frac{1}{2}$ hour head start, how long will it take Darlene to catch up with Lessa?

Your Solution:

Solutions to the WorkOuts

WorkOut 1. rate of first train = 55 miles per hour, rate of second train = 65 miles per hour

WorkOut 2. 400 miles

WorkOut 3. $1\frac{4}{7}$ hours or $\approx 1.57$ hours